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The Mechanism of Eliminating Harmonic Resonance with Controllers and Application to the Power Systems *

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Abstract: The phenomenon of anti-symmetrical bifurcation of periodic solutions occurring near an integral manifold is the intrinsic cause resulting in harmonic resonance over-voltage in power systems. Due to this discovery, the principle of eliminating resonance by using anti-bifurcation technique is presented, which makes that the theoretical bases of very measure to eliminate resonance are unified firstly from a point of view of basic theory. Our discussion models depend on a class of nonlinear control model. Using the direct Lyapunov method, a complete theoretical proof is given in accordance with the measure of eliminating resonance by connecting nonlinear resistor in series to the neutral point of P. T., and the feedback control law being applied. It comprises the action of parameters of resistor to eliminate resonance and the actual process of eliminating resonance, i.e., to go against bifurcation process which forces the big harmonic solutions to retreat to the integral manifold gradually and disappear eventually, which by using the nonlinear controllers. This makes it sure that the intrinsic cause of resonance is eliminated thoroughly. The obtained theory results and computing results are better than the presented results.

Key words: eliminate resonance, feedback control law, over-voltage, bifurcation, manifold

The resonant over-voltage which appears frequently in ungrounded systems brings enormous economical lose. Among these the accidents caused by harmonic resonance over-voltage make up certain proportion. From 1940s over-voltage experts began to pour their major vigor to search for all kinds of ways to eliminate resonance. The early researches on this topic can be found in Refs. [1 – 3]. The recent results can be found in Ref. [4]. The common weakness in these works is that the basic theory level was not raised and any complete theoretical proof was not given. Therefore, we lacked complete rational knowledge about applied range and the action of some related parameters to eliminate resonance etc. It makes that sometime ones have one-sidedness on these problems, even plunge into blindness. This paper is a sequel of the authors' recent works[5–9]. On the basis of discovering the secret of the harmonic resonance, we put the stress on the establishment of the theoretical basis of eliminating harmonic resonance. The core of this theory is eliminating resonance by anti-bifurcation, which the researched models are the nonlinear high-order control mathematical models. It makes that the mechanisms of every measure of eliminating resonance are unified theoretically. A complete theoretical proof is given in accordance with a kind of measures of eliminating resonance by connecting nonlinear resistor in series to the neutral point of P. T. (potential transformer) in this paper. The obtained results give a complete theoretical basis for power systems to avoid being damaged by harmonic resonance, and the given model is a kind of nonlinear control system. Therefore, the result submits an easy method to seek for eliminating resonance, and can be applied to the nonlinear systems. In this way, one of the basic theories for researches of eliminating resonance, which were never solved for a long time, is mainly solved.

1 Eliminating Resonance by Anti-Bifurcation

There exists a two-dimensional integral manifold, on which there is only one asymptotically stable harmonic solution, the four-dimensional phase space models of the power systems was disclosed[5,6]. This solution corresponds to the case of regular power supply. The saturation of the core of P. T. leads to the increase of nonlinearility. As a result, the anti-symmetrical bifurcation of periodic solutions occurs in the region near the

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integral manifold and in addition to the asymptotically stable harmonic solution, there are two big harmonic solutions bifurcating from the integral manifold. The occurrence of this makes up the internal causes resulting in over-voltage. If power systems are excited by external stimuli, for example, the operation of switches, it will result in harmonic resonance over-voltage directly. In order to differentiate the two kinds of harmonic solutions, one of them is located on S, which is called small harmonic solution. Apparently, any measure to eliminate harmonic resonance must preserve the only asymptotically stable harmonic solution on the integral manifold to guarantee the system continues to supply power normally. In the meantime the two big harmonic solutions induced by bifurcation must be confined or even eliminated to wipe out the intrinsic causes of harmonic resonance. This reverse bifurcation process makes the big harmonic solutions that retreat to the integral manifold gradually and disappear eventually. This idea is called the principle of eliminating resonance by anti-bifurcation. Any effective measure to eliminate resonance must accord with this principle. Then, every theoretical basis of the measures to eliminating resonance is unified completely.

