Absolute Exponential Stability of Generalized Dynamical Neural Networks *

Sun Changyin ** Fei Shumin Feng Chunbo

(Research Institute of Automation, Southeast University, Nanjing 210096, China)

Abstract: This paper investigates the absolute exponential stability of generalized neural networks with a general class of partially Lipschitz continuous and monotone increasing activation functions. The main obtained result is that if the interconnection matrix $\boldsymbol{T}$ of the neural system satisfies that $-\boldsymbol{T}$ is an $\boldsymbol{H}$-matrix with nonnegative diagonal elements, then the neural system is absolutely exponentially stable(AEST). The Hopfield network, Cellular neural network and Bidirectional associative memory network are special cases of the network model considered in this paper. So this work gives some improvements to the previous ones.

Key words: absolute exponential stability, partial Lipschitz continuity, neural networks

For an absolutely stable (ABST) neural network has the ideal characteristics that for any neuron activation in a proper class of sigmoid functions and other network parameters the network has a unique and globally asymptotically stable (GAS) equilibrium point. Recently, the analysis of absolute stability of neural networks has been studied[1-9]. The ABST property of neural networks is very attractive in their applications for solving optimization problems, such as linear and quadratic programming, because it implies that the optimization neural networks are devoid of the spurious suboptimal responses for any activation functions in the proper class and other network parameters. The ABST neural networks are thus regarded as the most suitable ones for solving optimization problems.

The existing ABST results of neural networks in Refs. [1 - 8] were obtained within the classes of bounded and differentiable activation functions. However, in practical optimization applications, it is not uncommon that the activation functions in optimization neural networks are unbounded and/or nondifferentially as demonstrated in previous work[9-12]. Moreover, it is desirable that the neural networks are globally exponential stable at any prescribed exponential convergence rate[11-16]. Motivated by these, the analysis of absolute exponential stability of neural networks is deemed necessary and rewarding[9]. An absolutely exponentially stable (AEST) neural network means that the network has a unique and globally exponentially stable equilibrium point for any activation functions in the proper class and other network parameters.

The main purpose of this paper is to follow the idea obtained in Ref. [9] and provide an AEST result for generalized dynamical neural networks, which can be stated as follows: if the interconnection matrix $\boldsymbol{T}$ of the network system satisfies that $-\boldsymbol{T}$ is an $\boldsymbol{H}$-matrix with nonnegative diagonal elements, then the network system is AEST with respect to a general class of partially Lipschitz continuous and monotone increasing activation functions. The obtained AEST result of the generalized dynamical neural networks in the paper is first proposed in the literature.

1 Neural Network Model and Preliminaries

Consider the generalized dynamical neural network model described by the system of differential equations in the form

$$\dot{x} = -\boldsymbol{D}f(x) + \boldsymbol{T}g(x) + \boldsymbol{I} \tag{1}$$

where $x = (x_1, x_2, \cdots, x_n)^{\mathrm{T}} \in \boldsymbol{R}^n$, $\boldsymbol{D}$ is an $n \times n$ constant diagonal matrix with diagonal elements $d_i > 0$, $i = 1, 2, \cdots, n$, $\boldsymbol{T} = (T_{ij})$ is an $n \times n$ constant interconnection matrix, $f(x) = (f_1(x_1), f_2(x_2), \cdots, f_n(x_n))^{\mathrm{T}} \in \boldsymbol{R}^n$ and $f_i(x_i)$ $(i = 1, 2, \cdots, n)$ is defined as

$$f_i(x_i) = \begin{cases} m(x_i - 1) + 1 & x_i \geqslant 1 \\ x_i & |x_i| < 1 \\ m(x_i + 1) - 1 & x_i \leqslant -1 \end{cases} \tag{2}$$

where $m \geqslant 1$. $g(x) = (g_1(x_1), g_2(x_2), \cdots, g_n(x_n))^{\mathrm{T}}$: $\boldsymbol{R}^n \to \boldsymbol{R}^n$ is a nonlinear vector-valued activation

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** Born in 1975, male, graduate.

function and $\boldsymbol{I} = (I_1, I_2, \cdots, I_n)^{\mathrm{T}} \in \boldsymbol{R}^n$ is a constant input vector.

