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Performance Comparison of Space-Time Trellis Codes and Space-Time Transmit Diversity under the Same Bandwidth Efficiency*

Wu Gang** Chen Ming Cheng Shixin

(National Mobile Communication Research Laboratory, Southeast University, Nanjing 210096, China)

Abstract: Space-time coding can provide high data rate and performance gain for wireless communication system. Performance comparison of space-time trellis codes and space-time transmit diversity is carried out under the same bandwidth efficiency in this paper. We also propose some optimum low rate space-time trellis codes in quasi-static Rayleigh fading channel. Performance analysis and simulation show that the low rate space-time trellis codes outperform space-time transmit diversity at the same bandwidth efficiency, and are more suitable for the power limited wireless communication system which has no strict requirement on bandwidth efficiency.

Key words: space-time trellis codes, space-time transmit diversity, bandwidth efficiency

Space-time coding is an attractive scheme to achieve high data rate and performance gain. For space-time trellis codes (STTC) in quasi-static Rayleigh fading channel, Tarokh et al. derived the fundamental tradeoff among bandwidth efficiency (called transmission rate or data rate in the paper), diversity gain, constellation size, and trellis complexity[1]. The best tradeoff achieves the maximum possible bandwidth efficiency at a given diversity gain for bandwidth constrained system. In the power limited system, which has no strict requirement on bandwidth efficiency, a low rate multi-dimensional space-time trellis code is also proposed by using multiple trellis coded modulation (MTCM) construction in Ref. [1]. However, MTCM construction is only valid in independent fading channel, which is ensured by ideal interleaving[2]. In quasi-static Rayleigh fading channel, MTCM construction cannot improve the system performance, on the contrary, it can increase the complexity of code construction.

In Ref. [3], performance comparison of different space-time coding schemes was carried out in quasi-static Rayleigh fading channel, including STTC and Space-Time Transmit Diversity (STTD). In this paper, concatenating space-time block codes[4,5] (STBC) with convolutional code and modulator is called STTD, which is used in current 3GPP standard[6]. However, the comparison of STTC and STTD in the paper is not under the same bandwidth efficiency. Therefore, although the conclusion of the paper points out that STTC reaches the best tradeoff of performance against complexity, the demonstration of the statement is not satisfied.

In this paper, we present the frame error probability of STTC and STTD, and point out that the optimum STTC has better performance compared with STTD at the same bandwidth efficiency. Then, we propose some optimum low rate STTC and carry out performance comparison between STTC and STTD at the same bandwidth efficiency. Analysis and simulation results show the better performance of STTC.

1 Performance Analysis of STTC and STTD

The system model of STTC is shown in Fig. 1. Two

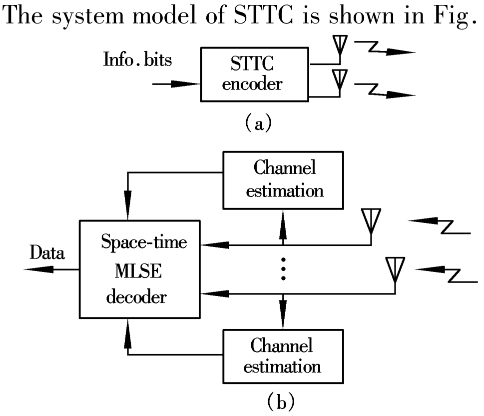

Fig. 1 The system model of STTC. (a) Transmitter; (b) Receiver

transmitting and M receiving antennas are considered. If the number of transmitting antenna becomes larger than 2, the bandwidth efficiency of STBC is lower, thus the comparison of STTC and STTD is difficult to be carried out. Information source data are encoded by an STTC encoder and sent out symbol sequences $c = c_{1,1}c_{2,1}c_{1,2}c_{2,2}\cdots c_{1,2L}c_{2,2L}$, where $2L$ is the length of

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** Born in 1970, male, graduate.

one frame. The symbol sequences are transmitted from two antennas at the same time. Assuming that the channel between each transmitting and receiving antenna is flat quasi-static Rayleigh fading channel, and that maximum-likelihood decoder is used and deciding out symbol sequence is $e = e_{1,1} e_{2,1} e_{1,2} e_{2,2} \cdots e_{1,2L} e_{2,2L}$, the upper bound on the pairwise error probability is approximated by [1]

$$P_{\mathrm{tc}}(c \to e) \leqslant \prod_{i=1}^{r} \left[1 + \lambda_i \frac{E_s}{4N_0} \right]^{-M} \tag{1}$$

where r is the rank of the matrix $\boldsymbol{B}$

$$\boldsymbol{B}(c,e) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & e_{2L}^1 - c_{2L}^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & e_{2L}^2 - c_{2L}^2 \end{bmatrix} \tag{2}$$

λ_i are the non-zero eigenvalues of the matrix $\boldsymbol{A}(c,e) = \boldsymbol{B}\boldsymbol{B}^*$; E_s is the energy per symbol; N_0 is the total variance of noise variable. The optimum STTC should make $\boldsymbol{B}(c,e)$ be full rank and minimize the maximum upper bound on pairwise error probability.

