

On-Off Intermittency Route to Chaos Synchronization and Spatial Periodic Synchronization of Chaos in Coupled Arrays of Chaotic Systems^{*}

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Abstract: Chaos synchronization of coupled nonlinear systems is ubiquitous in nature and science. Dynamic behaviors of coupled ring and linear arrays of unidirectionally coupled Lorenz oscillators are studied numerically. We find that chaos synchronization in circular arrays of chaotic systems can occur through the on-off intermittent synchronization with a power-law distribution of laminar phases. And in the coupled ring and linear array it is found that the chaotic rotating waves generated from the ring propagate with spatial periodic synchronization along the linear array.

Key words: Lorenz oscillator, chaos synchronization, intermittency chaos, spatio-periodic chaos

Early work on synchronous, coupled systems was done by H. Fujisaka and T. Yamada^[1]. Later, in 1990, L. M. Pecora and T. L. Carroll^[2] realized the chaos synchronization in a real set of electronic chaotic circuit. Thus, the field of chaotic synchronization has grown considerably since 1990. The understanding of the chaos synchronization in coupled chaotic oscillators is essential for a wide range of scientific investigations.

Ring and linear arrays are two main coupling models in the study of chaos synchronization since their potential applications in communication and neural process. So our work is mainly about ring and linear arrays. In this paper, we present numerical studies of chaos synchronization in the ring of coupled chaotic Lorenz systems, and show that the permanent chaos synchronization^[2-6] can be achieved through the on-off intermittent chaos synchronization^[7,8] in certain region of coupling constant. This relation is closely related to the intermittent route to chaos of a single Lorenz oscillator in the ring. Also we find that the chaotic rotating waves generated from the ring propagate with spatial periodic synchronization along the linear array.

1 On-off Intermittency Route to Chaos Synchronization in Circular Arrays^[9]

The relation of permanent and on-off intermittent chaos synchronization is the subject of this section.

The dynamical evolution equations for the unidirectionally coupled Lorenz oscillators in circular geometry can be written as^[4]

$$\left. \begin{aligned} \dot{x}_j &= \sigma(y_j - x_j) \\ \dot{y}_j &= R(x_j + \alpha(x_{j-1} - x_j)) - y_j - x_j z_j \\ \dot{z}_j &= x_j y_j - b z_j \end{aligned} \right\} \quad (1)$$

where $j = 1, \dots, N$; $0 \leq \alpha \leq 1$; and $x_0 = x_N$. Eq.(1) have been studied numerically for $N = 3$ and rich nonlinear dynamical behaviors were observed, such as stable periodic rotating waves, chaotic rotating waves, permanent chaos synchronization^[2-6] and transient chaotic rotating waves.

All of the nonlinear dynamical behaviors were computed with the values $\sigma = 20$, $b = 2.5$, and $R = 35$, for which the isolated Lorenz oscillator is in a chaotic state. A typical result of $x_1(t) - x_2(t)$ for $N = 3$ and $\alpha = 0.151$ is depicted in Fig.1. The key characteristic in this figure is that the chaos synchronization is intermittent. While the on-off intermittent

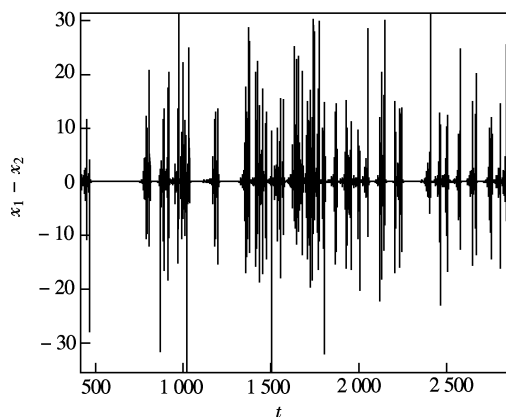


Fig. 1 The time evolution of $x_1 - x_2$ with $\sigma = 20$, $b = 2.5$, $R = 35$, and $\alpha = 0.151$

synchronizations are described by the power-law distributions of the laminar phases^[10]. The interleaved intervals of synchronized and unsynchronized dynamics can be saved and then sorted according to the duration of the individual segments. The probability distribution $P(\tau)$ of the laminar phase is shown in Fig.2. The fit to these numerical results gives a power-law

