Journal of Southeast University (English Edition) June 2001 Vol.17 No.1 ISSN 1003—7985

# A Single Neuron with Stochastic Resonance for Noisy Square Pulse Train Signal Transmission*

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**Abstract:** A nonlinear single neuron is demonstrated to exhibit stochastic resonance by theoretical analysis and numerical simulations. This single neuron is used for noisy periodic signal transmission, and significant performance of raising input-output SNR gain can be achieved. The research of this paper not only gives a very simple model of neuron with stochastic resonance, but also enlarges the application scope of neuron to the transmission of periodic signals.

**Key words:** neuron, nonlinear system, signal transmission, SNR gain

Stochastic resonance is a nonlinear phenomenon, which can be defined as an enhancement of the transmission of a coherent signal by certain nonlinear systems. This nonlinear phenomenon is obtained by means of noise addition to the system. This paradoxical effect was introduced about 15 years ago in the context of climate dynamics[1]. At present, it has gradually been reported in various systems, including lasers, electronic devices, neurons[2-4]. However, the existing neurons with stochastic resonance often have complicated nonlinear dynamics. In this paper, a very common single neuron described by a simpler nonlinear model than that in Ref.[4] is shown to exhibit stochastic resonance phenomenon and used for the square pulse train signal transmission.

## 1 A Single Neuron with Stochastic Resonance

Consider the single neuron in Fig.1, which is described by a nonlinear model as

$$y(t) = g[u] = \begin{cases} 0 & \text{for } u \leqslant \theta \\ A_y & \text{for } u > \theta \end{cases} \tag{1}$$

where $\theta$ is a hard threshold with $A_y > 0$. Fig.1 displays the input and output characteristics of this nonlinear neuron. The input voltage $u$ consists of the sum $s(t) + \eta(t)$, where $s(t)$ is a coherent periodic signal with the period $T_s$, $\eta(t)$ is a stationary white noise. These two signals from the input to this neuron produce the output

$$y(t) = g[s(t) + \eta(t)] \tag{2}$$

According to the theory of Ref.[5], the coherent part in the output signal $y(t)$ shows up in the output power spectral density as spectral lines at integer

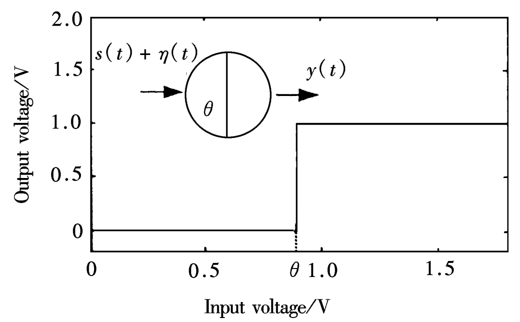


**Fig.1** Nonlinear signal neuron and its input-output characteristics

multiples of the coherent frequency $1/T_s$. The power contained in the coherent spectral line at frequency $n/T_s$ is given by $|\bar{Y}_n|^2$, where $\bar{Y}_n$ is the order $n$ Fourier coefficient of $T_s$ periodic non-stationary output mean $E[y(t)]$.

$$\bar{Y}_n = \frac{1}{T}\int_0^{T_s} E[y(t)]\exp\left(-in\frac{2\pi}{T_s}t\right)dt \tag{3}$$

For a nonlinear system $g(u)$, the mean $E[y(t)]$ and the variance $\mathrm{var}[y(t)]$ at a fixed time $t$ are computed as

$$E[y(t)] = \int_{-\infty}^{+\infty} g(u)f_\eta[u - s(t)]du \tag{4}$$

$$\mathrm{var}[y(t)] = \int_{-\infty}^{+\infty} g^2(u)f_\eta[u - s(t)]du - \left(\int_{-\infty}^{+\infty} g(u)f_\eta[u - s(t)]du\right)^2 \tag{5}$$

The coherent part at frequency $n/T_s$ in the noisy input $s(t) + \eta(t)$ is measured by the spectral line at frequency $n/T_s$ in the input power spectral density, which contains the coherent power $|S_n|^2$ with the order

Received 2000-09-08.

* The project supported by the National Nature Science Foundation of China (69875004) and Foundation for Key Teacher by the Ministry of Education.