When SNR is high, the frame error probability of optimum STTC is approximated by

$$P_{\mathrm{tc}} \cong \min_{c \subset \Omega} \left\{ p_{\mathrm{tc}}(c) \left[\prod_{i=1}^{2} (\lambda_{i,\mathrm{opt}}^{C}) \right]^{-M} \right\} \left(\frac{E_s}{4N_0} \right)^{-2M} \tag{3}$$

where $\lambda_{i,\mathrm{opt}}^{C}$ is the i-th eigenvalue of the matrix $\boldsymbol{A}(c,e)$ for the transmitting symbol sequence c when the optimum STTC is used; $p_{\mathrm{tc}}(c)$ is a priori probability of transmitting c; Ω is the set of all transmitting symbol sequences.

The system model of STTD is shown in Fig.2. The scheme of STTD is STBC concatenated with convolutional code and modulator. In fact, the scheme of STTD can also be STBC concatenated with conventional trellis coded modulation[7]. Two transmitting and M receiving antennas are considered. If the transmitting symbol sequence is $c = c_{1,1} c_{2,1} c_{1,2} c_{2,2} \cdots c_{1,2L} c_{2,2L}$, where $c_{2,2l-1} = -c_{1,2l}^*$, $c_{2,2l} = c_{1,2l-1}^*$, ($l = 1,2,\cdots,L$, $2L$ is the length of one frame), the combining output of the receivers are[4]:

$$\tilde{c}_{1,2l-1} = \sum_{j=1}^{M} \{ (|\alpha_{1,j}|^2 + |\alpha_{2,j}|^2) \times c_{1,2l-1} \sqrt{E_s} + \alpha_{1,j}^* n_{2l-1,j} + \alpha_{2,j} n_{2l,j}^* \} \tag{4}$$

$$\tilde{c}_{1,2l} = \sum_{j=1}^{M} \{ (|\alpha_{1,j}|^2 + |\alpha_{2,j}|^2) \times c_{1,2l} \sqrt{E_s} - \alpha_{2,j} n_{2l-1,j}^* + \alpha_{1,j}^* n_{2l,j} \} \tag{5}$$

where $n_{2l-1,j}$, $n_{2l,j}$ are the noise variables at time $2l-1$, $2l$ on the j-th receiving antenna respectively, and are independent complex Gaussian random variables with mean zero and variance 0.5 per dimension; $\alpha_{i,j}$ is the path gain which is constant during one frame.

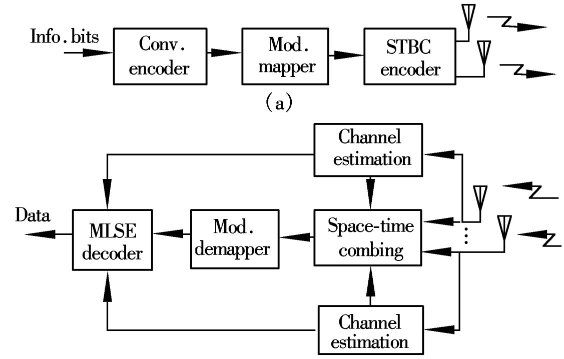

Fig.2 The system model of STTD. (a) Transmitter; (b) Receiver

After maximum-likelihood decoding, the output symbol sequences can be considered as $e = e_{1,1} e_{2,1} e_{1,2} e_{2,2} \cdots e_{1,2L} e_{2,2L}$, where $e_{2,2j-1} = -e_{1,2j}^*$, $e_{2,2j} = e_{1,2j-1}^*$. Then the pairwise error probability can approximately be

$$P_{\mathrm{td}}(c \to e \mid \alpha_{i,j}, i, j) \leqslant \exp\left(-d^2(c,e) \frac{E_s}{4N_0}\right) \tag{6}$$

where $d^2(c,e) = \sum_{j=1}^{M} \sum_{t=1}^{2L} [|c_{1,t} - e_{1,t}|^2 (|\alpha_{1,j}|^2 + |\alpha_{2,j}|^2)]$. If $\alpha_{i,j}$ can be modeled as independent complex Gaussian random variable with mean zero and variance 0.5 per dimension, the probability distribution function of $|\alpha_{i,j}|$ is

$$p(|\alpha_{i,j}|) = 2|\alpha_{i,j}| \exp(-|\alpha_{i,j}|^2) \tag{7}$$

By averaging (7) over the probability distribution of $\alpha_{i,j}$ in (6), we can obtain

$$P_{\mathrm{td}}(c \to e) \leqslant \left[1 + \frac{E_s}{4N_0} \sum_{t=1}^{2L} |c_{1,t} - e_{1,t}|^2 \right]^{-2M} \tag{8}$$

