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Stabilization of Dynamic Systems for Multiple Omni-Directional Mobile Robots *

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Abstract: This paper deals with the stabilization of dynamic systems for two omni-directional mobile robots by using the inner product of two vectors, one is from a robot's position to another's, the other is from a robot's target point to another's. The multi-step control laws given can exponentially stabilize the dynamic system and make the distance between two robots be greater than or equal to the collision-free safe distance. The application of it to two omni-directional mobile robots is described. Simulation result shows that the proposed controller is effective.

Key words: omni-directional mobile robot, dynamics, coordination, collision avoidance, stabilization

In recent years, the research on cooperative control for multiple mobile robots is both extensive and diverse. Many theoretical issues proposed are full of challenge. These researches mainly centralize on the "high level motion planning", of which the designs of collision-free scheme is the most representative one. These control methods can be divided into two categories, namely, model-based[1,2], and sensor-based[3]. The optimal control problem of coordinating multiple mobile manipulators translating a grasped object is addressed in [4]. Based on distributed traffic regulation, a controller for multiple autonomous mobile robots operating in discrete space was presented in [5]. A cooperative hunting behavior by mobile-robot troops was considered in [6]. But there are few results in the stabilization of dynamic systems for multiple mobile robots.

It is well known for a single robot system, that the study of stabilizing problems is very important, and so is it for multiple robot systems. But the latter is much more difficult than the former. Main drawback for the later, is that the systems have to satisfy a group of algebra inequalities besides a group of differential equations. Considered in this paper, is just one of such problems, i.e., the stabilizing problem of dynamic systems for multiple mobile robots by using the concept of inner products which are concerned with the distribution relation between current positions of robots and aim positions of robots.

1 Problem Formulation

An industrial mobile robot is required to have some functions such that it can freely travel within a confused factory and achieve accurate positioning in a work station. To this end, an active investigation is now focused on the study of omni-directional mobile robots[7], instead of a conventional mobile robot with two independent driving wheels, or with front-wheel handing and rear-wheel driving. If the system models are exactly known, by using state and input transformations, those dynamic systems can be transformed into the form

$$\dot{x}_1 = u_{11}, \quad \dot{u}_{11} = \tau_1, \quad \dot{y}_1 = u_{12}, \quad \dot{u}_{12} = \tau_2$$

where (x_1, y_1) and (x_2, y_2) denote the mass center coordinates of robot $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, respectively. τ_i with $i = 1,2$ are new control inputs.

Consider the hybrid system composed of the above two omni-directional robots(called $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$) of the form

$$\begin{cases}\dot{x}_1 = u_{11}, \dot{u}_{11} = \tau_{11} \\ \dot{y}_1 = u_{12}, \dot{u}_{12} = \tau_{12}\end{cases}, \quad \begin{cases}\dot{x}_2 = u_{21}, \dot{u}_{21} = \tau_{21} \\ \dot{y}_2 = u_{22}, \dot{u}_{22} = \tau_{22}\end{cases} \tag{1}$$

Let $(x_1(0), y_1(0))$ and $(x_2(0), y_2(0))$ denote the initial position coordinates of $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, respectively, $(x_1(f), y_1(f))$ and $(x_2(f), y_2(f))$ denote the final position coordinates of $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, respectively. Let

$$\boldsymbol{R}_1(0) = (x_1(0), y_1(0))$$
$$\boldsymbol{R}_2(0) = (x_2(0), y_2(0))$$
$$\boldsymbol{R}_1(\mathrm{f}) = (x_1(f), y_1(f))$$

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$$\boldsymbol{R}_2(f) = (x_2(f), y_2(f))$$

Under no confusion, one will use (x_1, y_1), (x_2, y_2) or $\boldsymbol{R}_1(t), \boldsymbol{R}_2(t)$ to denote this position coordinates respectively.

For convenience, define the distance between two robots in the following form.

Definition 1 The distance between the two robots $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ is defined by

$$\rho_1 = |x_1 - x_2| + |y_1 - y_2| \tag{2}$$

Property 1 Setting

$$\rho_2 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \tag{3}$$

then $\rho_1 \geqslant \rho_2$.

