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An Uncertainty Assessment Approach for Measuring Flatness Error in Close Way

Wen Xiulan* Zhou Xingpeng

(Department of Instrument Science and Engineering, Southeast University, Nanjing 210096, China)

Abstract: It is necessary for precise measurement to estimate the uncertainty of measurement result. When measuring flatness error in close way by pitch, usually the uncertainty of measurement result is independently estimated according to pitch points. By analyzing a concrete example, this paper proposed that the uncertainty should be evaluated by the correlation calculating method. This approach greatly improved the deficiencies of the assessment method according to independent measurement and enhanced measurement precision. It provides a reference value for uncertainty assessment in leveling a flat.

Key words: close way, flatness error, uncertainty

In general, when measuring flatness error by pitch in close way, the uncertainty of measurement result is usually estimated independently according to the pitch points[1], because the measurement data of every pitch are independent of one another. As showed in Fig.1, x_1, x_2 are the highest points, and x_3, x_4 are the lowest points, and after being measured by a horizontal instrument, the flatness error F of the flat should be calculated by[2]

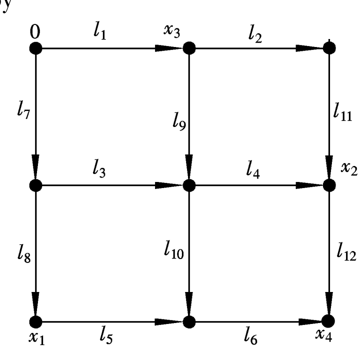

Fig.1 Arranging lines

$$F = \frac{1}{5}\left(Z_{x_1} + 4Z_{x_2} - 2Z_{x_3} - 3Z_{x_4}\right) \tag{1}$$

where Z_{x_1}, Z_{x_2}, Z_{x_3} and Z_{x_4} are the height differences between the pitch points x_1, x_2, x_3, x_4 and the starting point.

According to usual method, assuming Z_{x_1}, Z_{x_2}, Z_{x_3} and Z_{x_4} are independent, the uncertainty of measurement result u_F is given by[3-5]

$$u_F = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial F}{\partial x_i}\right)^2 u_{x_i}^2} \tag{2}$$

The partial derivative describes how the output quantity is related to each of the input quantities. The uncertainty of the measurement result is thus given by adding the uncertainty multiplied by the squares of the relevant partial derivatives, corresponding to the different point uncertainty. The square root of the uncertainty is a measure of the combined uncertainty of the output quantity F.

According to Eq.(2), the uncertainty of this example is expressed as

$$u_F = \frac{1}{5}\sqrt{u_{x_1}^2 + 16u_{x_2}^2 + 4u_{x_3}^2 + 9u_{x_4}^2} \tag{3}$$

Assuming the measurement data of every pitch is equivalent accuracy and its standard uncertainty is σ, the uncertainty of every measuring point can be estimated on the basis of the close way, then the uncertainty of x_1, x_2, x_3 and x_4 are as follows, respectively[1]: $u_{x_1} = 1.12\sigma$, $u_{x_2} = 1.10\sigma$, $u_{x_3} = 0.84\sigma$, $u_{x_4} = 1.22\sigma$.

Substitute them for the equivalents in Eq.(3), then

$$u_F = 1.214\sigma \tag{4}$$

The relation between Z_{x_1}, Z_{x_2}, Z_{x_3}, Z_{x_4} and the measurement data $\boldsymbol{L}$ is expressed by matrix

$$\boldsymbol{Z} = \begin{bmatrix} Z_{x_1} \\ Z_{x_2} \\ Z_{x_3} \\ Z_{x_4} \end{bmatrix} = \begin{bmatrix} l_7 + l_8 \\ l_3 + l_4 + l_7 \\ l_1 \\ l_5 + l_6 + l_7 + l_8 \end{bmatrix} \tag{5}$$

In Eq.(5), the measurement data of pitch l_1, l_2, l_3, l_4, l_5, l_6, l_7, and l_8 are independent of one another, whereas, Z_{x_1}, Z_{x_2}, Z_{x_3} and Z_{x_4} are by no

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* Born in 1966, female, graduate, associate professor.

means independent, they are correlative. When calculated in view of Eq.(2), the correlation items are neglected, therefore the value of u_F is usually a bit larger.