From the above inequality, the upper bound is mainly decided by the free Euclidean distance d_{free} between the transmitting symbols and decision output symbols. If convolutional code and 4PSK modulation are used in STTD, the upper bound is evaluated by the performance of the convolutional code. In order to get the best performance, we should choose the optimum convolutional code having largest d_{free}[8]. When SNR is high, the frame error probability is approximated by

$$P_{\mathrm{td}} \cong p_{\mathrm{td}}(c') \left(d_{\mathrm{free}}^2 \frac{E_s}{4N_0} \right)^{-2M} \tag{9}$$

where $p_{\mathrm{td}}(c')$ is the priori probability of the transmitting c' which achieves the largest d_{free} when optimum convolutional code is used.

From (3) and (9), if STTC and STTD have the same bandwidth efficiency, code rate, modulation and trellis complexity, we can see that the STTC can outperform STTD when the following inequality is satisfied

$$\min_{c \subset \Omega}\left\{p_{\mathrm{tc}}(c)\left[\prod_{i=1}^{2}(\lambda_{i,\mathrm{opt}}^{C})\right]^{-1}\right\} < p_{td}(c')(d_{\mathrm{free}}^{2})^{-2} \tag{10}$$

2 Low Rate STTC

In order to carry out performance comparison between STTC and STTD at the same bandwidth efficiency and code rate, we propose low rate STTC in this paper. Low rate means that code rate is less than 1, while full rate means that code rate is equal to 1. According to the proposition in Ref.[1], the low rate STTC in this paper can be called low-rate one-dimensional STTC. It is noticed that the STTC in Fig.4 – Fig.9 of Ref.[1] can be called full-rate one-dimensional STTC, and the STTC in Fig.21 – Fig.22 in Ref.[1] can be called low-rate multidimensional STTC. We will give some examples of low rate STTC in quasi-static Rayleigh fading channel. The condition is: 16 states, 1/2 code rate, 1?bit · s^{-1}/Hz, 4PSK. The code construction of the STTC we proposed is represented with generator matrix form, which is used in Ref.[9]. Assuming that the information source bits $I = (b_1, b_2, \cdots, b_K)$ are transmitted, $b_l(l = 1,2,\cdots, K)$ is 0 or 1, where $K = 2L\log_2 Q$, $2L$ is the length of frame transmitted, $Q = 4$ for 4PSK modulation. Generator matrix $\boldsymbol{G}$ has N rows and $m + s$ columns, where N is the number of transmitting antennas, m is the number of input information bits which is used for once STTC encoding, s is the number of memory elements in the STTC encoder. Let $\Phi(\cdot)$ be a mapping function that maps integer values to the 4PSK constellation $\Phi(x) = \exp(\pi j x/2)$. At time t $(t = 1, 2,\cdots,2L)$, the transmitting symbol $c_{1,t}c_{2,t}$ is obtained by

$$(c_{1,t}c_{2,t}) = \Phi((I_t \cdot \boldsymbol{G}^{\mathrm{T}})(\mathrm{mod}4)) \tag{11}$$

where the superscript T denotes the transpose of the matrix.

We propose two low rate STTC of 16 and 256 states. When 16 states are used, $m = 1$, $s = 4$. The generator matrix is obtained by exhaustive search guided by the rank/determinant criteria in Ref.[1].

$$\boldsymbol{G}_1 = \begin{bmatrix} 0 & 1 & 2 & 2 & 3 \\ 1 & 2 & 0 & 0 & 2 \end{bmatrix} \tag{12}$$

The scheme of STTD is an STBC encoder concatenated with a convolutional (2,1,5) encoder and 4PSK modulator. The code rate of the convolutional code is 1/2. The constraint length is 5. The convolutional code has maximum free distance, which can be found in Ref.[8]. It can be calculated out that $p_{\mathrm{tc}}(c) \approx p_{\mathrm{td}}(c')/2$, $\prod_{i=1}^{2}(\lambda_{i,\mathrm{opt}}^{C}) = (68 - 24\sqrt{2}) \approx 34$, $d_{\mathrm{free}}^{4} = 64$. Then, (10) is satisfied.

3 Simulation and Discussion

The upper bound of frame error probability can be only used to evaluate the performance roughly. In this section, we carry out performance evaluation by computer simulation.