2 Nonlinear Control Model

Some laboratory tests and computer-aided analysis have been undertaken[8]. Ferromagnetic resonance over-voltage is a lasting nonlinear resonance phenomena that appeared in tank circuit with the action of iron core inductance saturation magnetization. Resonance formed in circuit that related to three phases closely, it will be difficult to study the system's complex mechanism (such as theory, applications etc). However, as linearizing mathematical models are used, the analysis can be dealt with by the methods of linear superposition of effects of successive three-phase closing process (e.g., I.A.Wright had given the unsimplified reason and arranged in a line to prove it). Germand first set up three-phase nonlinear mathematical model in power system in 1975, even the model is found incomprehensive now, it will show the discussion to resonance mechanism of the switching over-voltage must be in high dimension space. Generally, over-voltage produced by closing of three-phase is a nonlinear phenomenon in nature. On the other hand, there are some inductive elements with core in power system, due to the operation or other causes cores tent to saturate and inductors with cores may become nonlinear. It is clear that in order to further research on the mechanism of over-voltage caused by three-phase closing of switches, the nonlinear mathematical model must be set up, and then the switching over-voltage to the system is analyzed. Summed up the above discussion, the key of the over-voltage depends on two factors (nonlinear, high-dimension). To study the mechanism of higher over-voltage caused by closing of switches in power system, we must set up nonlinear high-dimensional mathematical model to study it. By using the sensitivity coefficient method and the fractional iterative methods of functional differential equations, we have obtained sub-optimum and its designing process of the nonlinear control over-voltage model in [8]. We have given a characterization of stability margins achieved with the inverse optimal control law in [9].

If connecting a nonlinear resistor, which is characterized by $U = Ai^{\alpha}$ in series to the neutral point of P. T., effectively eliminating resonance can be realized. We will deal with this as an example to give a complete proof of the basic theory and make use of it to illustrate the connotation of eliminating resonance by anti-bifurcation concretely. The nonlinear control mathematical model connected with nonlinear resistor can be expressed as[9]

$$\left.\begin{aligned}\frac{\mathrm{d}^2\varphi_1(t)}{\mathrm{d}t^2}+(k+a+9b\varphi_1^8(t))\frac{\mathrm{d}\varphi_1(t)}{\mathrm{d}t}+k(a\varphi_1(t)+b\varphi_1^9(t))+W+\tilde{u}_1(t)&=\frac{\sqrt{3}}{2}E\cos\left(t+\frac{\pi}{6}\right)\\ \frac{\mathrm{d}^2\varphi_2(t)}{\mathrm{d}t^2}+(k+a+9b\varphi_2^8(t))\frac{\mathrm{d}\varphi_2(t)}{\mathrm{d}t}+k(a\varphi_2(t)+b\varphi_2^9(t))+W+\tilde{u}_2(t)&=-\frac{\sqrt{3}}{2}E\cos\left(t+\frac{\pi}{6}\right)\end{aligned}\right\}\quad(1)$$

where

$$\begin{aligned}W&=\beta[a(\varphi_1+\varphi_2)+b(\varphi_1{}^9+\varphi_2{}^9)]+kG[a(\varphi_1+\varphi_2)+b(\varphi_1{}^9+\varphi_2{}^9)]^{\alpha}+\\&\quad\alpha G[a(\varphi_1+\varphi_2)+b(\varphi_1^9+\varphi_2^9)]^{\alpha-1}\left[a\left(\frac{\mathrm{d}\varphi_1}{\mathrm{d}t}+\frac{\mathrm{d}\varphi_2}{\mathrm{d}t}\right)+9b\left(\varphi_1^8\frac{\mathrm{d}\varphi_1}{\mathrm{d}t}+\varphi_2^8\frac{\mathrm{d}\varphi_2}{\mathrm{d}t}\right)\right]\end{aligned}$$

Similar to the method of [7], then (1) is equivalent to

$$\left.\begin{aligned}\frac{dx_1}{dt}&=x_2-kx_1-f(x_1)-G[f(x_1)+f(x_3)]^\alpha\\\frac{dx_2}{dt}&=-kf(x_1)-\beta[f(x_1)+f(x_3)]-kG[f(x_1)+f(x_3)]^\alpha-\tilde{u}_1+\frac{\sqrt{3}}{2}E\cos\left(t+\frac{\pi}{6}\right)\\\frac{dx_3}{dt}&=x_4-kx_3-f(x_3)-G[f(x_1)+f(x_3)]^\alpha\\\frac{dx_4}{dt}&=-kf(x_3)-\beta[f(x_1)+f(x_3)]-kG[f(x_1)+f(x_3)]^\alpha-\tilde{u}_2-\frac{\sqrt{3}}{2}E\cos\left(t+\frac{\pi}{6}\right)\end{aligned}\right\}\tag{2}$$

where $x_1=\varphi_1, x_3=\varphi_2$. α is a non-integral positive number, $G=A\frac{(\varphi_m\omega)^{\alpha-1}}{r^\alpha}$, where ω is the frequency of power supply, r is wasted resistance of P. T., A is characteristic parameter of nonlinear resistor (constant), φ_1,φ_2 are the magnetic linkage and φ_m is the per unit value of magnetic linkage, $f(x_j)=ax_j+bx_j^9, j=1,3, a,b,k,G,\beta,$ E are positive constants which depend on the parameters of circuit. Sometimes, for the sake of convenience, the mathematical model (1) is equivalent to another form as follows:

