

Sliding Mode Controller Design for a Class of Nonlinear System

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Abstract: A sliding mode control methodology is presented for nonlinear systems represented by input-output models, which does not depend on the state variables. There are two parts in the controller design, one is the sliding controller design and the other is the design of linear feedback system. Simulation results demonstrate the validity of the control scheme.

Key words: nonlinear system, sliding mode control, adaptive control

The sliding mode control approach is based on switching functions of the state variables which are used to create a “sliding surface”. When this surface is attained, the switching functions keep the trajectory on the surface, thus yielding desired system dynamics^[1]. The sliding mode approach, which can obtain accurate tracking, good transient behavior, and insensitivity to parameter variations or disturbances, has found wide application in real control problems^[2]. However there are some drawbacks in the original SMC, such as the chattering in the control signal and the needing knowledge of the bounds of the disturbances and uncertainties in the controller design. One difficulty in applying the sliding mode control is the need for the knowledge of the full state vector. Ref. [3] presents a well-known control scheme based on only input and output measurements. It needs to augment two “state” filters in the course of the controller design. Since the introduction of this idea, it has been used in a number of studies^[4–7]. Ref. [8] considers a class of single-input-single-output nonlinear systems with unknown parameters and gives a semiglobal adaptive output controller. However the results require persistence of excitation and other restrictions.

In this paper, we consider a class of single-input-single-output nonlinear systems with unknown constant parameters and bounded disturbances. The “state” of the system can be expressed as a function of the input, the output, and their derivatives up to a certain order. Extending the dynamics of the system by augmenting a series of integrators at the input side makes the derivatives of the input available for feedback. There are two parts in the design of the adaptive sliding mode feedback controller. One is the SMC which uses the

output and its derivatives up to a certain order. Another is to control the integrators augmented in the input side. Since the integrators are linear, so the objective is to design a linear feedback controller (LFC). The adaptive sliding mode feedback controller can obtain better tracking results and has more robustness than other adaptive controllers, and does not need the condition of persistence of excitation.

1 Problem Statement

Consider a single-input-single-output system represented globally by the n -th-order differential equation

$$y^{(n)} = f_0(\cdot) + \sum_{i=1}^p f_i(\cdot)\theta_i + (g_0 + \sum_{i=1}^p g_i\theta_i)u^{(m)} + d(\cdot) \quad (1)$$

where u is the control input; y is the measured output; $y^{(i)}$ and $u^{(i)}$ denote the i -th derivative of y and u respectively, and $m < n$. The function f_i ($i = 0, 1, \dots, p$) is known bounded smooth nonlinearities, which can depend on $y, y^{(1)}, \dots, y^{(n-1)}, u, u^{(1)}, \dots, u^{(m-1)}$, e.g. $f_i(\cdot) = f_i(y, y^{(1)}, \dots, y^{(n-1)}, u, u^{(1)}, \dots, u^{(m-1)})$. The constant parameters g_0 to g_p are known, while the constant parameters θ_1 to θ_p are unknown, but the vector $\theta = [\theta_1, \dots, \theta_p]^T$ belongs to Ω , a known compact convex subset of R^p . $d(\cdot)$ denotes the disturbances.

Define $y_r(t)$ the given reference signal, $Y(t) \triangleq [y(t), y^{(1)}(t), \dots, y^{(n-1)}(t)]^T$ and $Y_r(t) \triangleq [y_r(t), y_r^{(1)}(t), \dots, y_r^{(n-1)}(t)]^T$. The objective of this paper is to design an adaptive output feedback controller that all variables of the closed-loop system

are bounded for all $t \geq 0$ and $\lim_{t \rightarrow \infty} |y(t) - y_r(t)| = 0$.

The following assumptions are made on system (1).

Assumption 1 $\exists k > 0$, let $|g_0 + \sum_{i=1}^p g_i \theta_i| \geq$

$k > 0$. Set $l \triangleq g_0 + \sum_{i=1}^p g_i \theta_i$. Without loss of generation, in this paper we assume $l > k > 0$.

