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Compete Matching Pursuits Algorithm*

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Abstract: Matching pursuits algorithm (MP), as an adaptive signal representation upon overcomplete fundamental waveforms, is a powerful tool in many applications. However, MP suffers from distinguishing a doublet structure. In this paper, the authors proposed an algorithm called compete matching pursuits (CMP), which can overcome this shortcoming and performance very well.

Key words: matching pursuit, compete matching pursuit, overcomplete representations

Signal expansion is one of the major approaches in many applications. The traditional way is to represent a signal by complete bases, where signal decomposition is well defined and unique. An alternative and potentially more general method of signal representation uses an overcomplete dictionary, which allows a greater number of bases than samples in the input signal. Under an overcomplete dictionary, the decomposition of a signal is not unique, because some bases in the dictionary have representations in terms of other bases, which give us the possibility of adaptation. In addition, it is possible for us to obtain a representation which possesses both sparsity and superresolution.

It is a challenging job to represent a signal upon an overcomplete dictionary. Here the common idea is to find the best matched atoms by evaluating the degree of matching between the signal and different combinations of atoms, which will result in insufferable computational burden. Therefore, we should seek some other methods which have both high performance and acceptable computational burden. Several methods are already available in finding the "optimal" representation of a signal by an overcomplete dictionary. These ranges from general approaches, such as frames[1], basis pursuit[2] (BP) and matching pursuit[3], or clever schemes derived for specialized dictionaries, such as the best orthogonal basis[4] and high-resolution pursuit[5]. All these methods have their advantages and shortcomings. For example, basis pursuit, based on global optimization, may have better performance than other methods but its implementation is computationally intensive. Though MP does not have the same good performance as BP, it also works well and its computational complexity is much less than BP. MP has been used in many areas successfully, especially in video coding. We will focus on the improvement of matching pursuit.

This paper is organized as follows. In section 1, the matching pursuit algorithm for achieving overcomplete expansions is reviewed and discussed. In section 2, we introduce compete matching pursuit, which will reduce the effect from drawback of MP. Section 3 presents numerical examples of the performance of compete matching pursuit.

1 Matching Pursuits

The concept of matching pursuits lies in decomposing a signal into superposition of basic functions (words) g_r ($\| g_r \| = 1$) that belongs to a redundant dictionary $\boldsymbol{D}$. After greedily choosing a $g_{r_0} \in \boldsymbol{D}$, the signal f can be decomposed into

$$f = \langle f, g_{r_0} \rangle g_{r_0} + Rf \tag{1}$$

where Rf is the residual vector after approximating f in the most matched direction. Clearly g_{r_0} is orthogonal to Rf, hence

$$\| f \|^2 = \left| \langle f, g_{r_0} \rangle \right|^2 + \| Rf \| \tag{2}$$

To minimize $\| Rf \|$, one might choose g_{r_0} such that $\left| \langle f, g_{r_0} \rangle \right|$ is maximum or suboptimal in some sense. Then, one can substitute this residual vector to original f to repeat this procedure. The operation continues iteratively until either a collection of expansion coefficients is generated or some energy threshold for the residual is reached. After m times iterations, the signal f can be decomposed into

$$f = \sum_{k=0}^{m-1} \langle R^n f, g_{r_k} \rangle g_{r_k} + R^m f \tag{3}$$

If we stop the algorithm at this stage and only

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record the partial structure book $(\langle R^n f, g_{r_k}\rangle, r_k)_{0\leqslant k<m}$ the summation of (3) recovers an approximation of f, with an error equal to $R^m f$.

MP algorithm works well for many types of signals. But the greedy optimization algorithm is myopic, which misses entirely the doublet structure. The following example illustrates this drawback in which we use cubic b-splines. Scared versions of this cubic b-spline are of the form $g(2^j x)$. The twin peaks function f, illustrated in Fig.1(a) with dashed, is the sum of two cubic b-splinesat the same scale but different position. Let the dictionary $\boldsymbol{D}$ consist of cubic b-splines at a wide range of translates and scales, including those used to construct f. Take the twin peaks as an example, the first element chosen by MP is the one which does not match either of the two functions that are the true componets of f. Fig.1(a) shows the original function dashed and the first element chosen by MP (solid). The first MP residual is shown in Fig.1(b) (solid). Since MP chose a "wrong" atom in the first iteration, it is forced to choose a serial of atoms to correct this error.

