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Optimal Maintenance Policy for a Storage System with Finite Number of Inspections

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Abstract: The expected cost per unit of time for a sequential inspection policy is derived. It still has some difficulties to compute an optimal sequential policy numerically, which minimizes the expected cost of a system with finite number of inspections. This paper gives the algorithm for an optimal inspection schedule and specifies the computing procedure for a Weibull distribution. Using this algorithm, optimal inspection times are computed as a numerical result. Compared with the periodic point inspection, the policies in this paper reduce the cost successfully.

Key words: storage system, sequential inspection, optimal time sequence

Missiles and spare parts of aircraft are stored for a long time after delivery until the usage and have to hold high mission reliability at their operations. Moreover, the reliability of a storage system goes down with time and it is impossible to judge whether the storage system is normal or not. So, inspection and maintenance of the storage system at suitable time are necessary to maintain the system in high reliability. However, the system cannot be inspected so frequently because each inspection costs and causes the degradation of system. Therefore, an optimal inspection policy of the storage system should be considered carefully.

Barlow and Proschan[1] summarized the optimal inspection policy, which minimizes the total expected cost until detection of failure. Luss and Kander[2] and Zacks and Fenske[3] extended it to many intricate systems. Martinez discussed the periodic inspection for storage electronic equipment and denoted how to calculate its reliability just before and after the inspection. Wattanapanom and Shaw[4] considered the optimal policy of system where inspections may hasten failures.

Missiles are composed of various kinds of mechanical, electric and electronic parts, and some parts have a short lifetime because they have to generate high power in a very short operating time. Such parts should be exchanged after reaching cumulative operating times of inspections, i.e. a prespecified time of quality warranty.

In this paper, we consider the inspection for a storage system with two kinds of units where unit 1 is checked and maintained at time $x_j (j = 1,2,\cdots,N)$ and unit 2 is continuously degraded with time. The system is replaced at detection of failure or at the N-th inspection. Under the above assumptions, we derive an expected cost per unit of time. Next, using Barlow's algorithm, we can compute an optimal inspection schedule $\{x_j^*\}$ in Weibull distribution case, and compare it with that of periodic times.

1 Expected Cost

Considering an inspection policy for the storage system with unit 1 and unit 2. The system is inspected and maintained at time $x_j (j = 0,1,\cdots)$ and $x_0 = 0$. Any failure is detected at the next inspection time and the system is replaced immediately. Supposing that a prespecified inspection number of warranty is N, i.e., the system is replaced at inspection time x_N. The time of any inspection and replacement is negligible.

The system is roughly divided into two kinds of units. Unit 1 is a new one after every inspection; however, unit 2 remains unchanged through any inspections. Unit i $(i = 1,2)$ has a hazard rate function $h_i(t)$, which is the function of instruments spoiling probability at time t. Then, the hazard rate function $h(t)$ of the system is

$$h(t) = h_1(t - x_{j-1}) + h_2(t) \tag{1}$$

where $x_{j-1} < t \leqslant x_j (j = 1,2,\cdots,N)$ and $x_0 = 0$. Thus, the cumulative hazard rate function $H(t)$ is

$$H(t) = \int_0^t h(u)\mathrm{d}u = \sum_{k=1}^{j-1} H_1(x_k - x_{k-1})$$

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$$+ H_1(t - x_{j-1}) + H_2(t) \tag{2}$$

where $H_i(t) \equiv \int_0^t h_i(u)\mathrm{d}u\ (i = 1,2)$, and hence, the reliability $\bar{F}(t)$ of the system is given by $\bar{F}(t) = \exp[-H(t)]$ at time t.

