

# Reconstruction Temperature Field of Flame by Optical Sectioning Tomography\*

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**Abstract:** Optical sectioning thermography is developed to measure 3-D flame temperature distribution. A 3-D flame can be regarded as a series of 2-D parallel sections. A camera moving along a line to shift focused section is used to capture overlapped section images. With these obtained images, image reconstruction algorithm is applied to rebuild each section. Temperature related to image parameter of flame can be three-dimensionally reconstructed. Experimental results are given finally.

**Key words:** optical sectioning tomography, reconstruction of temperature field, primary colors

Temperature is one of the most important parameters of flame. 3-D temperature measurement is essential to the investigation on whole field parameters in flame.

Non-invasive methods are widely studied because of its unique characteristics. Many non-contacted methods have been developed. As a 3-D noninvasive measuring technique, computerized tomography (CT)<sup>[1]</sup> has been widely applied to 3-D parameter reconstructions. There are many methods based on CT technique for flame temperature reconstruction such as Interferotic CT, Holographic CT, Morrie Deflection CT, Transmissive CT, Emissive CT, etc. Although these techniques have their unique feature, their complex nature limits their usage in measurement<sup>[2]</sup>.

In this paper, a novel technique called optical sectioning thermography is proposed to measure 3-D temperature in flame. The authors apply this technique to 3-D temperature reconstruction in flame<sup>[3]</sup>. The results demonstrate that this technique is a useful measuring tool with high temporal and spatial resolution.

## 1 Principle

### 1.1 Reconstruction of sectioning picture

As we know, any 3-D object can be considered as combination of many serial 2-D sections, and any picture captured by ordinary camera is a 2-D image which contains information of 3-D object field. In fact, the 2-D image denotes an overlapped image of many sections of the object. While one of these 2-D section images is focused, all others are defocused. Focusing each parallel section of the object one by one with a

single photograph system (See Fig.1), a group of 2-D images will be obtained, and each of them is the overlapped image of every 2-D section of the object. According to Fourier optics theory, an intensity distribution  $I(x', y')$  on image plane is the convolution of the irradiance distribution  $I_o(x, y)$  on object plane and point-spread function (PSF) of photograph lens system. So 2-D irradiance distribution of any section of the object, such as a flame, can be reconstructed by inversion of the sets of equations.

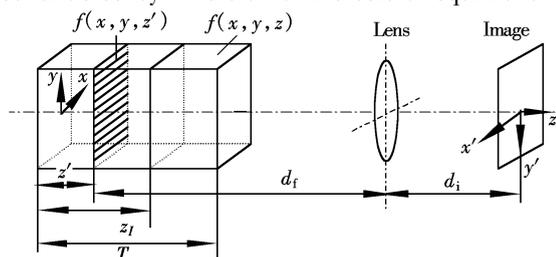


Fig.1 Optical system

As shown in Fig.1, 3-D luminous specimen with thickness  $T$  irradiates irrespective light, and its irradiance is  $f(x, y, z)$ . An optical system with focal length  $F$ , object distance  $d_f$ , and image distance  $d_i$  are utilized. Axis  $z$  is regarded as the main optical axis, left of the object is placed at  $z = 0$ , while flame space is at  $(x, y, z)$  and image space is at  $(x', y', z')$ . Focusing the section of  $z'$  which irradiance is  $g(x, y, z')$ , an overlapped image is captured, whose irradiance is  $g'(x', y', z')$ , so  $g'(x', y', z')$  is the overlapped image of the focused  $z'$  and other unfocused sections. Since we expect to get not bigger or smaller image, but the section itself, the image will project to the section along the fore way. For example,  $g'(x', y', z')$  projects to the section  $z'$  in reverse,

unchanged  $g(x, y, z')$  will be captured. Other section, such as  $z_l$ , which irradiance is  $f(x, y, z_l)$  and image is  $f'(x', y', z_l)$ , projects to focal section  $z'$  inversely, the equation  $g_1(x, y, z') = f(x, y, z_l) * h(x, y, z_l - z')$  will be obtained, where  $*$  is convolution,  $h(x, y, z_l - z')$  is the point spread function (PSF) of the optical system because of  $(z_l - z')$  away from focal section. Focusing on the section  $z'$ , the governing equation of image irradiance is thus as follows in the absence of noise.

$$g(x, y, z') = \int_0^l f(x, y, z) * h(x, y, z' - z) dz \quad (1)$$

When the above equation is transformed, it becomes the following governing equation

$$g(x, y, z') = \sum_{i=1}^N f(x, y, i\Delta z) * h(x, y, z' - i\Delta z) \Delta z \quad (2)$$

where  $N = T/\Delta z$ ,  $\Delta z$  is the distance between close sections. So a 3-D body can be regarded as  $N$  2-D parallel sections to be constructed, the image irradiance focusing on section  $z'$  is the summation of  $z'$  section focal image and the others are not focused section images. Then those captured 2-D images provide the 3-D information.