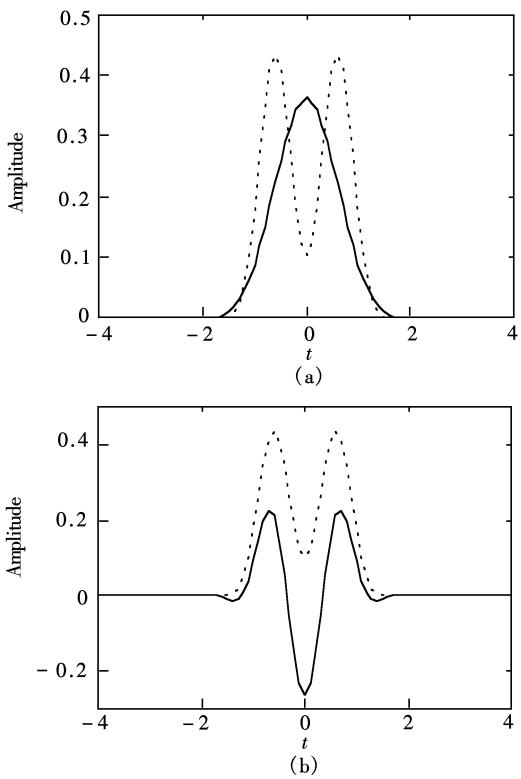

Fig.1 Process of MP for twin peaks function. (a) Twin peaks function and first element chosen by MP; (b) First residual generated by MP

2 Compete Matching Pursuits

We know that MP misses the doublet structure because of its myopia, an improved method, namely compete matching pursuits(CMP), is proposed here to help MP work correctly as the following.

Step 1 The first two most contributive atoms w_1^1 and w_2^1 are selected from the same dictionary as used in MP, where

$$w_1^1 = \arg\max_{g\in D} |\langle f, g_r\rangle|$$

$$w_2^1 = \arg\max_{g_r\in D, g_r\neq w_1^1} |\langle f, g_r\rangle|$$

After that, two residual signals, $R_1^1 f$ and $R_2^1 f$, can be attained

$$R_1^1 f = f - \langle f, w_1^1\rangle w_1^1$$

$$R_2^1 f = f - \langle f, w_2^1\rangle w_2^1$$

Step 2 w_{11}^2 and w_{12}^2 are selected for $R_1^1 f$ as the two most contributive atoms, while w_{21}^2 and w_{22}^2 for $R_2^1 f$. After that, four residual signals, $R_{11}^2 f$, $R_{21}^2 f$, $R_{21}^2 f$ and $R_{22}^2 f$, can be attained.

Step 3 The energy of the four residual signals is computed and evaluated, and then two residual signals, whose energy is the smallest one and the sub-smallest one, are selected. $R_1^2 f$ is used to represent the residual signal whose energy is the smallest, and $R_2^2 f$ for the sub-smallest one. Atoms w_1^2 and w_2^2 correspond to $R_1^2 f$ and $R_2^2 f$, respectively. Only $R_1^2 f$ and $R_2^2 f$ are kept for future decomposition.

Step 4 The two residual signals, $R_1^2 f$ and $R_2^2 f$, are further decomposed using the similar methods in step 2 and step 3. The rest may be deduced by analogy until the energy of one of the residual signals is smaller than some threshold set in advance or the number of atoms is just enough.

Fig.2 is an example of how CMP working. If we want to use one (two, three, respectively) atom to represent signal f, we should choose w_1^1 (w_1^1 and w_1^2, w_2^1; w_2^2 and w_1^3 respectively). The bold line in Fig.1 shows the choice of four atoms by CMP.

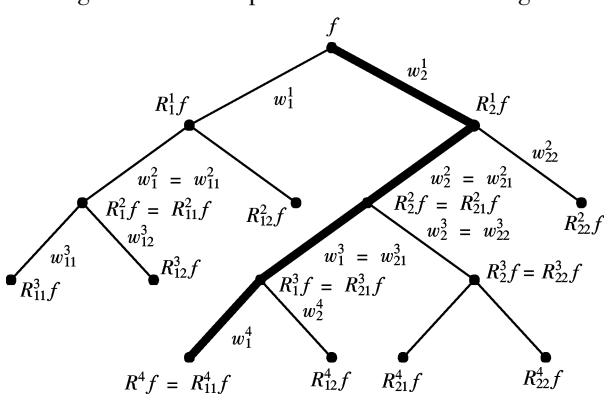

Fig.2 An example of compete matching pursuit

The algorithm mentioned above selects the first two most contributive atoms to decompose a signal respectively at every step. We call it two-branch CMP.

This algorithm can be easily expended to n-branch CMP by using first n contributive atoms to decompose signal respectively at every step. The complexity of the n-branch CMP is about n times that of MP.