Suppose d_0 is the smallest distance under the sense of ρ_1 between mass center coordinates (x_1, y_1) and (x_2, y_2) of two robots without collision (this is always called collision-free safe distance (CFSD)).

Elementary conditions, designing feedback control such that robot $\boldsymbol{R}_1$ moves from $\boldsymbol{R}_1(0)$ to $\boldsymbol{R}_1(f)$, and robot $\boldsymbol{R}_2$ from $\boldsymbol{R}_2(0)$ to $\boldsymbol{R}_2(f)$ without collision, is that

$$\rho_1(x_1(0), x_2(0)) \geqslant d_0 \tag{4}$$

$$\rho_1(x_1(f), x_2(f)) \geqslant d_0 \tag{5}$$

This kind of boundary conditions are called feasible boundary conditions(FBC).

Definition 2 System (1) is called stability to target points $\boldsymbol{R}_1(f)$ and $\boldsymbol{R}_2(f)$ satisfying (5) on the domain $D \subseteq \boldsymbol{R}^8$, if there exists a controller τ_{ij} with i, $j = 1,2$, such that, for arbitrary initial state $x(0)$ satisfying (4) in D, the solution of the system from this initial state has the properties with

(i) $x_i \to x_i(f), y_i \to y_i(f)(i = 1,2), u_{ij} \to 0$, $i, j = 1,2$, as $t \to \infty$;

(ii) $\rho((x_1(t), y_1(t)), (x_2(t), y_2(t))) \geqslant d_0$, $t \geqslant t_0$, in which d_0 is collision-free safe distance.

This type of the stabilization is local stabilization, and if $D \subseteq \boldsymbol{R}^8$, it is called global stabilization.

Main Problem For target states $\boldsymbol{R}_1(f)$ and $\boldsymbol{R}_2(f)$ satisfying (5), the controller τ_{ij} with $i = 1,2$; $j = 1,2$, is to be designed such that the closed system (1), for arbitrary initial state $\boldsymbol{R}_1(0)$ and $\boldsymbol{R}_2(0)$ satisfying (4), have the properties with

(i) $x_i(t) \to x_i(f), y_i(t) \to y_i(f), u_{ij}(t) \to 0$ $(i, j = 1,2)$, as $t \to \infty$;

(ii) $\rho((x_1(t), y_1(t)), (x_2(t), y_2(t))) \geqslant d_0$, $t \geqslant t_0$.

2 Controller Design

It is assumed that the vector from the target point to the initial point of robot $\boldsymbol{R}_1$, noted $\alpha_1 = (x_1(0) - x_1(f), y_1(0) - y_1(f))$, is parallel to the initial velocity vector $\beta_1(u_{11}(0), u_{12}(0))$. The same assumption is given for robot $\boldsymbol{R}_2$, and noted $\alpha_2(x_2(0) - x_2(f), y_2(0) - y_2(f))$ and $\beta_2 = (u_{21}(0), u_{22}(0))$. For example, robot at rest falls into this case. If collision is not considering, it is very easy to design state feedback controller such that the closed system is globally stabilized to the target points for the system (1). Let

$$\left.\begin{array}{l} z_1(t) = x_1(t) - x_1(f), z_2(t) = y_1(t) - y_1(f) \\ z_3(t) = x_2(t) - x_2(f), z_4(t) = y_2(t) - y_2(f) \end{array}\right\} \tag{6}$$

Lemma 1 Consider system (1) and choose $k_1 > 0, k_2 > 0$. If α_1 and β_1 is linearly dependent, the controller with the form

$$\tau_{11} = -k_1 z_1 - k_2 u_{11}, \quad \tau_{12} = -k_1 z_2 - k_2 u_{12}$$

$$\tau_{21} = -k_1 z_3 - k_2 u_{21}, \quad \tau_{22} = -k_1 z_4 - k_2 u_{22}$$

is such that robot $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ can be exponentially stabilized to target points $\boldsymbol{R}_1(x_1(f), y_1(f))$ and $\boldsymbol{R}_2(x_2(f), y_2(f))$ along straight line, respectively.