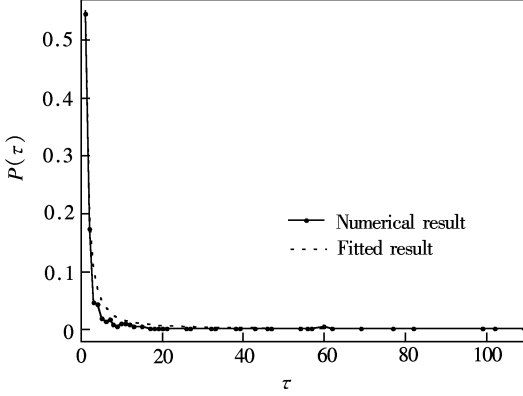


Fig.2 Distribution of laminar phases with the same parameters as in Fig.1, the solid line is the numerical result and the dashed line is the fitted result

distribution of τ , $P(\tau) \sim \tau^{-3/2}$, which is known as one of the signature of on-off intermittency^[10]. This power-law distribution holds in the coupling region of $0.120 \leq \alpha \leq 0.160$. When $\alpha \geq 0.170$, the permanent chaos synchronization appears, that is the on-off intermittency route to permanent chaos synchronization occurs when the coupling constant α changes increasingly from 0.100 to 1.000. Further studies show that the on-off intermittency route to chaos synchronization is closely related to the intermittency route to chaos of a single Lorenz oscillator in the ring. The behaviors of a single oscillator, for example x_1 , change with increasing α from modulated periodic wave ($\alpha = 0.100$), through intermittent chaos ($0.120 \leq \alpha \leq 0.160$), to strange attractor ($0.170 \leq \alpha \leq 1.000$).

In Fig.3, $x_1(t)$ oscillates around one stable point with quasiperiod, after a time interval, it changes to chaotic oscillations, and then it changes to one of the two stable points randomly, then to chaos, and so on. The time intervals for both quasiperiodic and chaotic oscillations are random. The chaos unsynchronizations of neighboring oscillators occur when chaotic oscillations change to quasiperiodic oscillations. These unsynchronous intervals are labeled in Fig.3 by arrows. From above discussions, we find that the sudden change of dynamical behaviors can result in chaotic unsynchronizations in any chaotic systems.

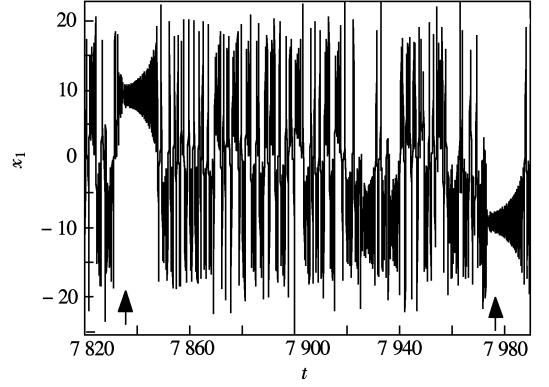


Fig.3 The time evolution of x_1 with the same parameters as in Fig.1

2 Spatial Periodic Synchronization in the Coupled Linear and Circular Arrays^[11]

In the above and many other studies on coupled chaotic oscillators mainly focused on the instability of the uniform synchronous state, in which either all oscillators synchronize to the drive or all of them do not synchronize in the long time region^[2,3,5]. In this section we couple a ring and a linear array and numerically study chaos synchronization of the linear array.

A scheme of our coupling geometry is shown in Fig.4. In the coupling geometry the ring can be treated

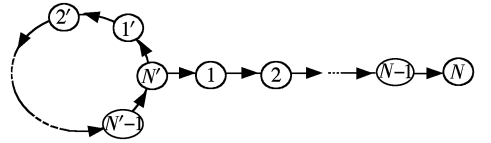


Fig.4 Geometry of the coupled ring and linear arrays of Lorenz systems, explained in the text

as the external drive and the linear array as the response system. All oscillators both in the ring and in the linear array are identical, and connected unidirectionally through variable x with the same coupling strengths. The arrow indicates an oscillator drives the one closely behind it, that is to say, chaotic waves unidirectionally propagate in the arrays. With these considerations we have the following evolutions for the system (just another formation of Eq.(1)):

$$\left. \begin{aligned} \dot{x}_j &= \sigma(y_j - x_j) \\ \dot{y}_j &= R(\alpha \bar{x}_j + (1 - \alpha) - x_j) - y_j - x_j z_j \\ \dot{z}_j &= x_j y_j - b z_j \end{aligned} \right\} \quad (2)$$

where $\bar{x}_j = x_{j-1}$ for $j \neq 1$, the coupling strength α allows one to control the stability of the connection, $j = 1, \dots, N; 1', \dots, N'$ and $0 \leq \alpha \leq 1$. The boundary conditions enter through \bar{x}_j , that takes the value $\bar{x}_{1'} = x_{N'}$ for circular arrays, while for linear arrays it is $\bar{x}_1 =$

x_N in our system. The oscillators are labeled by $k' = 1', 2', 3', \dots, N'$ in the ring, and $l = 1, 2, 3, \dots, N$ in the linear. Size of the ring is $m = N'$.