Assumption 2 $|d(\cdot)| \leq D(\cdot)$, where $D(\cdot)$ is a positive function.

Assumption 3 $y_r(t)$ is bounded and has bounded derivatives up to n -th order.

It has been proven theoretically that the feedforward neural network which has single hidden layer can approximate any complicated nonlinear function in arbitrary accuracy^[9]. So we can use a feedforward neural network to approximate the unknown parameter vector $\theta' = [\theta^T, l]^T$. Define $\hat{\theta}', \hat{\theta}, \hat{l}$ are the estimation values of θ', θ, l , respectively.

Assumption 4 For any given arbitrary small approximate error ϵ , there exists a neural network $\hat{\theta}'(p^*)$ which contains the optimum parameter p^* to satisfy $\max \|\hat{\theta}'(p^*) - \theta'\| \leq \epsilon$. Assume $\tilde{\theta}'$ is the approximation error of θ' , that is $\tilde{\theta}' = \hat{\theta}' - \theta'$, then $\max \|\tilde{\theta}'\| \leq \epsilon$. Assumptions 1 and 4 imply that $\hat{l} > 0$.

Now we augment a series of m integrators at the input side of the system and denote the state of these integrators by $z_1 = u, z_2 = u^{(1)}, \dots, z_m = u^{(m-1)}$, and set $v = u^{(m)}$ as the control input of the augmented system. By taking $x_1 = y, x_2 = y^{(1)}, \dots, x_n = y^{(n-1)}$, the extended system can be represented as

$$\left. \begin{aligned} \dot{x}_i &= x_{i+1} & 1 \leq i \leq n-1 \\ \dot{x}_n &= f_0(X, Z) + \theta^T f(X, Z) + lv + d(\cdot) \\ \dot{z}_i &= z_{i+1} & 1 \leq i \leq m-1 \\ \dot{z}_m &= v \\ y &= x_1 \end{aligned} \right\} \quad (2)$$

where $X = [x_1, \dots, x_n]^T, Z = [z_1, \dots, z_m]^T, f = [f_1, \dots, f_p]^T$. Take

$$\left. \begin{aligned} e_1 &= y - y_r = x_1 - y_r \\ e_2 &= \dot{y} - \dot{y}_r = x_2 - \dot{y}_r \\ &\vdots \\ e_n &= y^{(n-1)} - y_r^{(n-1)} = x_n - y_r^{(n-1)} \\ e &= [e_1, e_2, \dots, e_n]^T \end{aligned} \right\} \quad (3)$$

then (2) can be rewritten as

$$\begin{aligned} \dot{e} &= Ae + b\{f_0(e + Y_r, Z) \\ &\quad + \theta^T f(e + Y_r, Z) + lv + d(\cdot) - y_r^{(n)}\} \end{aligned} \quad (4a)$$

$$\dot{Z} = A_2 Z + b_2 v \quad (4b)$$

where (A, b) and (A_2, b_2) are controllable canonical pairs of the same form

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{n \times n}, b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{n \times 1} \\ A_2 &= \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{m \times m}, b_2 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{m \times 1} \end{aligned} \quad (5)$$

2 Design of the Output Feedback Controller

In this section, the design of an adaptive output feedback controller, including the SMC and the LFC, is discussed. Based on (4), we can separate the design of the adaptive sliding mode output feedback controller into two parts: the sliding mode controller design and the linear feedback controller design. Stability analysis and simulation results are given to show that this controller can achieve accurate tracking, good transient behavior and insensitivity to disturbances if the bound of the disturbances is known.

2.1 Linear feedback controller design

From (4b) and (5), it can be seen that this linear system is controllable but unstable. So we choose matrix K as the state feedback, let

$$A_m = A_2 - b_2 K = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & & 1 \\ -k_1 & -k_2 & -k_3 & \cdots & -k_m \end{bmatrix} \quad (6)$$

where $k_i > 0$ and the polynomial $\lambda^m + k_1 \lambda^{(m+1)} + \dots + k_{m-1} \lambda$ is Hurwitz. In this paper, we also assume that $z(0)$ is bounded.