$$\left.\begin{aligned}\frac{dx_1}{dt}&=x_2-kx_1-f(x_1)-G[f(x_1)+f(x_3)]^\alpha-u_1+p(t)\\\frac{dx_2}{dt}&=-kf(x_1)-\beta[f(x_1)+f(x_3)]-kG[f(x_1)+f(x_3)]^\alpha\\\frac{dx_3}{dt}&=x_4-kx_3-f(x_3)-G[f(x_1)+f(x_3)]^\alpha-u_2-p(t)\\\frac{dx_4}{dt}&=-kf(x_3)-\beta[f(x_1)+f(x_3)]-kG[f(x_1)+f(x_3)]^\alpha\end{aligned}\right\}\tag{3}$$

where

$$u_1=\int_{-\frac{\pi}{6}}^{t}\tilde{u}_1(s)ds+\tilde{u}_1\left(-\frac{\pi}{6}\right), u_2=\int_{-\frac{\pi}{6}}^{t}\tilde{u}_2(s)ds+\tilde{u}_2\left(-\frac{\pi}{6}\right), p(t)=\frac{\sqrt{3}}{2}E\int_{-\frac{\pi}{6}}^{t}\cos\left(s+\frac{\pi}{6}\right)ds.$$

The deduction is analogous in Ref.[5]. We will choose (2) and (3) according to our needs. For $G=0$, the corresponding system is denoted by (4) or (5), respectively, i. e.,

$$\left.\begin{aligned}\frac{dx_1}{dt}&=x_2-kx_1-f(x_1)\\\frac{dx_2}{dt}&=-kf(x_1)-\beta[f(x_1)+f(x_3)]-\tilde{u}_1+\frac{\sqrt{3}}{2}E\cos\left(t+\frac{\pi}{6}\right)\\\frac{dx_3}{dt}&=x_4-kx_3-f(x_3)\\\frac{dx_4}{dt}&=-kf(x_3)-\beta[f(x_1)+f(x_3)]-\tilde{u}_2-\frac{\sqrt{3}}{2}E\cos\left(t+\frac{\pi}{6}\right)\end{aligned}\right\}\tag{4}$$

and

$$\left.\begin{aligned}\frac{dx_1}{dt}&=x_2-kx_1-f(x_1)-u_1+p(t)\\\frac{dx_2}{dt}&=-kf(x_1)-\beta[f(x_1)+f(x_3)]\\\frac{dx_3}{dt}&=x_4-kx_3-f(x_3)-u_2-p(t)\\\frac{dx_4}{dt}&=-kf(x_3)-\beta[f(x_1)+f(x_3)]\end{aligned}\right\}\tag{5}$$

Let $S_1^0=\{x\mid x\in R^4 \text{ and } x_1+x_3=0\}$, $S_2^0=\{x\mid x\in R^4 \text{ and } x_2+x_4=0\}$, and $S=\{x\mid x\in R^4 \text{ and } x\in S_1^0\cap S_2^0\}$. Then S is the integral manifold of (4)[5].

3 The Invariance of the Integral Manifold Under

Let the control inputs be chosen by

$$u_1 = -k_1[x_1 + x_3] + l_1[f(x_1) + f(x_3)] \tag{6}$$

and

$$u_2 = -k_2[x_1 + x_3] + l_2[f(x_1) + f(x_3)] \tag{7}$$

Then the invariance of the integral manifold under the perturbation of the closed-loop system may be expressed as follows:

Case 1 [$G = 0$ — no nonlinear resistor] Here S is a two-dimensional integral manifold of the closed-loop system (5). There exists only one asymptotically stable harmonic solution on S, and this solution of the closed-loop system is also globally stable in the sense of two-dimension[5].

Case 2 [$G \neq 0$ — nonlinear resistor is joining-up in series] Here S is still a two-dimensional integral manifold of the closed-loop system (3). The qualitative structure of the phase trajectories on it keeps invariant, i.e., the same as that of case 1, since $f(x_1) + f(x_3) = 0$ on S. Then, the closed-loop system (3) degenerates into the closed-loop system (5) on S.

