

Simultaneous Extrapolation of RCS in Both Angular and Frequency Domains Based on AWE Technique^{*}

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Abstract: The radar cross section (RCS) of a target is dependent on frequency as well as observation angle. The method of moments (MOM) in conjunction with the asymptotic waveform evaluation (AWE) technique is applied to predict the mono-static RCS of an arbitrarily shaped two-dimensional cylinder in both frequency and angular domains. The electric field integral equation (EFIE) is solved using MOM to obtain the equivalent current on the cylinder. In the AWE technique, the equivalent current at a given frequency and angle is expanded in the desired frequency and angle band in a binomial Taylor's series. The binomial Taylor's series coefficients are then matched via the Pade approximation to a rational function. Using the rational function, the current is obtained at any frequency and angle within the interesting frequency and angular range, which is in turn used to calculate RCS of the cylinder. This method for extrapolation of RCS has at least two significant advantages: one is that RCS analytical formulae are obtained and the other is it can decrease the amount of time required for modeling problems with large computational domains.

Key words: MOM, AWE technique, extrapolation, RCS

The solution of the electric field integral equation (EFIE) via the method of moments (MOM) has been a very useful tool for accurately predicting RCS of an arbitrarily shaped cylinder in both frequency and angular domains^[1]. In MOM, the EFIE is reduced to a matrix equation of the equivalent currents. The RCS is then computed from the knowledge of the currents. The generation of the matrix equation, which is usually a dense matrix, and its solutions are two major computationally intensive operations in MOM.

RCS contains information in both frequency and angular domains simultaneously. To obtain RCS over a band of frequency or angle using MOM, one has to repeat calculation at each frequency or angle over the frequency or angular band of interest. If the RCS is high frequency or angular dependent, one needs to do calculation at finer increment of frequency or angle to get accurate representation of the frequency or angular response. This is computationally intensive.

The asymptotic waveform evaluation (AWE) technique with some numerical methods that leads to an efficient technique for solving EFIE has already been successfully used in some electromagnetic problems^[2-5]. On the basis of AWE technique, a Taylor series expansion is generated about a specific value of the system parameter (frequency, angle, etc.). In most cases, Taylor series, incorporating polynomial and binomial models for one- and two-dimensional curves and surfaces, respectively, are

able to give fairly good results. For some situations, however, the Taylor series model is not sufficient to represent the data to take into account for the physics of the problem because simple polynomial and binomial functions cannot model accurately the pole-zero structure of the frequency or angular response. The accuracy of the Taylor series is limited by the radius of convergence and would not converge beyond the range of convergence. In such cases, the rational function approach is used to improve the accuracy of the numerical solution. The coefficients of the Taylor series or moments are then matched via Pade approximation to the rational function. The AWE technique provides a reduced-order model using a rational function as opposed to what would be possible if a polynomial or other nonphysical function were used. A rational function is more useful than simple polynomials or binomial models in some cases because it is able to represent complicated pole-zero functional forms. Pade representations have a larger circle of convergence and can therefore provide a broader extrapolation when compared to Taylor series representations.

The AWE technique has been successfully applied to the extrapolation of RCS alone versus frequency or angular curves in appeared documents^[3-5], but there has not been any documented attempt to extend this technique to include extrapolation of RCS versus both the frequency and angle simultaneously. The purpose

of this paper is to present a Pade rational function model of two variables that are matched via a Taylor series and used to extrapolate both frequency and angular characteristics of RCS simultaneously.

1 MOM

Consider an arbitrarily shaped cylinder. Dividing the cylinder surface into sub-domains Δ_{s_m} ($m = 1, 2, \dots, M$) and employing impulse functions as the basis function and δ functions as the power function, the EFIE is dispersed into the matrix equation as

$$[Z_{mn}(f)][I_n(f, \theta)] = [V_m(f, \theta)] \quad (1)$$

where θ is an EM wave incident angle; $[Z_{mn}(f)]$ is a complex and dense matrix with only dependence on frequency f , $[V_m(f, \theta)]$ is the excitation column vector with dependence on both f and θ ; and $[I_n(f, \theta)]$ is the current density vector on the cylinder with also dependence on both f and θ .