Assume that g belongs to the class PLI (partially lipschitz and increasing) of activation functions defined by the property that $g \in$ PLI if for $i = 1, 2, \cdots, n$, $g_i(x_i): \boldsymbol{R} \to \boldsymbol{R}$ is a partially Lipschitz continuous and monotone increasing function. A function $h(\rho): \boldsymbol{R} \to \boldsymbol{R}$ is said to be partially Lipschitz continuous in $\boldsymbol{R}$ if for any $\rho \in \boldsymbol{R}$ there exists a positive number l_ρ such that

$$|h(\theta) - h(\rho)| \leqslant l_\rho |\theta - \rho| \quad \forall \theta \in \boldsymbol{R} \quad (3)$$

It can be seen that a function $g \in$ PLI may be unbounded and/or non-differentiable. In Ref. [9], Liang and Wang have shown that several classes of activation functions in common use are special ones of the PLI class.

Model (1) serves as a general framework for neural network models. The neural network model includes some well-known networks as its special cases, for example,

1) When g_i is the Sigmoid function, $f_i(x) = x$, $i \in \{1, 2, \cdots, n\}$, model (1) turns to a Hopfield neural network;

2) When g_i is piecewise linear function, $f_i(x) = x$, $i \in \{1, 2, \cdots, n\}$, model (1) turns to a Cellular neural network[18,19];

3) When $\boldsymbol{D}$ is an identity matrix, $f_i(x) = x$, n is an even number and the weight matrices $\boldsymbol{T} = \begin{pmatrix} 0 & T_1 \\ T_2 & 0 \end{pmatrix}$, and $\boldsymbol{T}_1$, $\boldsymbol{T}_2$ are $(n/2) \times (n/2)$ matrices, model (1) reduces to a Bidirectional associative memory (BAM) network[20].

If $g \in$ PLI, then the vector field defined by the right hand of system (1), $-\boldsymbol{D}f(x) + \boldsymbol{T}g(x) + \boldsymbol{I}$, satisfies a local Lipschitz condition. By the theorem of local existence and uniqueness for the solutions of ordinary differential equations (ODE)[21], for any $x_0 \in \boldsymbol{R}^n$, there exists a unique solution of the autonomous system (1) denoted by $x(t, x_0)$ for $t \in [0; t^*(x_0))$ satisfying $x(0; x_0) = x_0$, where $t^*(x_0) \in (0, +\infty)$ or $t^*(x_0) = +\infty$ such that $[0, t^*(x_0))$ is the maximal right existence interval of the solution $x(t; x_0)$. It will be found, in section 2, that the solution $x(t; x_0)$ is actually bounded for $t \in [0, t^*(x_0))$. By the continuation theorem for the solutions of ODE, we can conclude that $t^*(x_0) = +\infty$. In the following definitions of stability, we will denote $x(t; x_0)$ for $t \in [0, +\infty)$ as the global solution of system (1) uniquely determined by the initial condition $x(0; x_0) = x_0 \in \boldsymbol{R}^n$. Moreover, we will use two equivalent norms of vector x in $\boldsymbol{R}^n$, i.e., $\|x\| = \left(\sum_{i=1}^n x_i^2\right)^{1/2}$ and $\|x\|_1 = \sum_{i=1}^n |x_i|$.