Keeping the flame and optical system spatially invariant by focusing on each different section along the optical axis, a series of different images are obtained.

$$g(x, y, j\Delta z) = \sum_{i=1}^N f(x, y, i\Delta z) * h(x, y, j\Delta z - i\Delta z) \Delta z \quad j = 1, 2, \dots, N \quad (3)$$

And Eq. (3) can be simplified as

$$g_j = \sum_{i=1}^N f_i * h_{j-i} \quad j = 1, 2, \dots, N \quad (4)$$

By Fourier transformation, Eq. (4) can be written as

$$G_j = \sum_{i=1}^N F_i * H_{j-i}$$

Since  $G_j (j = 1, 2, \dots, N)$  described above can be produced by image Fourier transformation, and  $H_{j-i} (i = 1, 2, \dots, N; j = 1, 2, \dots, N)$  can be worked out by theory or measured experimentally,  $f_i (i = 1, 2, \dots, N)$  are thus obtained conclusively.

From the above analysis, a 3-D luminous body can be considered as the combination of many 2-D parallel luminous sections. Focusing on these discrete sections respectively by a single camera, a group of images can be captured to form governing equations of irradiance. After inversion procedure, the irradiance distribution of different sections can be decoded, thus 3-D luminous irradiance distribution can be reconstructed.

If every section absorbs the irradiation near the

section, and the absorbance of section I is  $a_i (i = 1, 2, \dots, N)$ , Eq.4 can be changed into

$$g_j = \sum_{i=1}^N f_i * h_{j-i} \times a_i \quad j = 1, 2, \dots, N \quad (5)$$

If the absorbance of each section is  $a$ , then

$$g_j = a \sum_{i=1}^N f_i * h_{j-i} \quad j = 1, 2, \dots, N \quad (6)$$

If flame body is insteaded by above specimen, it is obvious that the intensity of 3-D flame body can be reconstructed based on the above-described method.

This study establishes the theory of the focusing tomography to measure transparent, translucent, and opaque 3-D irradiance distribution. The advantage is: only a single fixed camera system moving along the optical axis is used to take pictures of each section, while the flame is spatially stationary.

## 1.2 Principle of irradiance for reconstruction of temperature field

In general, the radiation and distribution of the object is described by Planck function

$$E(\lambda, T) = \epsilon_\lambda \frac{C_1}{\lambda^5 \left[ \exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]} \quad (7)$$

where  $C_1, C_2$  are the Planck's constants;  $E(\lambda, T)$  is the spectrum emission power of the object;  $T$  is absolute temperature;  $\epsilon_\lambda$  is emissivity;  $\lambda$  is wavelength. The object's emissivity formula form is designed as

$$\epsilon_\lambda = f(\lambda, T) \quad (8)$$

It can be seen from the above two equations that object's spectrum emission power is determined by temperature and emissivity.

To reappear object's color, three respective primary colors (which are linearly independent) are selected casually and compounded not in proportion so as to lead to different sense of color, which is principle of primary colors. The wavelengths of primary colors selected by International Light Commission are:  $\lambda_R = 700.0 \text{ nm}$ ,  $\lambda_G = 546.1 \text{ nm}$ ,  $\lambda_B = 435.8 \text{ nm}$ .

When the three primary colors are chosen and their units are decided, RGB complex curve will be obtained from experiments arranged by different colors.

If the light's power is  $p(\lambda)$ , the coefficients of three colors are described as follows:

$$\left. \begin{aligned} R &= \int_{380}^{780} r(\lambda) P(\lambda) d\lambda \\ G &= \int_{380}^{780} g(\lambda) P(\lambda) d\lambda \\ B &= \int_{380}^{780} b(\lambda) P(\lambda) d\lambda \end{aligned} \right\} \quad (9)$$

Eq.(9) shows that object's color caused by radiation is determined by its spectrum emission. By

measuring object's colors coefficient and solving the above equation with method of minimum squares, object's temperature and radiation coefficients can be deduced, which is the principle of primary colors for temperature measurement.