Now, introducing the following costs: cost c_1 is required for one inspection; cost c_2 is for time elapsed failure; and cost c_3 is required for the replacement. Then, when a failure is detected and the system is replaced at $x_j (j = 1,2,\cdots,N)$, the expected cost is

$$\sum_{j=1}^{N}\int_{x_{j-1}}^{x_j}[jc_1 + (x_j - t)c_2 + c_3]\mathrm{d}F(t) \tag{3}$$

where $F \equiv 1 - \bar{F}$. Further, when the system is replaced without failure at time x_N, the expected cost is

$$\bar{F}(x_N)(Nc_1 + c_3) \tag{4}$$

Thus, from (3) and (4), the expected total cost function is

$$\begin{aligned}\sum_{j=1}^{N}\int_{x_{j-1}}^{x_j}&[jc_1 + (x_j - t)c_2 + c_3]\mathrm{d}F(t)\\ &+ \bar{F}(x_N)(Nc_1 + c_3)\\ =& \sum_{j=0}^{N-1}[c_1 + c_2(x_{j+1} - x_j)]\bar{F}(x_j)\\ &- c_2\int_0^{x_N}\bar{F}(t)\mathrm{d}t + c_3\end{aligned} \tag{5}$$

Next, the mean time that the system fails and will be replaced is

$$\sum_{j=1}^{N}\int_{x_{j-1}}^{x_j} x_j\mathrm{d}F(t) \tag{6}$$

The mean time to replacement without failure at time x_N is

$$\bar{F}(x_N)x_N \tag{7}$$

Thus, the mean time to system replacement is

$$\sum_{j=1}^{N}\int_{x_{j-1}}^{x_j} x_j\mathrm{d}F(t) + \bar{F}(x_N) = \sum_{j=0}^{N-1}(x_{j+1} - x_j)\bar{F}(x_j) \tag{8}$$

Therefore, from (5) and (8), the expected total cost per unit of time is

$$C(X) = \frac{\sum_{j=0}^{N-1}[c_1 + c_2(x_{j+1} - x_j)]\bar{F}(x_j) - c_2\int_0^{x_N}\bar{F}(t)\mathrm{d}t + c_3}{\sum_{j=0}^{N-1}(x_{j+1} - x_j)\bar{F}(x_j)} \tag{9}$$

where $X \equiv (x_1, x_2, \cdots, x_N)$.

2 Sequential Inspection Policy

Suppose that the system is inspected at optimal time $x_1 < x_2 < x_3 < \cdots < x_N$. Then, we introduce $D(\alpha, X)$ which is given by

$$D(\alpha, X) \equiv E_c(X) - \alpha E_t(X) \tag{10}$$

where

$$E_c(X) \equiv \sum_{j=0}^{N-1}[c_1 + c_2(x_{j+1} - x_j)]\bar{F}(x_j) - c_2\int_0^{x_N}\bar{F}(t)\mathrm{d}t + c_3 \tag{11}$$

$$E_t(X) \equiv \sum_{j=0}^{N-1}(x_{j+1} - x_j)\bar{F}(x_j) \tag{12}$$

We evidently have

$$D(0, X) = E_c(X) > 0 \tag{13}$$

$$D(c_2, X) = c_1\sum_{j=0}^{N-1}\bar{F}(x_j) - c_2\int_0^{x_N}\bar{F}(t)\mathrm{d}t + c_3 \tag{14}$$

There is optimal X^* that minimizes $C(X)$ when $D(c_2, X) \leqslant 0$ (See Ref.[1]), i.e.

$$c_1\sum_{j=0}^{N-1}\bar{F}(x_j) + c_3 < c_2\int_0^{x_N}\bar{F}(t)\mathrm{d}t \tag{15}$$

From (11) and (12), (10) is

$$D(\alpha, X) = \sum_{j=0}^{N-1}[c_1 + (c_2 - \alpha)(x_{j+1} - x_j)]\bar{F}(x_j) - c_2\int_0^{x_N}\bar{F}(t)\mathrm{d}t + c_3 \tag{16}$$

Putting $\partial D/\partial x_j = 0$, we have

$$\delta_j = \frac{F(x_j) - F(x_{j-1})}{f(x_j)} - \frac{c_1}{c_2 - \alpha} \tag{17}$$

where $\delta_j \equiv x_{j+1} - x_j$. Once x_1 is selected, $x_2, x_3, \cdots$ can be generated recursively from (17). Moreover, the inspection schedule X should satisfy the following stipulation.

$$\delta_j \leqslant \delta_{j-1} \qquad j = 1,2,\cdots,N \tag{18}$$

$$\delta_j \geqslant 0 \qquad j = 1,2,\cdots,N \tag{19}$$

In summary, to get an optimal in section policy, we may solve the following optimization problem.

$$\min|D(\alpha, X)| \tag{20}$$

s.t. Formulae (18) and (19).