2 AWE Implementation

If one needs the RCS over a frequency and angular ranges, the calculation must be repeated for all the frequencies and angles of interest of each excitation. Instead, AWE can be applied for fast calculation of RCS over frequency and angular ranges. The AWE technique involves expanding the unknown current density at the frequency and angle (f, θ) in a Taylor series at a given frequency and angle (f_0, θ_0) and then obtaining a rational function representation via the Pade approximation.

$$[I_n(f, \theta)] = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} [I_n^{(i,j)}(f_0, \theta_0)] (f - f_0)^i \times (\theta - \theta_0)^j \quad (2a)$$

$$[I_n^{(i,j)}(f_0, \theta_0)] = [Z_{mn}(f_0)]^{-1} \left(\frac{[V_m^{(i,j)}(f_0, \theta_0)]}{i!} - \sum_{p=1}^i \frac{[Z_{mn}^{(p)}(f_0)]}{p!} [I_n^{(i-p,j)}(f_0, \theta_0)] \right) \quad (2b)$$

where $V_m^{(i,j)}(f_0, \theta_0)$ is the i -th and j -th derivatives of $V_m(f, \theta)$ with respect to f and θ evaluated at (f_0, θ_0) ; $I_n^{(i,j)}(f_0, \theta_0)$ is the i -th and j -th derivatives of $I_n(f, \theta)$ with respect to f and θ evaluated at (f_0, θ_0) ; $Z_{mn}^{(p)}(f_0)$ is the p -th derivatives of $Z_{mn}(f)$ with respect to f evaluated at (f_0, θ_0) . For plane wave excitations, the moment $I_n^{(i,j)}(f_0, \theta_0)$ can be calculated and one can therefore increase the order of the expansion as needed to extend the validity of the approximation. This is better achieved by casting (4) into a Pade rational function. The Pade representation for each

$I_n(f, \theta)$ ($n = 1, 2, \dots, N$) is

$$I_n(f, \theta) \approx P \left(\frac{L_f, L_\theta}{M_f, M_\theta} \right) = \frac{\sum_{l_f=0}^{L_f} \sum_{l_\theta=0}^{L_\theta} a_n^{l_f l_\theta} (f - f_0)^{l_f} (\theta - \theta_0)^{l_\theta}}{\sum_{m_f=0}^{M_f} \sum_{m_\theta=0}^{M_\theta} b_n^{m_f m_\theta} (f - f_0)^{m_f} (\theta - \theta_0)^{m_\theta}} \quad (3)$$

where the integers (L_f, L_θ) and (M_f, M_θ) are the orders of the numerator and denominator expansions of the Pade rational function $P(L/M)$, respectively. According to (2) and (3), the coefficients $a_n^{l_f l_\theta}$ and $b_n^{m_f m_\theta}$ are then found from the moment $I_n^{(i,j)}(f_0, \theta_0)$ from Lutterodt approximation LAB^[6]

$$\sum_{m_f=0}^{M_f} \sum_{m_\theta=0}^{M_\theta} I_n^{(i-m_f, j-m_\theta)}(f_0, \theta_0) \cdot b_n^{m_f m_\theta} = 0, \quad (i, j) \in B \quad (\text{as shown in Fig.1}) \quad (4a)$$

$$\sum_{m_f=0}^{M_f} \sum_{m_\theta=0}^{M_\theta} I_n^{(i-m_f, j-m_\theta)}(f_0, \theta_0) \cdot b_n^{m_f m_\theta} = a_n^{ij}, \quad (i, j) \in A \quad (\text{as shown in Fig.1}) \quad (4b)$$

where $b_n^{00} = 1, I_n^{(i,j)} = 0$ ($i < 0$ or $j < 0$) and the concourses of the index (i, j) are shown as Fig.1.

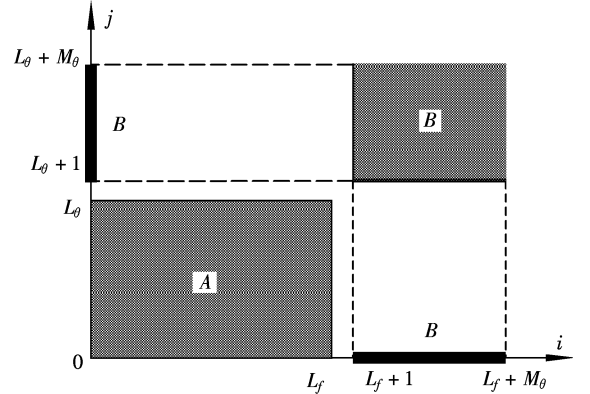


Fig.1 The concourses of the index (i, j) in (4)

3 Numerical Results

To validate the analysis presented in the previous section, we compute scattering of the 2-D dielectric cylinder slotted with $a = 2.7\text{cm}, b = 0.17\text{cm}, d = 0.57a, \epsilon_r = 4$ and the conducting cylinder slotted with $a = 0.914727\text{cm}$ as shown in Fig.2 and Fig.3. The