Definition 1 An equilibrium point $x^* \in \boldsymbol{R}^n$ of system (1) is a constant solution of (1), i.e., it satisfies the algebraic equation $-\boldsymbol{D}f(x^*) + \boldsymbol{T}g(x^*) + \boldsymbol{I} = 0$. The equilibrium x^* is said to be GES if there exist two positive constants $\alpha > 0$ and $\beta > 0$ such that for any $x_0 \in \boldsymbol{R}^n$ and $t \in [0, +\infty)$

$$\|x(t; x_0) - x^*\| \leqslant \alpha \|x_0 - x^*\| \exp(-\beta t)$$

Definition 2 System (1) is said to be AEST with respect to the class PLI if it possesses a GES equilibrium point for every function $g \in$ PLI, every input vector $\boldsymbol{I} \in \boldsymbol{R}^n$, and any positive diagonal matrix $\boldsymbol{D}$.

It is obvious that an AEST neural network system (1) is ABST because the GES property implies the GAS one.

For the proof of AEST result of neural network model (1) in section 2, we require some knowledge in matrix types with their characteristics and some concepts from degree theory. Their details can be found in Ref. [9] and its references therein.

2 AEST Result and Its Proof

In this section, we give the main result as follows.

Theorem 1 If $-\boldsymbol{T}$ is an $\boldsymbol{H}$-matrix with nonnegative diagonal elements, then the neural network system (1) is AEST with respect to the class PLI.

Proof Fix $g \in$ PLI, $I \in \boldsymbol{R}^n$ and the positive diagonal matrix $\boldsymbol{D}$. Suppose that $-\boldsymbol{T}$ is an $\boldsymbol{H}$-matrix with nonnegative diagonal elements. Then, its comparison matrix $\boldsymbol{M}(-\boldsymbol{T})$ is an $\boldsymbol{M}$-matrix which diagonal elements are $-\boldsymbol{T}_{ii}$ $(i = 1, 2, \cdots, n)$. Thus, for any positive diagonal matrix $\boldsymbol{K} = \mathrm{diag}(K_1, K_2, \cdots, K_n)$ the matrix $\boldsymbol{M}(-\boldsymbol{T}) + \boldsymbol{K}$ is a nonsingular $\boldsymbol{M}$-matrix. Therefore, its transposition $(\boldsymbol{M}(-\boldsymbol{T}) + \boldsymbol{K})^{\mathrm{T}}$ is also. It follows that there exists a positive diagonal matrix $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$ such that

$$\lambda_j T_{jj} + \sum_{i \neq j} \lambda_i |T_{ij}| < \lambda_j K_j \quad j = 1, 2, \cdots, n \quad (4)$$

Step 1 Let $\boldsymbol{H}(x) = \boldsymbol{D}f(x) - \boldsymbol{T}g(x) - \boldsymbol{I}$ $(x \in \boldsymbol{R}^n)$, then $x^* \in \boldsymbol{R}^n$ is an equilibrium of the network system the form $\boldsymbol{H}(x) = \boldsymbol{D}f(x) - \boldsymbol{T}G(x) + \boldsymbol{V}$, where the function $G(x) = (G_1(x_1), G_2(x_2), \cdots, G_n(x_n))^{\mathrm{T}} = g(x) - g(0) \in$ PLI satisfying $G(0) = 0$, and the vector $\boldsymbol{V} = (V_1, V_2, \cdots, V_n)^{\mathrm{T}} = -\boldsymbol{T}g(0) - \boldsymbol{I} \in \boldsymbol{R}^n$.

Since $g \in$ PLI, by (2) there exist positive constants $l_i > 0 (i = 1,2,\cdots,n)$ such that

$$|G_i(x_i)| = |g_i(x_i) - g_i(0)| \leqslant l_i |x_i|$$

for $x_i \in \boldsymbol{R}$ and $i = 1,2,\cdots,n$. We can select the positive diagonal matrix $\boldsymbol{K}$ as $K_j = d_j/2l_j > 0 (j = 1, 2,\cdots,n)$ for which the inequality (4) holds for some positive diagonal matrix $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$.