We mainly focus on the special case, where $m = 3$, $N \rightarrow \infty$, $\sigma = 20$, $b = 2.5$, and $R \geq 28$. Taking $\alpha = 1$, and $R = 35$, we find that three different chaotic rotating waves with a $2\pi/3$ phase difference of neighboring oscillators appear in the ring^[5], see Fig.5.

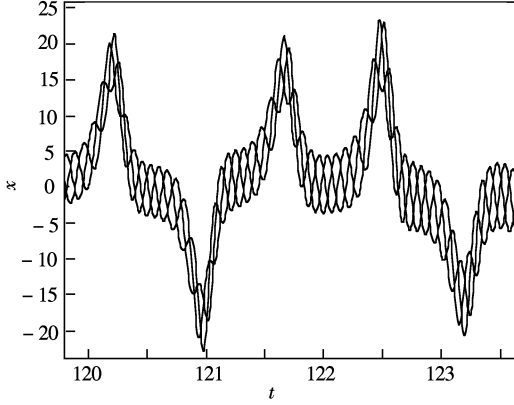


Fig.5 Diagram of x vs. t in the coupled ring and linear array calculated according to Eq. (2), showing chaotic rotating wave

Although the three CRWs are similar, the fine details such as the positions and heights of the chaotic wave peaks are different. Any three neighboring oscillators in the linear array response to the ring and exhibit the same behavior as that in the ring in the long time region. The noncontiguous oscillators k' and $l_0 + nm$ become synchronized with spatial period m ($m = 3$ in our special case) in the linear array, here $k' = 1', 2', 3'$; $l_0 = 1, 2, 3$; $n = 0, 1, 2, \dots$ and $k = l_0$, for example, oscillators $1', 1, 4, 7, 10, 13, \dots$ are all synchronized. Numerically calculations show that stabilities of the synchronization decrease with increasing n , that is, the stabilities of synchronization becomes weaker and weaker down the linear array.

Further study shows that this phenomenon can be observed for arbitrary size of the ring ($m \geq 3$, and we have tested this phenomenon up to the size $m = 60$). In each case, m different chaotic waves (not always CRWs) with a $2\pi/m$ phase difference of neighboring oscillators generated in the ring and propagating with spatial period m in the linear array are shown.

3 Conclusion

We have numerically studied the dynamical behaviors of unidirectionally coupled chaotic Lorenz oscillators in circular geometry, and show that the

system can result in intermittent chaos synchronization of neighboring oscillators with a power-law distribution $P(\tau) \sim \tau^{-3/2}$ of laminar phases. By changing the coupling constant α from 0.120 to 0.170, the intermittent synchronization approaches to permanent chaos synchronization. This on-off intermittency route to chaos synchronization is closely related to the intermittency route to chaos of a single Lorenz oscillator in the ring.

In the coupled ring and linear array system, chaotic synchronization occurs with spatial period m (the number of units in the ring) in the linear array due to the response of the linear array to the ring in which the chaotic rotating waves are generated. This result is correct for other waves.

Our results may have potential applications in neural processes, communications, and information processing systems. How to design chaos synchronizing systems and find their applications in science and technology remains an interesting topic for future research.

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耦合混沌系统中开关间歇性同步途径 趋于混沌同步及空间周期性混沌同步

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摘 要 自然界与科学研究中耦合的非线性系统的混沌同步普遍存在. 本文研究了单向耦合的环型和线型 Lorenz 振子的动力学行为. 我们发现环型耦合的混沌系统通过开关间歇性同步趋向混沌同步, 此间歇性同步符合层流相指数幂分布规律. 这种趋向混沌同步的转变与环型中单个 Lorenz 振子由间歇性混沌趋向混沌的途径有密切关系. 在环型、线型耦合中, 环型中产生的混沌旋转波与其在线型中传播产生的波空间周期性同步.

关键词 Lorenz 振子, 混沌同步, 间歇性混沌, 空间周期性混沌

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