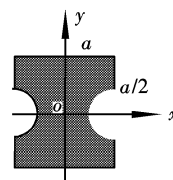


Fig.2 The conducting cylinder slotted

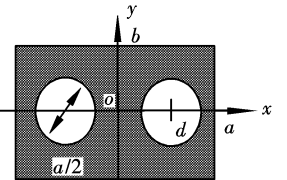


Fig.3 The dielectric cylinder slotted

numerical results from AWE expansion at (f_0, θ_0) (the dielectric cylinder (30GHz, 90°) and the conducting cylinder (30GHz, 45°)) are compared with the results from MOM calculated at each frequency and angle, respectively. The RCS responses of the dielectric

cylinder are matched with the reference RCS of only MOM as functions of frequency and angle as shown in Fig.4. The RCS responses in Fig.4 expressed the cross section at expansion (f_0, θ_0) are shown in Fig.5 (★ is AWE expansion position frequency and angle).

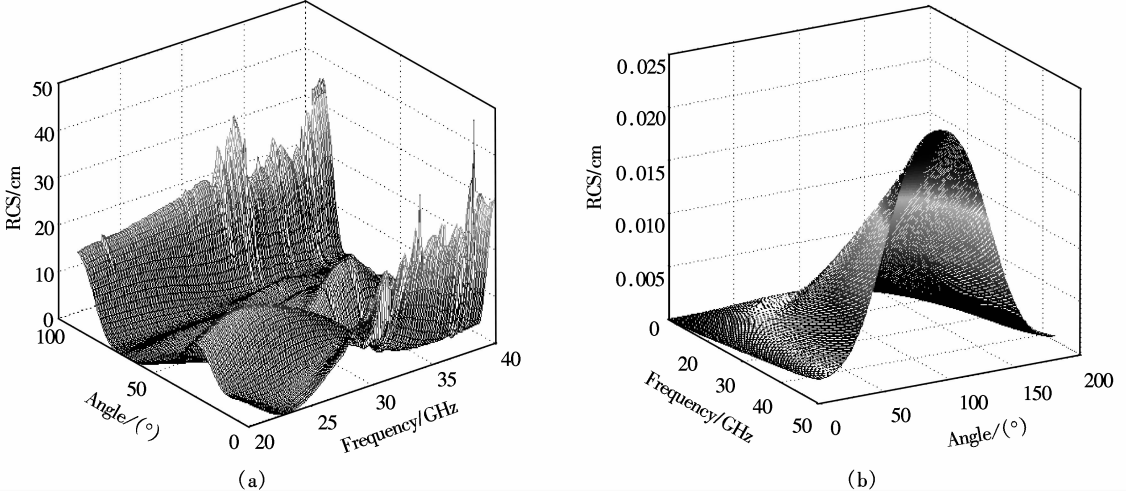


Fig.4 RCS frequency and angular responses of the cylinder. (a) The conducting cylinder; (b) The dielectric cylinder