Proof Omitted (It is easy to show by knowledge of linear system).

It may be said that, the control scheme, driving two robots both moving to their targets along straight line, is most directed and time-least one. However, if two robots travel together at the same workspace, using above convenient and fast method, collision may happen because of the inappropriate distribution between initial points and target points of two robots. This is required to consider cooperation and collision-free problem while stabilization is realized.

What ones call collision-free problem is that the distance between two robots must be greater than or equal to FBC required in the whole controlled procedure. It is easy to obtain the controller just satisfying collision-free conditions.

Lemma 2 Let the initial time $t_0 = 0$ and the initial velocities of two robots be zero, i.e. $u_{ij}(0) = 0$ with $i, j = 1,2,\cdots$. Let $\rho_1(0) = \rho_1(\boldsymbol{R}_1(0), \boldsymbol{R}_2(0)) \geqslant d_0$. Choose $0 < k < 1, k_1 > 0$, setting $w_1 = u_{11} - u_{21} - k(x_1 - x_2), w_1 = u_{12} - u_{22} - k(y_1 - y_2)$, the controller of the form

$$\left.\begin{array}{l} \tau_{11} - \tau_{21} = k(u_{11} - u_{21}) - k_1 w_1 \\ \tau_{12} - \tau_{22} = k(u_{12} - u_{22}) - k_1 w_2 \end{array}\right\} \tag{7}$$

is such that $\rho_1(\boldsymbol{R}_1, \boldsymbol{R}_2) \geqslant d_0, \forall t \geqslant 0$; $\rho_1(\boldsymbol{R}_1, \boldsymbol{R}_2) \to +\infty$, as $t \to +\infty$.

Proof The lemma can be proved by differenting w_1 and w_2 respectively and substituting (7) into (1).

We now consider under what conditions collision between two robots will not happen using the controller described by lemma 1. Furthermore, find the method to deal with collision-free problem when the conditions are not satisfied.

Suppose

$$\left.\begin{aligned} &s(\bar{T}) = x_1(\bar{T}) - x_2(\bar{T}), \upsilon(\bar{T}) = y_1(\bar{T}) - y_2(\bar{T}) \\ &b(\bar{T}) = u_{11}(\bar{T}) - u_{21}(\bar{T}), d(\bar{T}) = u_{12}(\bar{T}) - u_{22}(\bar{T}) \\ &\sigma_1 = x_1(f) - x_2(f), \sigma_2 = y_1(f) - y_2(f) \\ &a(\bar{T}) = s(\bar{T}) - \sigma_1, c(\bar{T}) = \upsilon(\bar{T}) - \sigma_2 \end{aligned}\right\} \tag{8}$$

where $\bar{T}$ is an arbitrary initial time for system (1).

Consider the behavior of the system moving from arbitrary time $\bar{T}$.

Lemma 3 Let the states of the system (1) satisfy FBC. The domain $\bar{D}$, in Euclidean space $\boldsymbol{R}^8$, is defined in the form

$$\bar{D} = \{x \mid s\sigma_1 + \upsilon\sigma_2 \geqslant \min\{s^2 + \upsilon^2, \sigma_1^2 + \sigma_2^2\}, x \in \boldsymbol{R}^8\}$$

Take k such that

$$k \geqslant \max\left\{\frac{|b(\bar{T})| + |d(\bar{T})|}{2(\sqrt{s^2(\bar{T}) + v^2(\bar{T})} - d_0)}, \frac{|b(\bar{T})| + |d(\bar{T})|}{2(\sqrt{\sigma_1^2 + \sigma_2^2} - d_0)}\right\} \tag{9}$$

The controller of the form

$$\left.\begin{aligned} &\tau_{11} = -2k^2 z_1 - 3ku_{11}, \ \tau_{12} = -2k^2 z_2 - 3ku_{12} \\ &\tau_{21} = -2k^2 z_3 - 3ku_{21}, \ \tau_{122} = -2k^2 z_4 - 3ku_{22} \end{aligned}\right\} \tag{10}$$

makes the closed loop system composed of (1) and (10) satisfy

(i) As $t \to \infty$, $x_i(t) \to x_i(f)$, $y_i(t) \to y_i(f)$, $u_{ij}(t) \to 0$, for $i, j = 1,2$;

(ii) $\rho((x_1, y_1), (x_2, y_2)) \geqslant d_0, t \geqslant \bar{T}$.