Case 3 [no saturable core] Consider the first approximate system of the closed-loop system of (5).

$$\left.\begin{aligned} \frac{dx_1}{dt} &= -(k + a - k_1)x_1 + x_2 + k_1 x_3 + p(t) \\ \frac{dx_2}{dt} &= -a(k + \beta)x_1 - a\beta x_3 \\ \frac{dx_3}{dt} &= k_1 x_1 - (k + a - k_1)x_3 + x_4 - p(t) \\ \frac{dx_4}{dt} &= -a\beta x_1 - a(k + \beta)x_3 \end{aligned}\right\} \tag{8}$$

Similar to [5], we can prove that S is still an integral manifold of the closed-loop system (8). The qualitative structure of phase trajectories on it is the same as that of the two cases above. On the other hand, the characteristic equation of the closed-loop system (8) has two negative roots and a pair of conjugate complex roots at least as follows:

$$\lambda_1 = -a, \lambda_2 = -k, \lambda_{3,4} = \left(k_1 - a - k \pm \sqrt{(a + k - 2k_1)^2 - 4ak - 8a\beta}\right)\Big/2 \tag{9}$$

When the feedback control laws (6) and (7) satisfy that

$$\frac{a+k}{2} - 2\sqrt{ak + 2a\beta} < k_1 < \frac{a+k}{2} + 2\sqrt{ak + 2a\beta} \tag{10}$$

the closed-loop system (8) has only harmonic solution in the sense of 4-dimension. Then the unique harmonic solution of the closed-loop system (8) on S in the sense of 4-dimension coincides with that of closed-loop system (8) on S in the sense on 2-dimension. If the closed-loop system (8) is regarded as an unperturbed equation, then case 1 means that S and the qualitative structure of phase trajectories on it remain invariant under the perturbance of the nonlinear terms bx_1^3, bx_3^3 on the right-hand sides of the closed-loop system (8). Case 2 means that S and the qualitative structure of phase trajectories on it also keep invariant under the perturbance of the nonlinear terms bx_1^9, bx_3^9 and $G[f(x_1) + f(x_3)]^\alpha$ on the right-hand sides of the closed-loop system (8). This invariance of integral manifold reflects that saturation of the core of P. T. The measure of eliminating resonance is given by the connecting nonlinear resistor in series to the neutral point of P. T., and the nonlinear controllers are applied, which do not damage the theoretical basis for the normal furnishing power of the system. Then it realizes the elementary requirement to eliminate resonance by anti-bifurcation in the first stage.

4 The Mechanism of Eliminating Resonance

In this section, we will illustrate the intrinsic mechanism of eliminating resonance with the nonlinear resistor and nonlinear controllers. We have the following theorem.

Theorem Let the nonlinear control input of (2) be given by

$$\tilde{u}_{1,2} = c[x_1 + x_3] + e[f(x_1) + f(x_3)] \tag{11}$$

where

$$\left.\begin{aligned}&e < \min\left(\frac{2\beta(k+\beta)k+3a\beta(k+2\beta)}{k(k+2\beta)},\sqrt{\frac{3\beta kb^2}{(2k^2+4)(k+2\beta)}}\right)\\&c+ae < \min\left(\frac{a\beta(k+a)}{k+2\beta},\sqrt{\frac{k}{4}-b^2}\right)\end{aligned}\right\}\tag{12}$$

then all solutions of closed-loop system of (2) are uniformly ultimately bounded and the ultimately bounded region of the solutions of closed-loop system of (2) must be a proper subset of Ξ, which the set Ξ is defined by

$$\Xi = \{x \| x_1| \leqslant M_2, |x_2| \leqslant M_1, |x_3| \leqslant M_2, |x_4| \leqslant M_1\}\tag{13}$$

Suppose $\alpha = \frac{q}{p}$, p and q are relatively prime odd numbers and there exist three constants $L_j (j = 1,2,3)$ such that

$$|f_2(x_{2i})| \leqslant L_1, |f_3(x_{2i-1})| \leqslant L_2, |f_4(x_{2i-1})| \leqslant L_3 \qquad i = 1,2\tag{14}$$

for any $|x_{2i}| \leqslant N_1$, $|x_{2i-1}| \leqslant N_2$ and $|x_{2i-1}| \leqslant N_3^{\frac{1}{8}}$.

Let $L = 2\sum_{j=1}^{3} L_j$, some parameters are denoted by

$$\left.\begin{aligned}&\Omega_1 = \frac{k}{2} - 2(ae+c)^2 + b^2), N_4 = \sqrt{\frac{2L}{\Omega_1}}\\&\Omega_2 = \frac{3\beta(k+2\beta)b^2}{2k} - (k+2\beta)^2e^2 - \frac{2(k+2\beta)^2e^2}{k^2}, N_5 = \left(\frac{L}{\Omega_2}\right)^{\frac{1}{18}}\\&M_1 = \max\{N_1, N_4\}, M_2 = \max\{N_2, N_3, N_5\}\end{aligned}\right\}\tag{15}$$

Proof Let the Lyapunov function be denoted as follows:

$$V = \frac{k+2\beta}{2k}(x_2^2+x_4^2) + \frac{k(k+2\beta)}{2}(x_1^2+x_3^2) + \left(\frac{\beta}{k}\right)^2(x_2-x_4)^2 +$$
$$\frac{2\beta(k+2\beta)}{k}[F(x_1)+F(x_3)] - (k+\beta)x_1x_2 - (k+\beta)x_3x_4 - \beta x_1x_4 - \beta x_2x_3\tag{16}$$

where $F(x_j) = \int_0^{x_j} f(s)\mathrm{d}s, j = 1,3$.