2.2 Sliding mode controller design

For nonlinear system (1), by the transforms of (2), (3) and (4a), define switching function $s(e) = \sum_{i=1}^{n-1} c_i e_i + e_n$, where the coefficients c_i are constants and satisfy Hurwitz polynomial $\lambda^{n-1} + c_{n-1} \lambda^{n-2} + \dots + c_2 \lambda + c_1$. The derivative of $s(e)$ is $\dot{s}(e) = \sum_{i=1}^{n-1} c_i e_{i+1}$

+ $\dot{e}_n = \sum_{i=1}^{n-1} c_i e_{i+1} + f_0(\mathbf{X}, \mathbf{Z}) + \boldsymbol{\theta}^T \mathbf{f}(\mathbf{X}, \mathbf{Z}) + \bar{l}\bar{v} - y_r^{(n)} + d(\cdot)$, where $\bar{v} = v - \mathbf{KZ}$. Then we take the control law as

$$v = \frac{1}{\hat{l}} \left(- \sum_{i=1}^{n-1} c_i e_{i+1} - f_0(\mathbf{X}, \mathbf{Z}) - \hat{\boldsymbol{\theta}}^T \mathbf{f}(\mathbf{X}, \mathbf{Z}) + y_r^{(n)} - \mathbf{KZ} - (D(\cdot) + \eta \text{sgn}(s)) \right) \quad (7)$$

where $\text{sgn}(s) = \begin{cases} 1 & s > 0 \\ 0 & s = 0 \\ -1 & s < 0 \end{cases}$, η is a positive

constant and satisfy the condition

$$\eta > \|\tilde{l}(\Phi + \boldsymbol{\theta}^T \mathbf{f})/l - \tilde{\boldsymbol{\theta}}^T \mathbf{f}\| \quad (8)$$

where $\Phi = \sum_{i=1}^{n-1} c_i e_{i+1} + f_0(\mathbf{X}, \mathbf{Z}) - y_r^{(n)} - \mathbf{KZ}$, $\tilde{l} = \hat{l} - l$, $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$. Since f is bounded and \mathbf{Z} is bounded too, and from the assumption 4, it is known that \tilde{l} and $\tilde{\boldsymbol{\theta}}$ can be taken arbitrary small, therefore the condition (8) is easy to be satisfied.

Thus, from the two steps 2.1 and 2.2 we complete the adaptive output feedback controller design. The block diagram of the closed-loop system is shown in Fig.1.

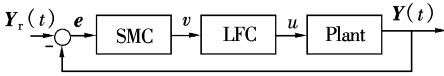


Fig.1 The structure of the SMC

2.3 Simulation results

Consider the nonlinear system

$$\begin{cases} \dot{\xi}_1 = \xi_2 + \theta_1 \xi_1^2 \\ \dot{\xi}_2 = u + \xi_3 \\ \dot{\xi}_3 = -\xi_3 + y + d(\cdot) \\ y = \xi_1 \end{cases}, \text{ where } \theta_1 \text{ is an unknown}$$

constant parameter. The system can be represented by the third-order differential equation $y^{(3)} = (u + y + \ddot{y}) + 2\theta_1(\gamma\dot{y} + \dot{y}^2 + y\ddot{y}) + d(\cdot) + \dot{u}$, which takes the form of (1) with $n = 3$, $m = 1$. We add an integrator at the input of the system, take $z = u$ as the state of the integrator and set $v = \dot{u}$ as the new control input of the augmented system. In the simulation, we choose $\theta_1 = 1$, $y(0) = 0.45$, $c_1 = 5$, $c_2 = 5$, $\eta = 1$, and $\mathbf{K} = [5, 5]$. Take $D(\cdot) = 5$ while $d(\cdot) = 5\sin(\gamma\dot{y})$. Simulation results are shown in Fig.2 and Fig.3.