Construct the nonempty, bounded and open subset $\boldsymbol{\Omega}_r = \{x \in \boldsymbol{R}^n \mid \|x\|_1 < r\} \supseteq \{0\}$ for some $r > 0$ and the homotopy $h(x;\lambda) = [h_1(x;\lambda), h_2(x;\lambda), \cdots, h_n(x;\lambda)]^{\mathrm{T}} \in \boldsymbol{R}^n$ defined as

$$h(x;\lambda) = \lambda f(x) + (1-\lambda)H(x)$$
$$x \in \overline{\boldsymbol{\Omega}}_r, \lambda \in [0,1]$$

where $\overline{\boldsymbol{\Omega}}_r = \{x \in \boldsymbol{R}^n \mid \|x\|_1 \leqslant r\}$. In the following, we will prove that for sufficiently large $r > 0$, $h(x;\lambda) \neq 0$ for $x \in \partial\boldsymbol{\Omega}_r = \{x \in \boldsymbol{R}^n \mid \|x\|_1 = r\}$ and $\lambda \in [0, 1]$.

Let the signum function $\mathrm{sgn}(\rho)(\rho \in \boldsymbol{R})$ be defined as 1 if $\rho > 0$; 0 if $\rho = 0$; and -1 if $\rho < 0$. Then, we have

$$\begin{aligned}
&\sum_{i=1}^{n} \lambda_i \mathrm{sgn}(x_i) h_i(x;\lambda) = \sum_{i=1}^{n} \lambda_i \mathrm{sgn}(x_i)[\lambda + (1-\lambda)d_i] f(x_i) + \\
&\quad \sum_{i=1}^{n} \lambda_i \mathrm{sgn}(x_i)\Big[(\lambda - 1)\sum_{j=1}^{n} T_{ij}G_j(x_j) + (1-\lambda)V_i\Big] \geqslant \\
&\quad \sum_{j=1}^{n} \lambda_j[\lambda + (1-\lambda)d_j]|x_j| - (1-\lambda)\sum_{j=1}^{n} \lambda_j T_{jj}|G_j(x_j)| - \\
&\quad (1-\lambda)\sum_{j=1}^{n}\sum_{i\neq j}\lambda_i |T_{ij}||G_j(x_j)| - \theta \geqslant \\
&\quad \sum_{j=1}^{n} \lambda_j[\lambda + (1-\lambda)d_j]|x_j| - (1-\lambda)\sum_{j=1}^{n} \lambda_j K_j |G_j(x_j)| - \\
&\quad \theta \geqslant \sum_{j=1}^{n} \lambda_j\Big[\lambda + \frac{1}{2}(1-\lambda)d_j\Big]|x_j| - \theta \geqslant \omega\|x\|_1 - \theta
\end{aligned}$$

where $\theta = (1-\lambda)\sum_{j=1}^{n}\lambda_j|V_j| \geqslant 0$ and $\omega = \min_{j=1,2,\cdots,n}(\min(\lambda_j/2, \lambda_j d_j/4)) > 0$.

Thus, if $r > \theta/\omega$, from the inequality above, then we can get that for $x \in \partial\boldsymbol{\Omega}_r$ and $\lambda \in [0,1]$, $\sum_{i=1}^{n}\lambda_i \mathrm{sgn}(x_i)h_i(x;\lambda) > 0$, which implies that $h(x;\lambda) \neq 0$. By the homotopy invariance property, we have that $d(h(z;0);0,\boldsymbol{\Omega}_r) = d(h(z;1);0,\boldsymbol{\Omega}_r)$, i.e., that $d(H;0,\boldsymbol{\Omega}_r) = d(id;0,\boldsymbol{\Omega}_r) = 1 \neq 0$. Thus, $\boldsymbol{H}(x) = 0$ has at least one solution in $\boldsymbol{\Omega}_r \subseteq \boldsymbol{R}^n$. Now, we show that there is at most one solution of $\boldsymbol{H}(x) = 0$ in $\boldsymbol{R}^n$ by the contradiction method. Assume that $x^{(1)} = (x_1^{(1)}, x_2^{(1)}, \cdots, x_n^{(1)})^{\mathrm{T}} \in \boldsymbol{R}^n$ and $x^{(2)} = (x_1^{(2)}, x_2^{(2)}, \cdots, x_n^{(2)})^{\mathrm{T}} \in \boldsymbol{R}^n$ be two different solutions of $\boldsymbol{H}(x) = \boldsymbol{0}$. This means