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$n$ Fourier coefficient of $s(t)$.

$$S_n = \frac{1}{T_s}\int_0^{T_s} s(t)\exp\left(-in\frac{2\pi}{T_s}\right)\mathrm{d}t \tag{6}$$

The ration of the amplitudes of the output and input coherent spectral lines at frequency $n/T_s$, which defines the input-output gain $G_{\mathrm{sig}}$ for the coherent component at frequency $n/T_s$, follows as

$$G_{\mathrm{sig}}\left(\frac{n}{T_s}\right) = \frac{|\bar{Y}_n|}{|S_n|} \tag{7}$$

The incoherent statistical fluctuations in the input $s(t) + \eta(t)$, which control the continuous noise background in the input power spectral density, are measured by the variance $\sigma_\eta^2$ of the input white noise $\eta(t)$. We defines the input-output gain $G_{\mathrm{noi}}$ for the amplitude of the noise fluctuations as

$$G_{\mathrm{noi}}\left(\frac{n}{T_s}\right) = \frac{\sqrt{\overline{\mathrm{var}(y)}}}{\sigma_\eta} \tag{8}$$

The ration of output and input SNRs, which defines the input-output SNR gain $G_{\mathrm{SNR}}$ as

$$G_{\mathrm{SNR}}\left(\frac{n}{T_s}\right) = \frac{G_{\mathrm{sig}}^2(n/T_s)}{G_{\mathrm{noi}}^2(n/T_s)} = \frac{|\bar{Y}_n|^2/\overline{\mathrm{var}(y)}}{|S_n|^2/\sigma_\eta^2} \tag{9}$$

**Lemma** 1[5] If the input-output SNR gain $G_{\mathrm{SNR}} > 1$ when noisy signal pass through a nonlinear system, then the stochastic resonance phenomenon exists in this nonlinear system.

## 2 Noisy Square Pulse Signal Transmission by Using the Single Neuron

Since the square pulse signal is an important element of digital signals, we choose a train of square pulse with amplitude $A_s > 0$ and duration $T$ for $T_s$ periodic signal $s(t)$:

$$s(t) = \begin{cases} A_s & \text{for } t \in [0, T] \\ 0 & \text{for } t \in [T, T_s] \end{cases} \tag{10}$$

Substituting Eq.(10) into Eqs. (2) - (9), lead to the input-output SNR gain $G_{\mathrm{SNR}}$ as

$$G_{\mathrm{SNR}}\left(\frac{n}{T_s}\right) = \frac{\sigma_\eta^2}{A_s^2}[F_\eta(\theta) - F_\eta(\theta - A_s)]^2 \times \left[\frac{T}{T_s}F_\eta(\theta - A_s)[1 - F_\eta(\theta - A_s)] + \left(1 - \frac{T}{T_s}\right)F_\eta(\theta)[1 - F_\eta(\theta)]\right]^{-1} \tag{11}$$

where the input noise $\eta(t)$ is zero-mean Gaussian noise with the distribution function.

$$F_\eta(u) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{u}{\sqrt{2}\sigma_\eta}\right)\right] \tag{12}$$

where the error function

$$\mathrm{erf}(u) = \frac{2}{\sqrt{\pi}}\int_0^u \exp(-u'^2)\mathrm{d}u' \tag{13}$$

## 3 Numerical Simulation Experiment Result

Fig.2 and Fig.3 present the numerical simulation results of the nonlinear signal neuron when a noisy square pulse train $s(t) + \eta(t)$ input to it, which reveal the non-monotonic evolutions of the input-output SNR gain $G_{\mathrm{SNR}}$ with root-mean-square(rms) amplitude $\sigma_\eta$ of the input zero-mean Gaussian noise $\eta(t)$ and with hard threshold $\theta$ of the nonlinear single neuron. Fig.2 clearly shows a range where the SNR gain $G_{\mathrm{SNR}}$ increase as the input noise level $\sigma_\eta$ increase, up to an optimal noise level where $G_{\mathrm{SNR}}$ is maximized. Fig.2 also shows the $G_{\mathrm{SNR}}$ gets higher when the filling factor $T/T_s$ of $s(t)$ gets small. Fig.3 shows the $G_{\mathrm{SNR}}$ gets a maximum for an optimal hard threshold $\theta$, and shows the peak height is a function of the input noise level $\sigma_\eta$. It is remarkable in Fig.2 and Fig.3 that there exists a wide range of SNR gain $G_{\mathrm{SNR}} > 1$. Therefore, according to lemma 1, the stochastic resonance phenomenon exits in this nonlinear single neuron.