Proof The lemma is easy to be proved by using linear system theory and linear algebra theory.

Remark 1 Note that $s(\bar{T})\sigma_1 + \upsilon(\bar{T})\sigma_2$ is the inner product of vector $(s(\bar{T}), \upsilon(\bar{T}))$ and (σ_1, σ_2), which, from (8), just denotes the inner product of the two vectors, one is from the position of a robot to another's, the other from the target point of a robot to another's. Lemma 3 shows that, if the inner product is greater than the norm's square of any one of the two vectors, the controller defined by (10) can exponentially stabilize two robots to their targets without collision.

The controller expressed by lemma 3 is dependent on the initial positions of the two robots. Consider next the design independent of the original positions of the two robots.

Corollary Suppose that the states of the system (1) satisfy FBC at the initial time $t_0 = 0$. In Euclidean space $\boldsymbol{R}^8$, define a domain of the form

$$D = \{x \mid s\sigma_1 + \upsilon\sigma_2 \geqslant \min\{s^2 + \upsilon^2, \sigma_1^2 + \sigma_2^2\}, u_{i,j} = 0, i, j = 1,2\}$$

Arbitrarily take positive number $k > 0$, when the initial value belongs to the domain D, the controller of the form

$$\left.\begin{aligned} &\tau_{11} = -2k^2 z_1 - 3ku_{11}, \tau_{12} = -2k^2 z_2 - 3ku_{12} \\ &\tau_{21} = -2k^2 z_3 - 3ku_{21}, \tau_{22} = -2k^2 z_4 - 3ku_{22} \end{aligned}\right\} \tag{11}$$

is such that the closed loop system composed of (1) and (11) has the properties with

(i) The two robots can be stabilized to their target points $\boldsymbol{R}_1(f)$ and $\boldsymbol{R}_2(f)$ on the domain D;

(ii) $\rho((x_1, y_1), (x_2, y_2)) \geqslant d_0$, $t \geqslant 0$.

Proof The proving method is completely similar to that used in lemma 4. Here omit.

We now consider that, when the conditions required by lemma 3 are not satisfied, how the information of target points is employed to design the feedback control such that the states of the system satisfy these conditions. Once these conditions hold in the given time, system (1) will be surely stabilized by the controller expressed in lemma 3.

Suppose that the initial velocities of the two robots are nonzero. If the collision-free measure is taken just when the distance between two robots equals CFSD d_0, the collision may not be avoided due to the inertia of the robots. Therefore, it is necessary to take a measure in advance in order to avoid collision. For convenience, the collision-free measure may be taken when the distance between two robots equals $2d_0$.

Let ρ_0 be an arbitrary position number with the property $\rho_0 > 2d_0$ and the initial velocities of the two robots are nonzero. The conditions for collision-free measure to be taken are defined as

d_1: $\rho_1((x_1, y_1), (x_2, y_2)) = \rho_0$

d_2: $\mathrm{d}\rho_0/\mathrm{d}t < 0$

d_3: $s\sigma_1 + \upsilon\sigma_2 \leqslant \min\{s^2 + \upsilon^2, \sigma_1^2 + \sigma_2^2\}$

The conditions d_1 and d_2 show that the distance between the robots will be rigorously smaller than ρ_0, unless the measure is taken at the present. By Lemma 3, d_3 shows that the controller given in lemma 3 cannot be insured that the distance between the robots will be smaller than the safe distance do and collision may happen. Next, the design of the feedback control with no collision is under consideration when the above three conditions are all satisfied.