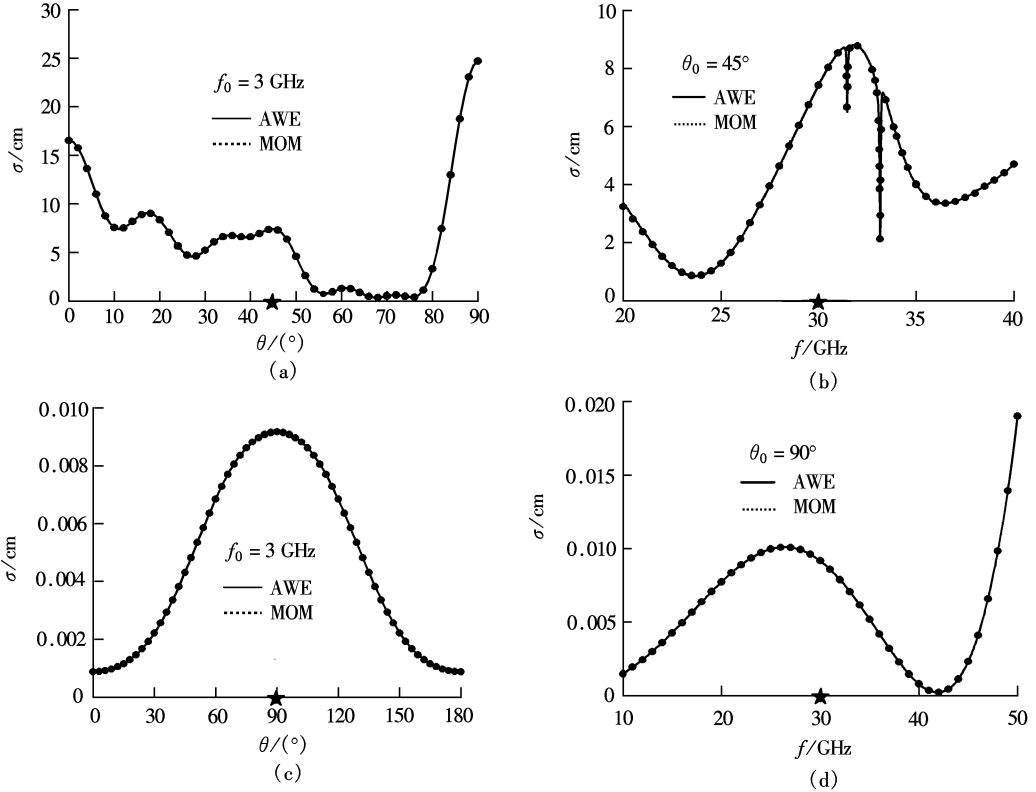


Fig.5 RCS frequency or angular responses expressed the cross section at expansion (f_0, θ_0) in Fig.4. (a) The dielectric cylinder; (b) The conducting cylinder; (c) The dielectric cylinder; (d) The conducting cylinder

The rational functions are of the numerator order ($L_f = 8, L_\theta = 6$) and the denominator order ($M_f = 8, M_\theta = 6$) for the dielectric cylinder and the numerator order ($L_f = 7, L_\theta = 8$) and the denominator order ($M_f =$

$= 8, M_\theta = 8$) for the conducting cylinder.

For CPU comparisons, MOM was carried out calculation of RCS frequency and angular responses with 0.01GHz frequency and 0.1° angular

increments. On the other hand, AWE with the same as increments was carried out calculation of RCS frequency and angular responses about 1/10 of the CPU time used by the MOM. This comparison clearly shows how RCS frequency and angular responses can be obtained much faster by means of AWE.

4 Conclusion

In this paper, the RCS scattering pattern in a broad frequency and angular bands are calculated using AWE technique. agree the results well with the MOM exact solution. The AWE decreases considerable CPU time consuming.

References

1 R. F. Harrington, *Field Computation by Moment Methods*,

Macmillian Company, 1968

2 Y. E. Erdemli, C. J. Reddy, and J. L. Volakis, AWE technique in frequency domain electromagnetics, *Journal of Electromagnetic Waves and Applications*, vol.47, no.3, pp. 359 – 378, 1999

3 Y. E. Erdemli, J. Gong, C. J. Reddy, and J. L. Volakis, Fast RCS pattern fill using AWE technique, *IEEE Trans. Antennas and Propagation*, vol.46, no.11, pp. 1752 – 1753, 1998

4 C. J. Reddy, and M. D. Deshpande, Fast RCS computation over a frequency band using method of moments in conjunction with asymptotic waveform evaluation technique, *IEEE Trans. Antennas and Propagation*, vol.46, no. 8, pp.1229 – 1233, 1998

5. C. M. Tong, H. X. Zhou, and W. Hong, Fast RCS pattern computation based on AWE technique, *Asia Pacific Conference on Electronic Communication Systems*, Australia, 2000

6. C. H. Lutterodt, A two-dimensional analogue of Pade approximant theory, *J. Phys. A*, vol. 22, no. 7, pp. 1027 – 1037, 1974

基于 AWE 技术的 RCS 角度和频率同时外推

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摘 要 目标的雷达散射截面 (RCS) 与照射频率和照射角有关. 本文采用矩量法 (MOM) 并结合渐近波形估计 (AWE) 技术在频域和角度上预测了任意形状二维柱体的单站 RCS. 采用 MOM 法求解电场积分方程获取柱体的等效电流. 在 AWE 技术中, 首先将等效电流在给定频率和角度处按双变量 Taylor 级数展开, 然后使该级数与 Pade 逼近表示的有理分式函数匹配, 最后利用该有理分式函数获取任意频率和角度处的等效电流, 进而计算柱体的 RCS. 这种 RCS 的外推法至少有 2 个明显的优点: 其一是能获得 RCS 的解析结果, 其二是能降低计算时间.

关键词 矩量法, 渐近波形估计技术, 外推, 雷达散射截面

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