Then V is a continuous positive infinitely large function and there exist positive continuous functions $A(r)$ and $B(r)$ such that

$$A(\|x\|) \leqslant V(x) \leqslant B(\|x\|), \lim_{r\to\infty} A(r) = +\infty\tag{17}$$

The proof is analogous to the work in Ref. [6], here we do not explain any more.

Let $g = [f(x_1) + f(x_3)]^\alpha$, then

$$\begin{aligned}\dot V|_{(2)} = {}& \frac{k+2\beta}{k}x_2\left(-kf(x_1) - \beta(f(x_1)+f(x_3)) - kGg - \tilde u_1 + \frac{\sqrt3}{2}E\cos\left(t+\frac{\pi}{6}\right)\right) +\\& \frac{k+2\beta}{k}x_4\left(-kf(x_3) - \beta(f(x_1+f(x_3)) - kGg - \tilde u_2 - \frac{\sqrt3}{2}E\cos\left(t+\frac{\pi}{6}\right)\right) +\\& [k(k+2\beta)]x_1[x_2 - kx_1 - f(x_1) - Gg] + [k(k+2\beta)]x_3[x_4 - kx_3 - f(x_3) - Gg] +\\& 2\left(\frac{\beta}{k}\right)^2(x_2-x_4)\left(-kf(x_1) + kf(x_3) + \sqrt3 E\cos\left(t+\frac{\pi}{6}\right) - \tilde u_1 + \tilde u_2\right) +\\& \frac{2\beta(k+2\beta)}{k}f(x_1)[x_2 - kx_1 - f(x_1) - Gg] + \frac{2\beta(k+2\beta)}{k}f(x_3)[x_4 - kx_3 - f(x_3) - Gg] -\\& (k+\beta)x_1\left(-kf(x_1) - \beta(f(x_1)+f(x_3)) - kGg - \tilde u_1 + \frac{\sqrt3}{2}E\cos\left(t+\frac{\pi}{6}\right)\right) -\\& (k+\beta)x_2[x_2 - kx_1 - f(x_1) - Gg] - (k+\beta)x_4[x_4 - kx_3 - f(x_3) - Gg]\\& -(k+\beta)x_3\left(-kf(x_3) - \beta(f(x_1)+f(x_3)) - kGg - \tilde u_2 - \frac{\sqrt3}{2}E\cos\left(t+\frac{\pi}{6}\right)\right) -\\& \beta x_1\left(-kf(x_3) - \beta(f(x_1)+f(x_3)) - kGg - \tilde u_2 - \frac{\sqrt3}{2}E\cos\left(t+\frac{\pi}{6}\right)\right) -\\& \beta x_4[x_2 - kx_1 - f(x_1) - Gg] - \beta x_2[x_4 - kx_3 - f(x_3) - Gg]\end{aligned}$$

$$- \beta x_3\left(- kf(x_1) - \beta(f(x_1) + f(x_3)) - kGg - \tilde{u}_1 + \frac{\sqrt{3}}{2}E\cos\left(t + \frac{\pi}{6}\right)\right) =$$

$$\dot{V}|_{(4)} - \frac{2\beta(k + 2\beta - 2l)}{k}G[f(x_1) + f(x_3)]^{1+\alpha} \tag{18}$$

Putting this in order, we have

$$\dot{V}|_{(4)} \leqslant - f_1(x) - f_2(x_2) - f_3(x_1) - f_2(x_4) - f_3(x_3) - f_4(x_1) - f_4(x_3) \tag{19}$$

where

$$\left.\begin{aligned}
& f_1(x) = \left[\frac{k}{2} - 2((ae + c)^2 + b^2)\right]x_2{}^2 + \left[\frac{3\beta(k + 2\beta)b^2}{2k} - (k + 2\beta)^2e^2 - \frac{2(k + 2\beta)^2e^2}{k^2}\right]x_1{}^{18} + \\
& \left[\frac{k}{2} - 2((ae + c)^2 + b^2)\right]x_4{}^2 + \left[\frac{3\beta(k + 2\beta)b^2}{2k} - (k + 2\beta)^2e^2 - \frac{2(k + 2\beta)^2e^2}{k^2}\right]x_3{}^{18} \\
& f_2(x_{2i}) = \frac{k}{6}x_{2i}{}^2 - \left[\frac{\sqrt{3}E(k + 2\beta)}{2k} + \frac{2\sqrt{3}E\beta^2}{k^2}\right]|x_{2i}| \\
& f_3(x_{2i-1}) = [\beta(k + \beta)a - (k + 2\beta)(c + ae)]x_{2i-1}^2 - \frac{\sqrt{3}Ek}{2}|x_{2i-1}| \\
& f_4(x_{2i-1}) = \left[2\beta(k + \beta)b + 3ab\frac{\beta(k + 2\beta)}{k} - (k + 2\beta)be\right]x_{2i-1}^{10} - \\
& \left\{\frac{3k(2k + 3\beta)^2}{2} + \frac{3k\beta^2}{2} + \frac{2k\beta(k + \beta)^2}{k + 2\beta} - \beta(k + \beta)a - \right. \\
& \left.\frac{3\beta(k + 2\beta)a^2}{2k} - k^2(k + 2\beta) + \frac{2(k + 2\beta)^2}{k^2} + (k + 2\beta)^2(ae + c)^2 + b^2\right\}x_{2i-1}^2 \qquad i = 1,2
\end{aligned}\right\} \tag{20}$$