Let T denote the time satisfying condition d_1, d_2, d_3. From d_1, (2) and (8) gives

$$\rho_1(T) = |x_1(T) - x_2(T)| + |y_1(T) - y_2(T)| = |s(T)| + |\upsilon(T)| > 2d_0$$

which means

$$|s(T)| > d_0 \text{ or } |\upsilon(T)| > d_0 \tag{12}$$

The following definition, for convenience, is introduced.

Definition 3 A function $g(\sigma): R \to R \setminus \{0\}$ is defined as

$$g(\sigma) = \begin{cases} \sigma & \sigma \neq 0 \\ 1 & \sigma = 0 \end{cases}$$

It is easily seen that g(σ) has the properties with ① $g(\sigma) \neq 0$, $\forall \sigma \in R$; ② $\sigma g(\sigma) \geqslant 0$, $\forall \sigma \in R$. The equality holds, if and only if $\sigma = 0$.

Lemma 4 Assume the system (1) satisfies the conditions d_1, d_2, d_3 at the initial time $t_0 = T$. By (12), without loss of generality, let $|s(T)| > d_0$, and choose the positive number m_1, m_2 such that

$$m_1 < \frac{m_2|g(\sigma_2)|(|s(T)| - d_0)}{|g(\sigma_1)|(|\upsilon(T)| + d_0)} \tag{13}$$

Let $w_1 = u_{11} - u_{21} - m_1 g(\sigma_1)$, $w_2 = u_{12} - u_{22} - m_2 g(\sigma_2)$. Take the positive number k_1 such that

$$k_1 \geqslant \frac{(|w_2(T)| + |w_2(T)|)\sqrt{m_1^2 g^2(\sigma_1) + m_2^2 g^2(\sigma_2)}}{|m_1 g(\sigma_1)\upsilon(T) - m_2 g(\sigma_2)s(T)| - d_0\sqrt{m_1^2 g^2(\sigma_1) + m_2^2 g^2(\sigma_2)}} \tag{14}$$

where $s(T)$ and $\upsilon(T)$ is defined by (8). The controller of the form

$$\left.\begin{aligned} \tau_1 &= \tau_{11} - \tau_{21} = -k_1 w_1 \\ \tau_2 &= \tau_{12} - \tau_{22} = -k_1 w_2 \end{aligned}\right\} \tag{15}$$

is such that

i) $\rho_1((x_1, y_1), (x_2, y_2)) \geqslant d_0$, $\forall t \geqslant T$;

ii) Setting

$$\hat{T} = \frac{\sigma_1^2 + \sigma_2^2 - \sigma_1 s(T) - \sigma_2 \upsilon(T) + \frac{1}{k_1}|\sigma_1 w_1(T) + \sigma_2 w_2(T)|}{m_1 g(\sigma_1)\sigma_1 + m_2 g(\sigma_2)\sigma_2}$$

when $t > T + \hat{T}$, one obtains $(x_1 - x_2)\sigma_1 + (y_1 - y_2)\sigma_2 > \sigma_1^2 + \sigma_2^2$.

Proof Firstly, $x_1 - x_2$ and $y_1 - y_2$ can be solved by differenting w_1 and w_2 respectively and substituting (15) into (1). Secondly, using algebra theory can prove it.

Remark 2 In the previous lemma, just $\tau_{11} - \tau_{21}$ and $\tau_{12} - \tau_{22}$ are designed. Therefore, there is much free space to design $\tau_{ij}(i, j = 1,2)$. The manner avoiding other robots depends on how to use such a space. For example, if τ_{11} and τ_{12} are indepently designed and τ_{11} and τ_{12} are accordingly chosen, this implies robot $\boldsymbol{R}_2$ avoiding robot $\boldsymbol{R}_1$ without condition. Conversely, so is it. If τ_{11} and τ_{22} are indepently designed and τ_{21} and τ_{12} are accordingly chosen, this implies to turn right or turn left when robot $\boldsymbol{R}_1$ approaches robot $\boldsymbol{R}_2$.