Considering factors of radiation transmission media<sup>[4]</sup>, optical image system and photoelectric conversion, the color coefficient formulas of the object with spectrum emission power  $E(\lambda, T)$ , which is measured by apparatus based on the principle of primary colors<sup>[5]</sup>, can be written as

$$\left. \begin{aligned} R &= K_0 \frac{V \exp(-kL)}{4(f/d)^2 + (1 + m/p)^2} \cos^4 \theta \int_{0.38}^{0.78} \bar{r}(\lambda) E(\lambda, T) d\lambda \\ G &= K_0 \frac{V \exp(-kL)}{4(f/d)^2 + (1 + m/p)^2} \cos^4 \theta \int_{0.38}^{0.78} \bar{g}(\lambda) E(\lambda, T) d\lambda \\ B &= K_0 \frac{V \exp(-kL)}{4(f/d)^2 + (1 + m/p)^2} \cos^4 \theta \int_{0.38}^{0.78} \bar{b}(\lambda) E(\lambda, T) d\lambda \end{aligned} \right\} \quad (10)$$

where  $\lambda$  is wavelength;  $T$  is absolute temperature;  $\bar{r}(\lambda)$ ,  $\bar{g}(\lambda)$ ,  $\bar{b}(\lambda)$  are distributing color coefficients of the RGB secondary color system<sup>[5]</sup>,  $K_0$  is photoelectric conversion factor;  $k$  is absorption coefficient of the air media (as a constant in visible range);  $L$  is distance between the object and the measuring apparatus;  $f$ ,  $d$ ,  $V$ ,  $m$ ,  $p$  are focus length, effective aperture, lens coefficient, reappearing modulus and aperture amplifying rate of the optical image system, respectively;  $\theta$  is angle between the main light axis of the object and the light axis of the lens.

The product left to the integrals is defined as  $\Phi = K_0 \frac{V \exp(-kL)}{4(f/d)^2 + (1 + m/p)^2} \cos^4 \theta$  and is called the system parameter of the radiation measurement apparatus.

Some methods and techniques will be used to simplify Eq.(10) so that we can obtain relation between true color and  $T$  directly. To every obtained sectioning picture, we will achieve temperature of every dot. As we know, there exists the color difference between real and black bodies described as  $e_b(\lambda, T)$ . To a section which color coordinate is  $r$ ,  $g$ ,  $b$ , there exists

$$\frac{r_0 - r}{r_0^* - r\Omega} = \frac{g_0 - g}{g_0^* - g\Omega} \quad (11)$$

where

$$\begin{aligned} r_0 &= \frac{\int_{0.38}^{0.78} \bar{r}(\lambda) E_b(\lambda, T) d\lambda}{\int_{0.38}^{0.78} [\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)] E_b(\lambda, T) d\lambda} \\ g_0 &= \frac{\int_{0.38}^{0.78} \bar{g}(\lambda) E_b(\lambda, T) d\lambda}{\int_{0.38}^{0.78} [\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)] E_b(\lambda, T) d\lambda} \end{aligned}$$

$$\begin{aligned} r_0^* &= \frac{\int_{0.38}^{0.78} \bar{r}(\lambda) e_b(\lambda, T) d\lambda}{\int_{0.38}^{0.78} [\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)] E_b(\lambda, T) d\lambda} \\ g_0^* &= \frac{\int_{0.38}^{0.78} \bar{g}(\lambda) e_b(\lambda, T) d\lambda}{\int_{0.38}^{0.78} [\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)] E_b(\lambda, T) d\lambda} \\ \Omega &= \frac{\int_{0.38}^{0.78} [\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)] e_b(\lambda, T) d\lambda}{\int_{0.38}^{0.78} [\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)] E_b(\lambda, T) d\lambda} \end{aligned}$$

and  $r = \frac{R}{R + G + B}$ ,  $g = \frac{G}{R + G + B}$ ,  $E_b(\lambda, T) = \frac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]}$ , so only parameter  $T$  is unknown.

## 2 Experimental Setup

The experimental setup is shown in Fig.2. The optical lens is composed of two lenses with a conjunction, the depth of field is of the order millimeter. CCD and lens, pushed by a leading screw, are established on a sliding plate. A stepper motor controls the leading screw. When the motor rotates a cycle, the screw goes forward a pitch of 0.8mm. If the stepper frequency is 400Hz, the minimum control precision is 2μm. The image-grabbing speed of the image card is 25frame/s.