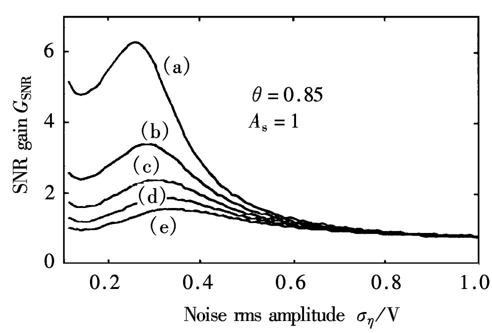


**Fig**.2 SNR gain $G_{\mathrm{SNR}}$ against rms amplitude $\sigma_\eta$ of the input noise $\eta(t)$. (a) $T = 0.025T_s$; (b) $T = 0.05T_s$; (c) $T = 0.075T_s$; (d) $T = 0.1T_s$; (e) $T = 0.125T_s$

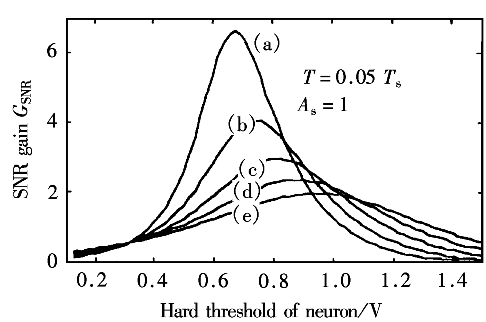


**Fig**.3 SNR gain $G_{\mathrm{SNR}}$ against hard threshold $\theta$ of the nonlinear signal neuron. (a) $\sigma_\eta = 0.25$; (b) $\sigma_\eta = 0.3$; (c) $\sigma_\eta = 0.35$; (d) $\sigma_\eta = 0.4$; (e) $\sigma_\eta = 0.45$

Fig.4 represents the square pulse train $s(t)$, input noisy signal $s(t) + \eta(t)$, and the output signal $y(t)$. Fig.4 shows well performance of noise reducing

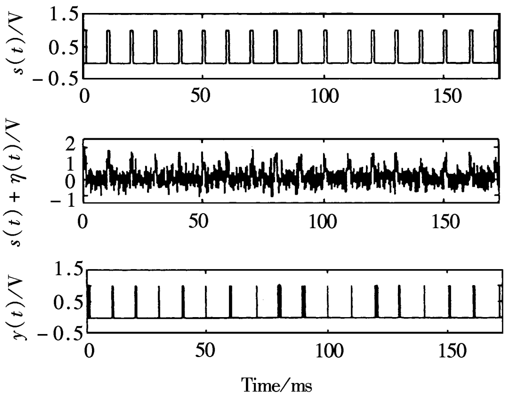


**Fig.**4 Input square pulse train $s(t)$, noisy signal $s(t) + \eta(t)$, and output signal $y(t)$. $\sigma_\eta = 0.38$, $T = 0.1T_s$, $A_s = 1$, $\theta = 1.1$

when this signal neuron with stochastic resonance is used for periodic signal transmission.

## 4 Conclusion

A single neuron with simple nonlinear model is demonstrated to exhibit stochastic resonance phenomenon. When it is used for noisy square pulse signal transmission, the noise can be greatly reduced. This neuron can serve as a simple and useful model for future investigation of the nonlinear effect of stochastic resonance.

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# 具有随机共振的单神经元用于含噪声方波脉冲信号传输

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**摘　要**　利用理论分析和数值仿真的方法证明了一种简单的非线性神经元存在随机共振现象，并将该神经元用于含噪声方波脉冲信号的传输，结果表明该方法可以有效提高信号传输系统的输入输出比增益，从而大大地抑制了信号中的噪声．本文的研究不仅给出了一种具有随机共振现象的简单神经元模型，而且将神经元的应用推广到周期性脉冲信号的传输领域．

**关键词**　神经元，非线性系统，信号传输，信噪比增益

**中图分类号**　TN911.72