Then $f_2(x_{2i}) \geqslant 0$ for every $|x_{2i}| \geqslant N_1$; $f_3(x_{2i-1}) \geqslant 0$ for every $|x_{2i-1}| \geqslant N_2$; and $f_4(x_{2i-1}) \geqslant 0$ for every $|x_{2i-1}| \geqslant N_3^{\frac{1}{8}}, i = 1,2$. One or more of $f_2(x_{2i}), f_3(x_{2i-1})$ and $f_4(x_{2i-1})$ may be negative if $|x_{2i}| \leqslant N_1$, $|x_{2i-1}| \leqslant N_2$ and $|x_{2i-1}| \leqslant N_3^{\frac{1}{8}}$ are involved. In this case, those have maximal value respectively according to the property of continuous functions. Suppose that there exist three constants $L_j(j = 1,2,3)$ such that

$$|f_2(x_{2i})| \leqslant L_1, |f_3(x_{2i-1})| \leqslant L_2, |f_4(x_{2i-1})| \leqslant L_3 \qquad i = 1,2$$

the above-mentioned results imply that

$$\begin{aligned}
\dot{V}|_{(4)} \leqslant & \sum_{i=1}^{2}[|f_2(x_{2i})| + |f_3(x_{2i-1})| + |f_4(x_{2i-1})|] - \left[\frac{3\beta(k + 2\beta)b^{22}}{2k} - (k + 2\beta)^2e^2 - \right. \\
& \left.\frac{2(k + 2\beta)^2e^2}{k^2}\right]x_1{}^{18} - \left[\frac{k}{2} - 2((ae + c)^2 + b^2)\right]x_4{}^2 - \left[\frac{k}{2} - 2((ae + c)^2 + b^2)\right]x_2{}^2 - \\
& \left[\frac{3\beta(k + 2\beta)b^2}{2k} - (k + 2\beta)^2e^2 - \frac{2(k + 2\beta)^2e^2}{k^2}\right]x_3{}^{18}
\end{aligned} \tag{21}$$

Note that $\frac{\Omega_1}{2}x_{2i}{}^2 \geqslant L$ for every $|x_{2i}| \geqslant N_4$; $\Omega_2 x_{2i-1}{}^{18} \geqslant L$ for every $|x_{2i-1}| \geqslant N_5$.

Therefore, we have

$$\dot{V}|_{(4)} \leqslant - \frac{\Omega_1}{2}(x_2^2 + x_4^2) - \frac{\Omega_2}{2}(x_1^{18} + x_3^{18}) \leqslant - C(r)\|x\| \leqslant 0 \tag{22}$$

On the complementary set Ξ^c of $\Xi = \{x \| x_1| \leqslant M_2, |x_2| \leqslant M_1, |x_3| \leqslant M_2, |x_4| \leqslant M_1\}$, where Ξ is a bounded set and $C(r)$ is a certain positive continuous function. Then all solutions of the closed-loop system (4) are uniformly ultimately bounded[8] and the bounded set Ξ is an estimation of the ultimately region of the solutions of the closed-loop system (4). Suppose $1 + \alpha = \frac{q + p}{p}, p + q$ is an even number, i.e., $- \frac{2\beta(k + 2\beta)G}{k}\left[f(x_1) + f(x_3)\right]^{1+\alpha} \leqslant 0$. Therefore, according to (19) and (22), the solutions of the closed-loop system of (2) are uniformly ultimately bounded and the ultimately bounded region of the solutions of the closed-loop system of (2) must be a proper subset of Ξ, are obtained. This completes our proof.