Remark 1 For comparison, the example is taken from [8](example 3). But in [8] there is no disturbance term $d(\cdot)$ while the algorithm needs persistence of excitation, so it does not do the simulations where the given reference signal is step signal. In this paper it can be seen that the proposed

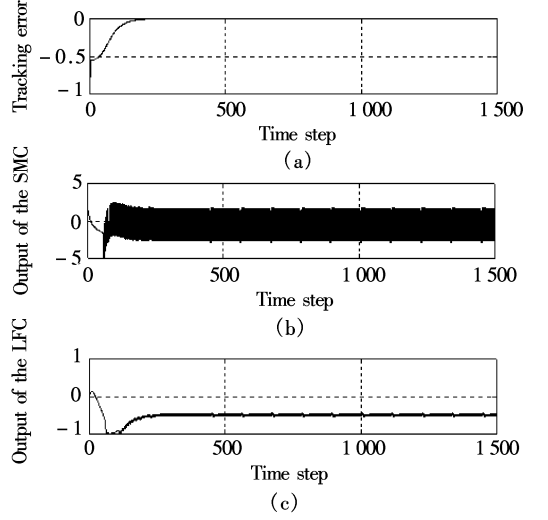


Fig.2 Simulation results with $y_r(t) = 1(t)$, $d(\cdot) = 5\sin(\gamma\dot{y})$

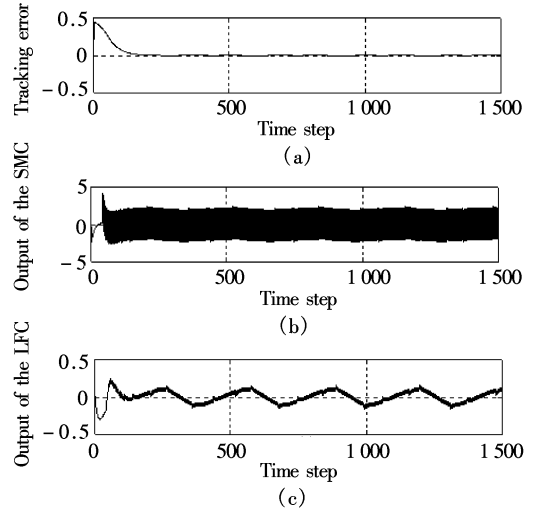


Fig.3 Simulation results with $y_r(t) = 0.1\sin(t)$, $d(\cdot) = 5\sin(\gamma\dot{y})$

controller can achieve accurate tracking and has more robustness and better transient behavior. Fig.2 shows the case that the given reference is step signal.

Remark 2 There exists chattering in the control signal. From the simulations, it can be seen that the chattering is diminished through LFC because the LFC acts as a low-pass filter.

Remark 3 In the design, the controller requires the knowledge of the output and its first $(n - 1)$ derivatives. In practice we can use the filters $1/F$ to obtain the derivatives^[10], where Hurwitz polynomial $F \triangleq h^n + a_1 h^{n-1} + \dots + a_{n-1} h + a_n$, $h \triangleq d/dt$.

3 Conclusion

In this paper, we present the adaptive output controller for nonlinear systems represented by

input-output models. The controller has more robustness and can obtain better tracking results than other controllers, especially it does not need the condition of persistence excitation. Simulation results on examples have shown the effectiveness of the control scheme.

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一类非线性系统的滑模控制器设计

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摘 要 对基于输入输出模型的一类非线性系统提出了滑模控制方法,解决了一般滑模控制设计需要依赖于系统状态的问题.通过对系统输入输出模型的分解,将控制器的设计分为两部分,一部分是由滑模控制器实现,另一部分是进行线性反馈控制器的设计.仿真结果表明了控制方法的有效性.

关键词 非线性系统,滑模控制,输出反馈控制

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