The system models in Fig.1 and Fig.2 are used in the simulation. Two transmitting and one receiving antenna are considered. Every frame has 192 information bits. At the beginning and the end of each frame, encoder is set to zero state. The length of one frame is 20?ms. In the quasi-static Rayleigh fading channel, Fig.3 shows the curves of bit error rate versus

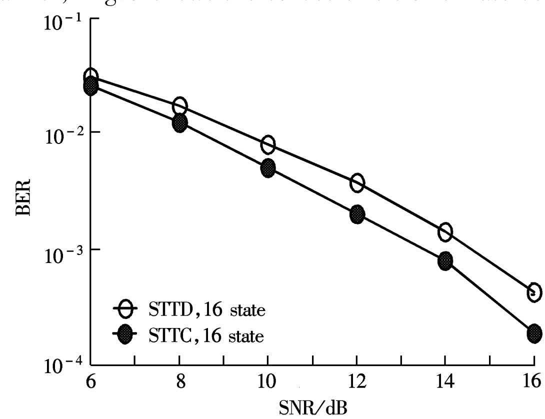

Fig.3 Performance comparison of low rate STTC and STTD in quasi-static Rayleigh fading channel

SNR (SNR $= 2E_s/N_0$, where $2E_s$ is the total transmitting energy of the two transmitting antennas) of STTC and STTD in section Ⅲ. The simulation result shows that the performance of STTC is superior to that of STTD at the same bandwidth efficiency.

Performance comparison at high bandwidth efficiency between STTC and STTD is also performed. The bandwidth efficiency is 2bit · s^{-1}/Hz; code rate is 1/1. The simulation is carried out in quasi-static Rayleigh fading channel. The STTC scheme is the same as 16 state STTC in Fig.5 of Ref.[1]. The scheme of STTD is realized by concatenating STBC with a 4PSK modulator. In order to ensure the comparison under the same bandwidth efficiency and code rate, we cannot concatenate STBC with convolution code or conventional trellis coded modulation[7]. Other conditions are the same as that of low rate case. It is easy to obtain that $p_{\mathrm{tc}}(c) \approx p_{\mathrm{td}}(c')/2$, $\prod_{i=1}^{2}(\lambda_{i,\mathrm{opt}}^{C'}) = 12$, $d_{\mathrm{free}}^{4} = 4$. Fig.4 shows the curves of frame error rate (FER) versus SNR. The simulation result shows that the performance of STTC is better than that of STTD.

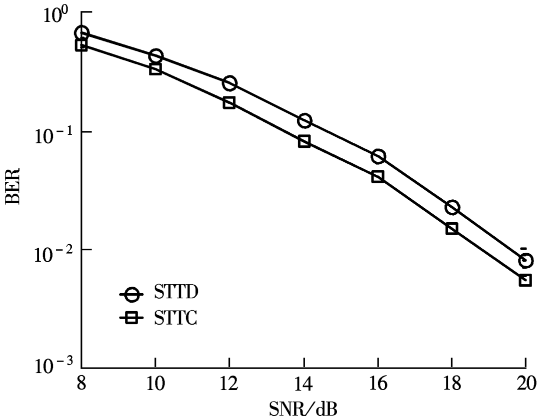

Fig.4 Performance comparison of STTC and STTD at high bandwidth efficiency (2?bit · s^{-1}/Hz)

As we know, STTC in Ref.[1] can be applied in bandwidth constrained system to achieve high bandwidth efficiency and performance gain[10]. In power limited system, which has no strict requirement on bandwidth efficiency, STTD can offer good performance[3]. However, low rate STTC proposed in this paper has better performance than STTD, at the same bandwidth efficiency, code rate, modulation, trellis complexity and the number of transmitting and receiving antennas. This is because that combination diversity and coded modulation together is the scheme of STTC, which can be optimized by exhaustive search, while STTD is just the scheme concatenating diversity with coded modulation.

4 Conclusion

Under the same bandwidth efficiency, data rate, modulation mapper and the number of transmitting and receiving antennas, STTC has higher code gain compared with STTD. Furthermore, the schemes of low rate STTC we proposed in this paper are more suitable for the power limited wireless communication system which has no strict requirement on bandwidth efficiency.

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空时格码与空时发射分集在相同频带利用率下的性能比较

邬　钢　陈　明　程时昕

(东南大学移动通信国家重点实验室,南京 210096)

摘　要　空时编码能够为无线通信系统提供高数据速率及性能.本文在相同频带利用率下将空时格码与空时发射分集的性能进行了比较.同时,针对准静态瑞利衰落信道,提出了几种优化的低码率空时格码.理论分析和仿真表明,在相同的频带利用率下,该低码率空时网格码可具有比空时发射分集更好的误码率性能,更适合于对频带利用率要求不高的功率受限无线通信系统.

关键词　空时网格码, 空时发射分集, 频带利用率

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