$$\begin{aligned}
&\boldsymbol{D}f(x^{(1)}) - \boldsymbol{T}g(x^{(1)}) - \boldsymbol{I} = \\
&\quad \boldsymbol{D}f(x^{(2)}) - \boldsymbol{T}g(x^{(2)}) - \boldsymbol{I} = \boldsymbol{0}
\end{aligned}$$

and hence

$$\begin{aligned}
&(-\boldsymbol{T})[g(x^{(2)}) - g(x^{(1)})] = \\
&\quad \boldsymbol{D}[f(x^{(1)}) - f(x^{(2)})] \neq \boldsymbol{0}
\end{aligned}$$

Let $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n)^{\mathrm{T}} = g(x^{(2)}) - g(x^{(1)}) \in \boldsymbol{R}^n$, then $(-\boldsymbol{T})\tilde{x} \neq \boldsymbol{0}$ and hence $\tilde{x} \neq \boldsymbol{0}$. Since $-\boldsymbol{T} \in P_0$, there exists an index $i \in \{1,2,\cdots,n\}$ such that $\tilde{x}_i = g_i(x_i^{(2)}) - g_i(x_i^{(1)}) \neq 0$ and $\tilde{x}_i(-T\tilde{x})_i = d_i\tilde{x}_i[f(x_i^{(1)}) - f(x_i^{(2)})] \geqslant 0$. The last inequality is equivalent to $\tilde{x}_i[x_i^{(1)} - x_i^{(2)}] \geqslant 0$ because of $d_i > 0$ and $[f(x_i^{(1)}) - f(x_i^{(2)})][x_i^{(1)} - x_i^{(2)}] \geqslant 0$. Moreover, noting the inequality $x_i^{(1)} - x_i^{(2)} \neq 0$ from $\tilde{x}_i \neq 0$, we know that $\tilde{x}_i[x_i^{(1)} - x_i^{(2)}] \neq 0$. Therefore, we should have $\tilde{x}_i[x_i^{(1)} - x_i^{(2)}] > 0$, i.e., that $[g_i(x_i^{(2)}) - g_i(x_i^{(1)})][x_i^{(1)} - x_i^{(2)}] > 0$. This is in contradiction with the monotone increasing property of $g_i(x_i)$.

At this point, we have shown that the network system (1) has a unique equilibrium point which can be denoted by $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T \in \boldsymbol{R}^n$.

Step 2 For any $x_0 \in \boldsymbol{R}^n$, let $x(t;x_0)$ for $t \in [0, t^*(x_0))$ be the unique solution of the autonomous system (1) satisfying the unique solution of the autonomous system (1) satisfying the initial condition $x(0;x_0) = x_0$, where $t^*(x_0) \in (0, +\infty)$ or $t^*(x_0) = +\infty$ such that $[0, t^*(x_0))$ is the maximal right existence interval of the solution $x(t;x_0)$. Let $z(t) = (z_1(t), z_2(t), \cdots, z_n(t))^{\mathrm{T}} = x(t;x_0) - x^* \in \boldsymbol{R}^n$ for $t \in [0, t^*(x_0))$. Then, $z(t)$ satisfies the following ODE of the form

$$\begin{aligned}
&\mathrm{d}z(t)/\mathrm{d}t = -D\bar{f}(z(t)) + T\bar{g}(z(t)) \\
&\quad \forall\, t \in [0, t^*(x_0)) \qquad (5)
\end{aligned}$$