3 Solution of the Model

On the basis of the model (20), we can apply mathematical methods to gain the optimal policy of maintenance. Before the actual utilization, we should conduct a concrete numerical calculation according to the economical, technical and systematic state at that time. If the method of numerical calculation to find the solution of the model (20) is used, it will involve the following difficulties.

1) Discrete decision variables and continuous decision variables will mix.

2) The boundary of the feasible region of the

feasible sets is irregular. Most of the boundaries have not definite analytic equation; or one part of them has constraint, the other has not. And, the optimal solution always doesn't existence in the boundary.

3) The equations of the mathematical model are tangling. They are commonly comprised in the generalized sum, multiply in succession or integral equation. This increases the errors in numerical calculation.

4) Because of the diversity of the decision variable, there are several local minimums in the geometrical toroidal of the objective function.

To solve the constrained optimization problem (20), there are many methods, such as transformation method, projection method, successive quadratic programming and direct search method, etc. These methods make requisition for the fluxionary probability and continuous the connectedness of the objective function or the constraint function and the properties of the convex and concave. But, it is very difficult to store whether these conditions are contended in this problem or not. In the procedures of the actual numerical calculation, the authors adopted the algorithm as follows to get the results successfully.

Given $\bar{\alpha}, x_1$ while x_1^0 in the point, we can prosecute the exploratory move. If $\exists k \in Z^+$ make $\delta_k < \delta_{k+1}$, then reduce x_1, and if $\delta_k < 0$, then increase x_1. If $(\bar{\alpha}, X_{\bar{\alpha}})$ is the feasible and minimizes $|D(\bar{\alpha}, X_{\bar{\alpha}})|$ in the line $\alpha = \bar{\alpha}$, we call the case as success while the other cases are called failure. The proceeding of the question's resolution is as follows.

Step 1 $\varepsilon > 0, i = 0, h = c_2/10$.

Step 2 Set $\alpha_i = c_2/2 + (-1)^i ih$, prosecute the exploratory move. If it is success, then set $\alpha_A = \alpha_i$ and $k = 0$, go to step 4.

Step 3 If $i < \mathrm{INT}(c_2/2h)$, then set $i = i + 1$, go to step 2. Otherwise, set $h = h/2$ and $i = 0$, go to step 2.

Step 4 Set $\alpha_k = \alpha_A + h/2^k$, prosecute the exploratory move.

Step 5 If it is success and $|D(\alpha_k, X_{\alpha_k})| < |D(\alpha_A, X_{\alpha_A})|$, then set $\alpha_A = \alpha_k$ and $k = 0$, go to step 4. If $h/2^k \geqslant \varepsilon$ then set $k = k + 1$, go to step 4. Otherwise, $\alpha_B = \alpha_A$ and $j = 0$.

Step 6 Set $\alpha_j = \alpha_B + h/2^j$, do exploratory move.

Step 7 If it is success and $|D(\alpha_j, X_{\alpha_j})| < |D(\alpha_B, X_{\alpha_B})|$, then set $\alpha_B = \alpha_j$ and $j = 0$, go to step 6. If $h/2^j \geqslant \varepsilon$ then set $j = j + 1$, go to step 6. If $\alpha_A = \alpha_B$, then stop. Otherwise, set $\alpha_A = \alpha_B$ and $k = 0$, go to step 4.

4 Numerical Illustrations

As an example, if failure time obeys the Weibull distribution with the parameter λ, we can calculate the optimal non-periodic inspecting time alignment $X^* = (x_1^*, x_2^*, \cdots, x_N^*)$ for the storage system with finite inspection. The assigned parameters and calculation results of the example are listed in Tab.1. From model(20), the dimensions of C_1, C_2 and C_3 don't affect the optimization of X^*. As a result, we only take into account the relative value of C_1, C_2 and C_3 in computing. As shown in Tab.1, the cost of storage increases with m, while the alignment interval of sequential inspection δ_i has no clear regularly.