Remark 3 The statement 2 of lemma 4 shows that the distribution relation between the robot's positions(which are the functions of time t) and their target points will surely satisfy the condition expressed in lemma 3 in a given time. Therefore, the system can move to its targets without collision by using the controller given in lemma 3. However, it is clear that the condition expressed in lemma 3 also holds if $(x_1 - x_2)\sigma_1 + (y_1 - y_2)\sigma_2 > (x_1 - x_2)^2 + (y_1 - y_2)^2$. Unlike the previous condition, this one is uneasily estimated in analytic manner. But it can be regarded as a condition to be checked if the condition in lemma 3 holds. In the following theorem, the whole condition expressed in lemma 3 is regarded as the checked condition to determine the time of switching controllers.

In lemma 4, the design of controllers is given just under the assumption $|s(T)| > d_0$. For the case $|\upsilon(T)| > d_0$, we have the following result.

Corollary Let $t = T$ denote the initial time of the system (3) satisfying the condition d_1, d_2, d_3. Let $|\upsilon(T)| > d_0$ and take the positive number m_1 and m_2 such that

$$m_2 < \frac{m_1|g(\sigma_1)|(|\upsilon(T)| - d_0)}{|g(\sigma_2)|(|s(T)| + d_0)} \tag{16}$$

Setting $w_1 = u_{11} - u_{21} - m_1 g(\sigma_1)$, $w_2 = u_{12} - u_{22} - m_2 g(\sigma_2)$, choosing the positive number k_1 of the same form expressed in lemma 4, we have that all the results stated in lemma 4 hold by using the controller given in (19).

Proof The method proving this corollary is completely similar to that used in the proof of lemma 4. Here omit.

Summary the previous statements, the scheme to stabilize the system (3) can be written as follows.

Theorem Suppose that $t_0 = 0$ is the initial time of the system (1) satisfying FBC. Choose $\rho_1 > 2d_0$, and $\bar{D}$, D are defined by lemma 3 and its corollary respectively. The domain $D_1 \subseteq \boldsymbol{R}^8$ is defined as

$$D_1 = \{x | \rho_1(\boldsymbol{R}_1(x_1, y_1), \boldsymbol{R}_2(x_2, y_2)) \geqslant \rho_1, x \in \boldsymbol{R}^8\}$$

The algorithm to stabilize the system (1) on the domain D_1 can be written as

Step 1 If $x(0) \in D$, the controller is taken in the form

$$\left.\begin{aligned}\tau_{11} &= -2k^2 z_1 - 3ku_{11}, \tau_{12} = -2k^2 z_2 - 3ku_{12} \\ \tau_{21} &= -2k^2 z_3 - 3ku_{21}, \tau_{22} = -2k^2 z_4 - 3ku_{22}\end{aligned}\right\} \quad (17)$$

where z_i is defined by (6) for $(i = 1,\cdots,4)$, $k > 0$, $s(0), \upsilon(0), a(0), b(0), c(0), d(0)$ by (8). Stop.

Step 2 If $x(0) \in D$, take the controller of (17), and then check the following conditions.

Condition 1 When $x(t) \in \bar{D}$, let T denote the present time, choose k such that (9). The controller is taken in the form (10). Stop.

Condition 2 Suppose $x(t) \notin \bar{D}$ and the conditions d_1, d_2, d_3 are all satisfied. Let T denote this time. If $|s(T)| > d_0$, choose the positive number m_1 and m_2 such that (13). If $|\upsilon(T)| > d_0$, choose the positive number m_1 and m_2 such that (16). Setting $w_1 = u_{11} - u_{21} - m_1 g(\sigma_1)$, $w_2 = u_{12} - u_{22} - m_2 g(\sigma_2)$, and choosing the positive number k_1 such that (14). Take the controller of the form (9).

Step 3 When $(x_1 - x_2)\sigma_1 + (y_1 - y_2)\sigma_2 > \min\{\sigma_1^2 + \sigma_2^2, s^2 + \upsilon^2\}$, go to check condition 1 of step 2.