Remark 1 In order to grasp the relevant relation between the proper subset of the related parameters of the characteristic of nonlinear resistor and control terms, for any $M_3 > 0, M_4 > 0$ such that $M_3 < M_1, M_4 < M_2$, consider the subset Ξ_1 of Ξ as follows:

$$\Xi_1 = \{x \| x_1| \leqslant M_4, |x_2| \leqslant M_3, |x_3| \leqslant M_4, |x_4| \leqslant M_3\}$$

On $\Xi \setminus \Xi_1$, $-f_2(x_2) - f_3(x_1) - f_2(x_4) - f_3(x_3) - f_4(x_1) - f_4(x_3)$ has a maximal value $\vec{L}$ and $[f(x_1) + f(x_3)]^{1+\alpha}$ has a minimal value $\underleftarrow{L}$. If there holds

$$G \geqslant \frac{k\vec{L}}{2\beta(k + 2\beta)\underleftarrow{L}} \tag{23}$$

then

$$-f_2(x_2) - f_3(x_1) - f_2(x_4) - f_3(x_3) - f_4(x_1) - f_4(x_3) - \frac{2\beta(k + 2\beta)G}{k}[f(x_1) + f(x_3)]^{1+\alpha} \leqslant \vec{L} - \frac{2\beta(k + 2\beta)G\underleftarrow{L}}{k} \leqslant 0$$

On the complementary set Ξ_1^c of Ξ_1, this inequality implies that

$$\dot{V}|_{(2)} = \dot{V}|_{(4)} - \frac{2\beta(k + 2\beta)G}{k}[f(x_1) + f(x_3)]^{1+\alpha} \leqslant - f_1(x) = -\Omega_1({x_2}^2 + {x_4}^2) - \Omega_2({x_1}^{18} + {x_3}^{18}) \leqslant -\frac{\Omega_1}{2}({x_1}^2 + {x_2}^2) - \frac{\Omega_2}{2}({x_1}^{18} + {x_3}^{18}) \leqslant 0 \tag{24}$$

This shows that the proper subset Ξ_1 is an estimation of the ultimately bounded region of the solutions of closed-loop system of (2). There exists a large enough A in $G = \frac{A(\varphi_m \omega)^{\alpha-1}}{r^\alpha}$ such that Eq.(24) holds.

To the matter of linear controllers, we have the following corollary.

Corollary Let the linear control input of (2) be given by

$$\tilde{u}_{1,2} = c[x_1 + x_3] \tag{25}$$

where

$$c < \min\left(\frac{a\beta(k + a)}{k + 2\beta}, \sqrt{\frac{k}{4} - b^2}\right) \tag{26}$$

then all solutions of closed-loop system of (2) are uniformly ultimately bounded and the ultimately bounded region of the solutions of closed-loop system of (2) must be a proper subset of Ξ_0, on which the set Ξ_0 is defined by

$$\Xi_0 = \{x \| x_1| \leqslant \tilde{M}_2, |x_2| \leqslant \tilde{M}_1, |x_3| \leqslant \tilde{M}_2, |x_4| \leqslant \tilde{M}_1\} \tag{27}$$

where

$$\left.\begin{aligned} &\dddot{N}_1 = \frac{3\sqrt{3}E(k + 2\beta)}{k^2} + \frac{12\sqrt{3}E\beta^2}{k^3}, \tilde{N}_2 = \frac{\sqrt{3}Ek}{2\beta(k + \beta)a - 2(k + 2\beta)c} \\ &\tilde{N}_3 = \frac{\frac{3k(2k + 3\beta)^2}{2} + \frac{3k\beta^2}{2} + \frac{2k\beta(k + \beta)^2}{k + 2\beta} + \frac{2(k + 2\beta)^2}{k^2} + (k + 2\beta)^2}{2\beta(k + \beta)b + \frac{3ab\beta(k + 2\beta)}{k}} - \\ &\qquad \frac{\beta(k + \beta)a + \frac{3\beta(k + 2\beta)a^2}{2k} + k^2(k + 2\beta)}{2\beta(k + \beta)b + \frac{3ab\beta(k + 2\beta)}{k}} \end{aligned}\right\} \tag{28}$$

Suppose $\alpha = \frac{q}{p}$, p and q are relatively prime odd numbers and there exist three constants $L_j (j = 1,2,3)$ such that

$$|f_2(x_{2i})| \leqslant L_1, |f_3(x_{2i-1})| \leqslant L_2, |f_4(x_{2i-1})| \leqslant L_3 \qquad i = 1,2$$

for any $|x_{2i}| \leqslant \tilde{N}_1$, $|x_{2i-1}| \leqslant \tilde{N}_2$ and $|x_{2i-1}| \leqslant \tilde{N}_3^{\frac{1}{8}}$.