1 Correlation Calculating Method of Uncertainty

As the height difference $\boldsymbol{Z}$ between each pitch point and the starting point can be indicated as linear function of the measurement data of pitch $l_1, l_2, \cdots, l_n$, then it can be expressed as

$$\boldsymbol{Z} = \boldsymbol{B}\,\boldsymbol{L} \tag{6}$$

where $\boldsymbol{B}$ is the coefficient matrix of the measurement data $\boldsymbol{L}$; $\boldsymbol{L}$ is the $n \times 1$ vector of the measurement data.

$$\boldsymbol{L} = [\,l_1, l_2, \cdots, l_n\,]^{\mathrm{T}} \tag{7}$$

In comparison with Eqs. (5) and (6), apparently, here

$$\boldsymbol{Z} = \begin{bmatrix} Z_{x_1} \\ Z_{x_2} \\ Z_{x_3} \\ Z_{x_4} \end{bmatrix} = \begin{bmatrix} 0&0&0&0&0&0&1&1&0&0&0&0 \\ 0&0&1&1&0&0&1&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&1&1&0&0&0&0 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_{12} \end{bmatrix} \tag{8}$$

So the coefficient matrix $\boldsymbol{B}$ of the measurement data in this example is

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \\ \boldsymbol{B}_3 \\ \boldsymbol{B}_4 \end{bmatrix} = \begin{bmatrix} 0&0&0&0&0&0&1&1&0&0&0&0 \\ 0&0&1&1&0&0&1&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&1&1&1&0&0&0&0 \end{bmatrix} \tag{9}$$

If the height value Z_i of a pitch point x_i is the linear function of $\boldsymbol{L}$, its formula of vector is

$$Z_i = \boldsymbol{B}_i\boldsymbol{L} \tag{10}$$

The height value Z_j of another pitch point x_j is also the linear function of $\boldsymbol{L}$, its formula of vector is

$$Z_j = B_j\boldsymbol{L} \tag{11}$$

The covariance between x_i and x_j can be expressed as[6]

$$\mathrm{cov}(x_i, x_j) = \boldsymbol{B}_i\,\boldsymbol{D}(\boldsymbol{L})\,\boldsymbol{B}_j^{\mathrm{T}} \tag{12}$$

where $\boldsymbol{D}(\boldsymbol{L})$ is the variance matrix of the measurement data $\boldsymbol{L}$. On the condition that the measurement data $l_1, l_2, \cdots, l_n$ are equivalent accuracy independent, then the variance matrix $\boldsymbol{D}(\boldsymbol{L})$ is

$$\boldsymbol{D}(\boldsymbol{L}) = \begin{bmatrix} \sigma_1^2 &&& \\ & \sigma_2^2 && \\ && \ddots & \\ &&& \sigma_n^2 \end{bmatrix} = \begin{bmatrix} \sigma^2 &&& \\ & \sigma^2 && \\ && \ddots & \\ &&& \sigma^2 \end{bmatrix} = \boldsymbol{E}\sigma^2 \tag{13}$$

where $\boldsymbol{E}$ is a unitary matrix; σ is the variance of the measurement data of every pitch. Then the covariance matrix can be computed from[7]

$$\mathbf{cov} = \boldsymbol{BD}(\boldsymbol{L})\boldsymbol{B}^{\mathrm{T}} \tag{14}$$

Substitute $\boldsymbol{B}$ in the above formula with Eq.(9), we have

$$\mathbf{cov} = \begin{bmatrix} 2&1&0&2 \\ 1&3&0&1 \\ 0&0&1&0 \\ 2&1&0&4 \end{bmatrix}\sigma^2 \tag{15}$$

Judging from Eq.(15), we know that the covariance items between x_1, x_2, x_3, and x_4 are not all zeros, so the uncertainty u_F of the measurement result of flatness should be calculated as[3]

$$u_F = \left[\sum_{i=1}^{N}\left(\frac{\partial F}{\partial x_i}\right)^2 u_{xi}^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial F}{\partial x_i}\frac{\partial F}{\partial x_j}\mathrm{cov}(x_i, x_j)\right]^{\frac{1}{2}} \tag{16}$$

According to Eq. (16), the uncertainty of this example should be expressed as

$$u_F = \frac{1}{5}\Big[\,u_{x_1}^2 + 16u_{x_2}^2 + 4u_{x_3}^2 + 9u_{x_4}^2 + 2(4\mathrm{cov}(x_1, x_2) - 3\mathrm{cov}(x_1, x_4) - 12\mathrm{cov}(x_2, x_4))\Big]^{\frac{1}{2}} = 0.594\sigma \tag{17}$$

Compare the result of Eq.(4) with Eq.(17), its ratio is

$$1.214/0.594 = 2.04$$

So it is obvious that the accuracy lost is too large if the uncertainty is estimated according to independent measurement.

2 Conclusion

From the above analysis, we know that when flatness error is measured by pitch in close way, though the measurement of every pitch is independent, the height differences between every pitch and starting point are not independent, for all of them are the function of the measurement data $\boldsymbol{L}$. So it's better to be evaluated by correlation method. It can minimize the measurement result uncertainty. The correlation method can also be used in the uncertainty assessment when measuring straight linearity error by pitch.

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封闭法测平面度误差时不确定度的计算方法

温秀兰　　周杏鹏

（东南大学仪器科学与工程系,南京 210096）

摘　要　在精密测量中要求对测量结果的不确定度进行评估.本文通过实例分析,指出用封闭法测平面度误差时,测量结果不确定度应该按相关计算法求得.这一方法大大改善了按独立测量计算不确定度存在的问题,提高了测量精度.该法对平板检定具有一定的参考价值.

关键词　封闭法, 平面度误差, 不确定度

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