Remark 4 Suppose condition expressed in step 1 holds. From the corollary of lemma 3, we know that system (1) can be stabilized using the controller of (11) without collision. Suppose condition 1 expressed in step 2 holds. We know from lemma 3, that system (1) can be stabilized using the controller of (11) without collision. When condition 2 in step 2 is satisfied, from lemma 4 and its corollary, one deduces that the condition expressed in lemma 3 is surely satisfied in a given time, and then come back to step 2 to check condition 1, which is what step 3 is said. Of course, the two conditions in step 2 are just sufficient. If these conditions are not always satisfied, system (1) can be stabilized by the controller proposed in (17) without switching controllers.

3 Simulation

Let $d_0 = 0.5$ m, the initial positions be $(x_1, y_1) = (1,4)$, $(x_2, y_2) = (-1,0)$, the initial velocities be $u_{11}(0) = 0$ m/s, $u_{12}(0) = 0$ m/s, $u_{21}(0) = 0$ m/s, $u_{22}(0) = 0$ m/s, the target points be $(x_1(f), y_1(f)) = (2,0.5)$, $(x_2(f), y_2(f)) = (3,3.5)$. The controller is taken as the form expressed in the theorem. The response of the system (1) is illustrated in Figs. 1 – 4.

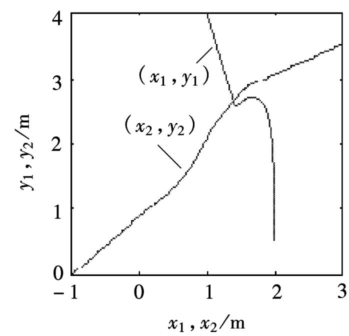

Fig. 1 Movement trajectories of robot $\boldsymbol{R}_1$ and robot $\boldsymbol{R}_2$ on plane

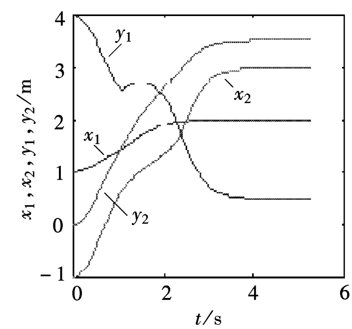

Fig. 2 The responses of the position coodinates x_i and y_i of two robots with respect to time

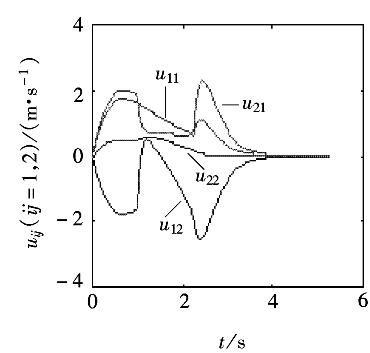

Fig. 3 The responses of the velocities of two robots with respect to time

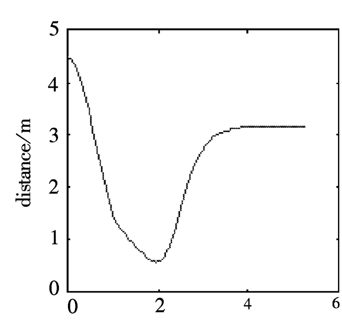

Fig. 4 The responses of the distance between two robots with respect to time

4 Conclusion

For the dynamic system composed of two omni-directional mobile robots, we give a stabilizing controller such that the two robots can move to their target points from arbitrary initial positions satisfying the feasible boundary conditions to target points satisfying the feasible boundary conditions without collision. The inner product used in the paper is the key to deal with the problem. Its optimistic that the method proposed in this paper can be expanded to the dynamic systems composed of three or more omni-directional mobile robots.

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一类多全方位移动机器人动力学系统的镇定

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摘　要　本文考虑了由2个全方位移动机器人组成的混合动力学系统的协调拟镇定问题.利用机器人位置之间的向量与机器人目标之间向量的内积,设计了多步拟镇定律,该控制律能够在避碰后按指数速率运动到目标点,且在整个过程中两机器人之间的距离不小于避碰的安全距离.最后对2个全方位移动机器人进行了仿真,验证了所给方法的有效性.

关键词　全方位机器人, 动力学, 协调, 避碰, 拟镇定

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