with the initial condition $z(0) = x_0 - x^*$, where the vector-valued function $\bar{f}(z) = (\bar{f}_1(z_1), \bar{f}_2(z_2), \cdots, \bar{f}_n(z_n))^{\mathrm{T}} \in \boldsymbol{R}^n$ is defined by $\bar{f}(z) = \bar{f}(z + x^*) - \bar{f}(x^*)$ and $\bar{g}(z) = (\bar{g}_1(z_1), \bar{g}_2(z_2), \cdots, \bar{g}_n(z_n))^{\mathrm{T}} \in \boldsymbol{R}^n$ is defined by $\bar{g}(z) = \bar{g}(z + x^*) - \bar{g}(x^*)$ for $z = (z_1, z_2, \cdots, z_n)^{\mathrm{T}} \in \boldsymbol{R}^n$ satisfying $\bar{f}(0) = 0$ and $\bar{g}(0) = 0$, respectively. Similarly, from the assumption of $g \in$ PLI, by (3) there exist positive constants $\mu_i > 0 (i = 1,2,\cdots,n)$ such that

$$|\bar{g}_i(z_i)| = |g_i(z_i + x_i^*) - g_i(x_i^*)| \leqslant \mu_i |z_i|$$

for $z_i \in \boldsymbol{R}$ and $i = 1,2,\cdots,n$. In what follows, we will take the positive diagonal matrix $\boldsymbol{K}$ as $K_j = d_j/(2\mu_j) > 0 (j = 1,2,\cdots,n)$ for which there exists a positive diagonal matrix $\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$

satisfying the inequality (4).

Construct the Lyapunov function $V(z) = \sum_{i=1}^{n} \lambda_i |z_i|$ for $z \in \boldsymbol{R}^n$. Define the right and upper Dini derivative of $V(z)$ along the solution $z(t)$ by

$$\frac{\mathrm{d}^+ V(z(t))}{\mathrm{d}t} = \limsup_{h \to 0^+} [V(z(t+h)) - V(z(t))]/h$$

Computing the Dini derivative of $V(z)$ along the solution $z(t)$ for $t \in [0, t^*(x_0))$, we have

$$\frac{\mathrm{d}^+ V(z(t))}{\mathrm{d}t} = \sum_{i=1}^{n} \lambda_i \operatorname{sgn}(z_i(t))[-d_j \bar{f}(z_i(t)) + \sum_{j=1}^{n} \times T_{ij} \bar{g}_j(z_j(t))] \leqslant -\sum_{i=1}^{n} d_j \lambda_j |\bar{f}(z_j(t))| + \sum_{j=1}^{n} \lambda_j T_{jj} |\bar{g}_j(z_j(t))| + \sum_{j=1}^{n} \Big(\sum_{i \neq j} \lambda_i |T_{ij}| \Big) \times |\bar{g}_j(z_j(t))| \leqslant - \sum_{j=1}^{n} d_j \lambda_j |z_j(t)| + \sum_{j=1}^{n} \lambda_j K_j |\bar{g}_j(z_j(t))| \leqslant -\frac{1}{2} \sum_{j=1}^{n} d_j \lambda_j |z_j(t)| \leqslant -(d_{\min}/2) V(z(t)) \leqslant 0 \quad t \in [0, t^*(x_0))$$

where $d_{\min} = \min_{j=1,2,\cdots,n} d_j > 0$.