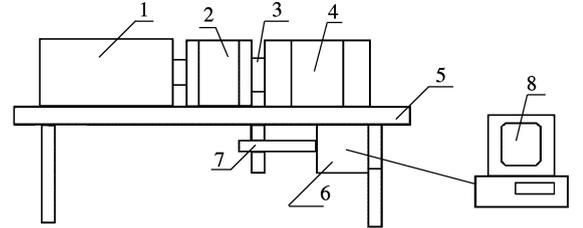


Fig.2 Experimental setup. 1. CCD camera; 2. Lens 1; 3. Conjunction; 4. Lens 2; 5. Lead stick; 6. Stepper motor; 7. Bolt; 8. Computer

In order to verify the correctness of this technique, some simulation experiments are performed. A candle flame is used as the experimental flame. To keep the stability of the flame, a windshield is employed. The dimension of flame is about 10mm. Optical sectioning is beginning from the margin of the flame, and the sectioning interval is 2mm. 5 sectioning images are captured. These images are very similar and difficult to distinguish.

## 3 Experimental Result

Fig.3 is the captured candle color image. It can be seen that the flame has been sectioned evidently. Fig.4 is the inverted result. It can be found that these images are different. The forms of these images from

left to right coincide with those of sectioned images of candle. Fig.5 is temperature distribution of the candle flame from the inverse result according to Plank's law mentioned above.

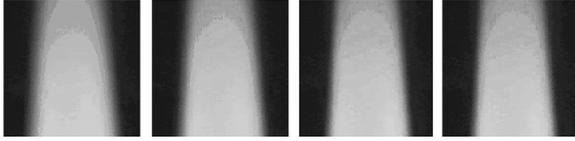


Fig.3 Original sectioning flame

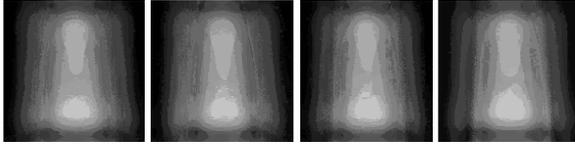


Fig.4 Inversed sectioning flame

## 4 Conclusion

This paper describes the theory of the optical sectioning thermography to reconstruct 3-D temperature distribution in flame. From the experimental results we can conclude that this technique has many advantages,

such as only one CCD camera system is used so that the system has simple and reliable configuration, focusing interval is  $2\text{?}\mu\text{m}$ , so the method has a very high spatial resolution and the temporal resolution is related to image capturing speed and focusing speed. After further development, the system will be a useful instrument for measurement.

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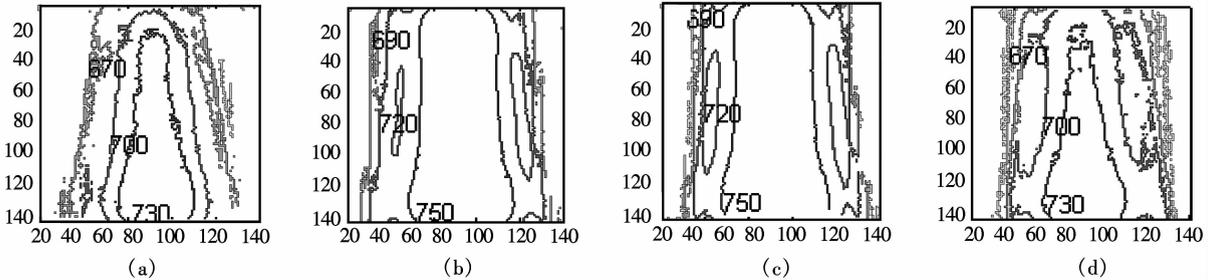


Fig.5 The reconstructed temperature field. (a) The first reconstructed temperature field; (b) The second reconstructed temperature field; (c) The third reconstructed temperature field; (d) The fourth reconstructed temperature field (Unit: pixel)

# 光学分层成像法重建火焰温度场的理论和实验研究

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**摘要** 提出了一种测量三维火焰温度的新方法.根据光学分层成像理论,三维火焰可以看成若干个互相平行的二维发光断层的组合.用高速摄像控制系统沿着某一固定方向对火焰进行分层聚焦摄像,得到一组辐射图像,每个图像都是其对应断层的聚焦像和其它断层离焦像的叠加像,运用图像反演算法,即可重建各断层的原始图像,再用彩色三基色测温方法,处理所得到的原始图像,即可建立火焰的三维温度场.通过蜡烛火焰的试验,验证了该方法的可行性.

**关键词** 光学分层成像, 温度场重建, 三基色

**中图分类号** TK311