As a reference, the optimal inspection time alignment for the storage system with periodicity was also calculated under the same condition and the results are given in Tab.2. By comparison, the economic target for the non-periodic storage system is better than the periodic one, which is consistent with our expectation. Nevertheless, the non-periodic sequential inspection is not so operative in its practical use, In order to solve the application problem in enterprise, how to apply the segmental quasi-period strategy when N is relatively large will be our future research project.

Tab.1 Non-periodicity optimal sequential inspection times X^*

m		1.1		1.2		1.3	
α^*		0.394 7		0.481 8		0.655 4	
$C(X^*)$		0.394 7		0.481 8		0.655 4	
X^*		x_j^*	δ_j	x_j^*	δ_j	x_j^*	δ_j
	0	0.00	147.23	0.00	178.03	0.00	161.45
	1	147.23	134.95	178.03	173.38	161.45	155.13
	2	282.18	121.97	351.41	171.03	316.58	151.57
	3	404.15	107.44	522.43	169.11	468.16	148.38
	4	511.60	91.36	691.54	167.27	616.54	144.79
j	5	602.96	73.96	858.81	165.28	761.32	140.04
	6	676.92	55.66	1 024.09	162.96	901.37	133.07
	7	732.58	36.97	1 187.05	160.07	1 034.44	122.36
	8	769.55	18.48	1 347.12	156.30	1 156.80	106.06
	9	788.03	0.73	1 503.42	151.29	1 262.86	82.68
	10	788.76	—	1 654.71	—	1 345.53	—

Note: $N = 10, a = 0.9, c_1 = 10, c_2 = 1, c_3 = 100$ and $\lambda = 1.0 \times 10^{-3}$.

Tab.2 Periodicity optimal sequential inspection times X^*

	m	1.1		1.2		1.3	
	α^*	0.443 8		0.621 8		0.656 2	
	$C(X^*)$	0.443 8		0.621 8		0.656 3	
	X^*	x_j^*	δ_j	x_j^*	δ_j	x_j^*	δ_j
	0	0.00	62.15	0.00	42.65	0.00	50.28
	1	62.15	62.15	42.65	42.66	50.28	50.28
	2	124.30	62.16	85.31	42.65	100.56	50.28
	3	186.46	62.15	127.96	42.65	150.84	50.28
	4	248.61	62.15	170.61	42.66	210.12	50.28
j	5	310.76	62.16	213.27	42.66	251.40	50.28
	6	372.92	62.15	255.92	42.65	301.68	50.28
	7	435.07	62.15	298.57	42.65	351.96	50.28
	8	497.22	62.16	341.22	42.66	402.24	50.28
	9	559.38	62.15	383.88	42.65	452.52	50.27
	10	621.53	—	426.53	—	502.79	—

Note: $N = 10, a = 0.9, c_1 = 10, c_2 = 1, c_3 = 100$ and $\lambda = 1.0 \times 10^{-3}$.

5 Conclusion

The expected cost per unit of time for a sequential inspection policy is derived. It has some difficulties to compute an optimal sequential policy numerically that minimizes the expected cost of the system with finite number of inspections. This paper gives the algorithm for an optimal inspection schedule and specifies the computing procedure for a Weibull distribution. With this, optimal inspection times are computed as a numerical example, and some useful discussions about these results are made. The mathematical model and algorithm in the paper will find application in the reliable maintenance for the storage of aircraft missiles. At the same time, the method included in this paper can also be used in other ways.

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具有有限检查次数的库存系统最优点检策略

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摘 要 首先导出了点检时间序列相对应的期望成本.由于使具有有限点检次数库存系统期望成本降低到最小的最优点检时间序列的数值计算比较困难,本文给出了计算一般最优点检时间序列的算法和 Weibull 分布情况下的计算步骤实例.据此计算出最优点检策略的数值结果.与周期点检策略相比,本文策略有效地降低了成本.

关键词 库存系统,点检,最优时间序列

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