By the comparison principle, from the above differential inequality we have

$$V(z(t)) \leqslant V(z(0)) \exp(-d_{\min} t/2) \quad t \in [0, t^*(x_0)) \tag{6}$$

Let the two constants $\lambda_{\max} = \max_{j=1,2,\cdots,n} \lambda_j > 0$ and $\lambda_{\min} = \min_{j=1,2,\cdots,n} \lambda_j > 0$, then we get $\lambda_{\min} \| z \|_1 \leqslant V(z) \leqslant \lambda_{\max} \| z \|_1$ for $z \in \boldsymbol{R}^n$. Thus, note that $z(t) = (z_1(t), z_2(t), \cdots, z_n(t))^{\mathrm{T}} = x(t; x_0) - x^*$, it can be inferred from (6) that for $t \in [0, t^*(x_0))$

$$\| x(t; x_0) - x^* \|_1 \leqslant \frac{\lambda_{\max}}{\lambda_{\min}} \| x_0 - x^* \|_1 \exp(-d_{\min} t/2) \tag{7}$$

The above inequality implies that the solution $x(t; x_0)$ is bounded for $t \in [0, t^*(x_0))$. By the continuation theorem for the solutions of ODE, we can conclude that $t^*(x_0) = +\infty$ and the inequality (7) still holds for $t \in [0, \infty)$. In view of the equivalence of the norms $\| x \|_1$ and $\| x \|$, by definition 1 and (7), x^* is the GES equilibrium of the system (1).

Integrating the above results, we have completed the proof of AEST result of the network system (1).

Remark 1 The inequality (7) implies that the exponential convergence rate of any network trajectory has a lower bound of $d_{\min}/2$. On the other hand, putting $\boldsymbol{T}$ equal to the zero matrix in the network model (1), we can easily see that the possible lower bound for the exponential convergence rate of the network trajectory cannot be greater than $d_{\min}$. When the network model (1) is used for solving optimization problem and the larger exponential convergence rate of network trajectories is desired, we can use the following modified network model

$$\tau \mathrm{d}x/\mathrm{d}t = -\boldsymbol{D} f(x) + \boldsymbol{T} g(x) + \boldsymbol{I}$$

where $\tau > 0$ is a time constant. It is obvious that the exponential convergence rate of any trajectory for the above modified network model has a lower bound of $d_{\min}/(2\tau)$. Thus, the exponential convergence rate of the network trajectory can be made arbitrarily large by tuning downward the time constant $\tau > 0$.

Remark 2 Let $g(x)$ is piecewise linear activation functions, then the network system (1) is a VLSI-oriented continuous-time cellular neural networks (CNNs) proposed by [23]. So, the GES result of the VLSI-oriented continuous-time CNNs is obtained simultaneously.

Remark 3 Let $m = 1$ in (1) and $g(x) \in$ PLI, then it is obviously shown that the AEST result of [9] is special case of this paper.

3 Conclusion

In this paper, we have obtained a new AEST result of the neural networks (1) with partially Lipschitz activation functions. Without assuming the boundedness and differentiability of the activation functions, the conditions ensuring the existence and uniqueness of the equilibrium are obtained. The network model considered here is general and includes Hopfield neural networks, Bidirectional associative memory network and Cellular neural networks as its special cases. The condition for global exponential stability, which demonstrates that the network system has the stronger global exponential stability, is easily checked in practice by simple algebraic methods.

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广义动态神经网络的绝对指数稳定性

孙长银　费树岷　冯纯伯

(东南大学自动化研究所,南京 210096)

摘　要　本文研究了一类具偏李普希兹连续和单调增加激活函数的神经网络绝对指数稳定性问题.所得结果归结为如果联接矩阵 $\boldsymbol{T}$ 的负矩阵是一个非负对角元的 $\boldsymbol{H}$ 矩阵,那么在任意输入向量和网络参数的条件下,所选激活函数只要是偏李普希兹连续且单调增加的,广义动态神经网络绝对指数稳定.该广义动态神经网络包含常用的 Hopfield 神经网络,双向联想记忆神经网络和细胞神经网络作为其特殊情形,所得结论推广了现有文献中的有关结论.

关键词　绝对指数稳定性, 偏李普希兹连续性,神经网络

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