3 Compare CMP with MP

The general MP algorithm cannot distinguish doublet structures. We then use CMP to decompose the same twin peaks function with the same dictionary used in MP and see the performance of CMP. Fig.3 illustrates the first two elements chosen by two-branch CMP and four-branch CMP, where the CMP can capture the main feature of twin peaks function and four-branch CMP has much better performance than that of two-branch CMP in this example. Since CMP chooses two reasonable elements in the first two stages, subsequent elements serve to refine the fit rather than to correct mistakes from previous stages. When branches increase in CMP algorithm, it can overcome the drawback of matching pursuit of missing doulet structures eventually.

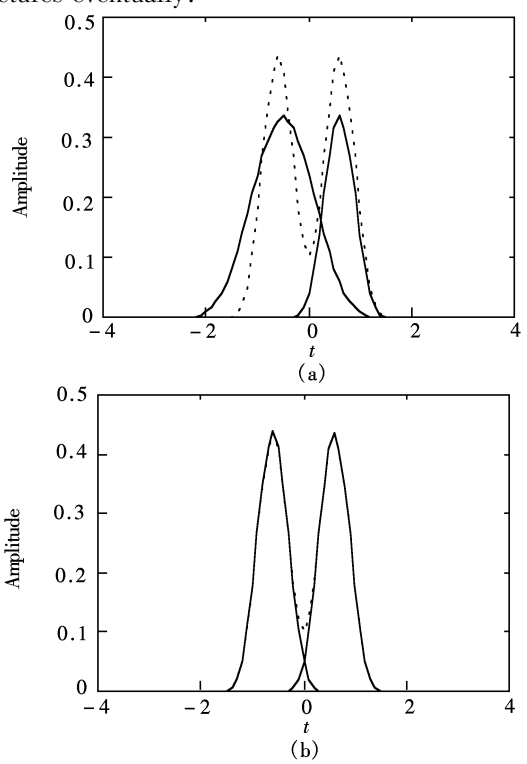

Fig.3 Process of CMP for twin peaks function. (a) Twin peaks function and first two elements chosen by two-branch CMP; (b) Twin peaks function and first two elements chosen by four-branch CMP

To illustrate the energy compaction property of MP and CMP, consider the following situation. A source, which is randomly generated according to a uniform distribution on the N-dimension unit sphere, is to be transform coded. The dictionaries are also randomly generated on the N-dimension unit sphere. Fig.4 shows the results of a simulation with $N = 8$ and using 4 terms to represent the source. The plot shows the fraction of the signal energy in the residual when using MP(dashed), two-branch CMP(solid) and six-branch CMP(doted). Dictionary size M ranges from 8 to 2 048.

We can see from Fig.4 that the performance of CMP is higher than that of MP. The performance of CMP is improved with the increase of branches.

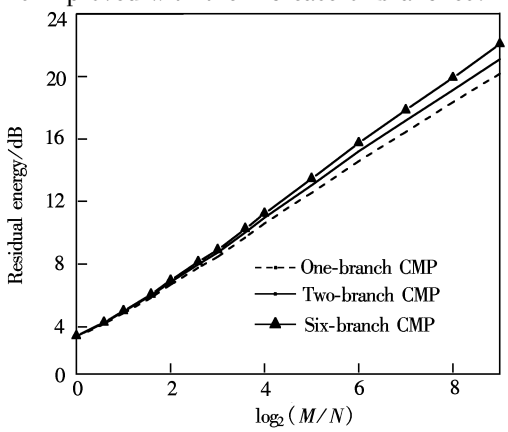

Fig.4 Comparison of energy compaction property with different branches of CMP

4 Conclusion

MP cannot resolve closely spaced features and basis pursuit is computationally intensive. A revised version of MP, CMP is developed and demonstrated in this paper. MP uses greedy algorithm to pick the most contributive atom at each step. CMP still uses greedy algorithm, but it will use the result of the current step to decide which atom it will use in the last step. By this way, CMP can see faraway than MP, so it overcomes the drawback of MP in some sense.

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竞争匹配追踪算法

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摘　要　匹配追踪算法是一种从一个极度冗余的词典中选择出某些基向量来叠加出一个特定的信号的算法.这种算法已经成功地用于视频压缩和其它领域中.但由于匹配追踪算法本质上是一种贪心算法,它的主要缺陷是无法分辨出信号中存在的双峰结构.本文提出了一种改进算法,称为竞争匹配追踪算法.这种算法能够克服上述匹配追踪算法的缺陷,并且在同等情况下能达到更优的性能.

关键词　匹配追踪,竞争匹配追踪,超完备表示

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