Let $L = 2\sum_{j=1}^{3} L_j$, some parameters are denoted by

$$\left.\begin{aligned} &\Omega_1 = \frac{k}{2} - (2c^2 + b^2), \tilde{N}_4 = \sqrt{\frac{2L}{\Omega_1}} \\ &\Omega_2 = \frac{3\beta(k + 2\beta)b^2}{2k}, \tilde{N}_5 = \left(\frac{L}{\Omega_2}\right)^{\frac{1}{18}} \\ &\tilde{M}_1 = \max\{\tilde{N}_1, \tilde{N}_4\}, \tilde{M}_2 = \max\{\tilde{N}_2, \tilde{N}_3, \tilde{N}_5\} \end{aligned}\right\} \tag{29}$$

Remark 2 The proof of theorem shows clearly that the mechanism of eliminating resonance of the nonlinear resistor connected in series to the neutral point of P. T. and the effect of eliminating resonance of the parameters of nonlinear resistor and nonlinear controllers can be summarized as follows:

1) Whatever measures of eliminating resonance are, first of all, they do not influence the normal furnishing power of the system. Before the nonlinear resistor is connected in series to the neutral point of P. T., and the suitable nonlinear controllers applied, power systems were "dissipative" (D-systems). Therefore, all solutions of the closed-loop systems of its mathematical model were uniformly ultimately bounded. This led to the existence of harmonic solutions[8], which provide the qualitative basis for the existence of harmonic solution on integral manifold, thus, provide the qualitative basis for normal furnishing power. When we let the characteristic parameter α of nonlinear resistor be $\frac{q}{p}$, where p and q are relatively prime odd numbers, the dissipative property of the systems remains invariant after the nonlinear resistor is connected in series and the nonlinear controllers are applied. Then the uniformly ultimate boundedness of all solutions of the corresponding mathematical model of the closed-loop systems also remains invariant. This finishes the preparation of the invariance of integral manifold and the qualitative structure of phase trajectories on it under the perturbation of system. Therefore, it makes sure that the measure by connecting resistor in series and the nonlinear controllers applied does not influence the normal furnishing power of the systems.

2) The nonlinear control mathematical model of power systems, before the fact that nonlinear resistor is connected and the nonlinear controllers is applied, has three harmonic solutions when bifurcation occurs, two of which are large and the third is small[6], however, they can coexist only in the ultimately bounded region.

3) After the neutral point of P. T. is connected in series to a nonlinear resistor, the ultimately bunded region of the closed-loop system varies facing on reduction in range. But when the nonlinear controllers are applied, the ultimately bounded region be charged. The larger the characteristic parameter A of the nonlinear resistor is, the smaller ultimately bounded region shrinks. Sufficiently large value of A is sufficient to make the ultimately bounded region shrink into positive direction attractive domain of the small harmonic solution, i.e., it destructs the base on which the large harmonic solutions continue to exist and thus eliminates the intrinsic cause of resonance thoroughly.

4) Note that $x_1 = \varphi_1, x_3 = \varphi_2$, where φ_1 and φ_2 are magnetic linkages. To reduce the range of the ultimately bounded region means to decrease x_1 and x_3. In engineering application, it reflects that it limits the saturation of the core. In power systems, resonance relates closely to the saturation of the core. So the physical meaning of the mechanism of eliminating resonance by connecting the nonlinear resistor in series to the neutral point of P. T. may be simply summarized as anti-saturation of the core. In this paper, we consider the nonlinear controllers function simultaneously. This simple summary can be easily understood by the numerous operators in power systems. They have apprehended the meaning of eliminating resonance just so.

5) Consider that the application to power systems, the numerical computation is given as follows. Consider the sampled-data

$k = 0.003\,27, a = 0.003\,565, E = 380, b = 0.000\,231\,9, \beta = 10.787\,429$, (12) implies that $e < 0.000\,007\,611\,1, c < 0.019\,20$. (13), (14) and (16) imply that the positive feedback controllers will result in the range of the ultimately bounded region in increasing and negative feedback controllers will result in its decrease.

The experimental results offered by Jiangsu Electric Test Institute, show that resonance is eliminated completely when $A = 12\,034.349\,71, e = 0.000\,358, c = 0.006\,573$. Those results provide substantial evidence for the demonstration of the paper.

The obtained theoretical results and computing results are better than all the previous results.

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控制消谐机理及其在电力系统中的应用

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摘 要 发生在积分流形上的周期解反对称分叉现象是导致电力系统谐振过电压的原因之一.利用反分叉技巧消谐的原理已经有结果.本文讨论的模型是电力系统中出现的非线性过电压控制模型.利用 Lyapunov 直接方法和反馈控制,给出了非线性电阻中性不接地爆破情况下的消谐方法.将参数电阻的消谐与消谐的实际过程相比较,即利用非线性控制率,我们的结果使得发生在积分流形上高频谐逐渐消失.理论和计算所得到的结论经过实际验证比已经出现的结论要好.

关键词 消谐,过电压,反馈控制,分叉,流形

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