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Spin-Orbit Scattering Effects on Hall Conductivity in a Layered Disordered Electron System*

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Abstract: Spin-orbit scattering effects in a layered quasi-2D disordered electron system have been investigated by the diagrammatic techniques in perturbation theory. The expression of Cooperon (propagator in particle-particle channel) is obtained as the function of interlayer coupling. The analytical result for the quantum correction to Hall conductivity has been obtained as functions of elastic, inelastic and spin-orbit scattering times. It is shown that the strong and weak couplings correspond, respectively, to the 3D and 2D situations. The Hall coefficient is shown to vanish. The relevant dimensional crossover behavior from 3D to 2D with decreasing the interlayer coupling has been discussed, and the condition for the crossover has been obtained. The present theory is expected to apply for the electronic transport in tunneling superlattices.

Key words: layered system, weak-localization, Hall conductivity, spin-orbit scattering

Anderson localization of disordered electron systems by elastic scattering from static impurities has been a topic of serious study for the last two decades[1,2]. According to the scaling theory of the pioneering work of Abrahams et al[3]. all electronic states in one- and two-dimensional (1D and 2D) disordered systems are localized irrespective of the degree of randomness, while in three-dimensional(3D) systems there exist metal-insulator transitions due to Anderson localization. In recent years, however, quasi-2D electron systems have attracted a great deal of attention because of their unique physical properties. A positive magnetoresistance due to suppression of antilocalization in a $CdTe/Hg_{1-y}Cd_yTe$ superlattice has been studied experimentally by Moyle, Cheung and Ong[4]. Szott, Jedrzejek and Kirk have completed the measurements and made extended studies of negative magnetoresistance effects in a $GaAs/Al_xGa_{1-x}As$ superlattice[5]. Another example of quasi-2D electron system is the layered high-T_c cuprates. The logarithmic increase of resistivity with decreasing temperature in a magnetic field suppressing superconductivity in $La_{2-x}Sr_xCuO_4$[6] and La-doped $Bi_2Sr_2CuO_7$[7], is attributed to weak-localization effects[2]. These experimental results provide a motive for theoretical investigation of weak-localization effects in quasi-2D disordered electron systems[8-12]. Recently, Abrikosov calculated the quantum interference corrections in a quasi-2D metal to conductivity as a function of temperature and magnetic field[12], and discussed the dimensional crossover from 3D to 2D behavior with decreasing the interlayer hopping energy.

In this work, we will study theoretically the spin-orbit scattering effects on weak-localization in a layered quasi-2D disordered electron system, which are not involved in above mentioned theoretical works. Weak-localization is a quantum effect that results from constructive interference between closed electron paths and their time-reversed counterparts. This constructive interference increases the probability of backscattering and results in an increase in resistivity over the classical Drude value. In the presence of spin-orbit scatterings the interference becomes suppressed due to the rotation of the electron spin[13]. Therefore spin-orbit scatterings must have very important influences on the transport properties of a quasi-2D system, as well as on the dimensional crossover behavior from 3D to 2D. By means of the diagrammatic techniques in the perturbation theory, we will calculate the Hall conductivity in a layered quasi-2D disordered electron system in the presence of spin-orbit scatterings, and discuss the relevant dimensional crossover behavior from 3D to 2D with decreasing the interlayer hopping energy.

In section 2, we will present the model for a layered quasi-2D disordered electron system, and calculate the Boltzmann conductivities of this model. In the perturbation theory, the so-called Cooperon (particle-particle propagator) is responsible for weak-localization effects, therefore we will, in section 3, derive the expression for Cooperon in the presence of spin-orbit scatterings and calculate the weak-localization corrections to conductivities.

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In section 4, the Hall conductivity and its quantum correction will be evaluated, and the relevant dimensional crossover behavior from 3D to 2D will be discussed. Finally, a brief summary is given in section 5.

1 The Model for a Layered Quasi-2D Disordered Electron System

The energy spectrum of a quasi-2D disordered electron system is given by

$$\epsilon_{\boldsymbol{k}} = k_{\parallel}^2/2m - t\cos(k_z a) \tag{1}$$

where $k_{\parallel} = (k_x, k_y)$ and k_z are wave vectors along the planar and z directions respectively; m is the in-plane effective mass; a is the period of the structure along z axis; t is the interlayer hopping energy which is assumed to be much smaller than the Fermi energy ϵ_F. It is easily shown that the Fermi surface of this model is a slightly corrugated cylinder, the density of states per spin at the Fermi energy is $N = m/2\pi a$, and the electron density is given by $n = k_F^2/2\pi a$ with $k_F = mv_F = \sqrt{2m\epsilon_F}$.

Let us consider spin-orbit scatterings. If an electron with spin $\boldsymbol{\sigma}$ is scattered by a potential $u\delta(r)$ from the state $\boldsymbol{k}$ into the state $\boldsymbol{k}'$, the Born amplitude of the scattering is given by $u[1 + i\eta(\boldsymbol{k} \times \boldsymbol{k}') \cdot \boldsymbol{\sigma}]$ with η being a small constant. The impurity-averaged retarded and advanced Green's function for the conduction electrons are given by

$$G^{RA}(\boldsymbol{k}, \omega) = (\omega - \xi_{\boldsymbol{k}} \pm i/2\tau)^{-1} \tag{2}$$

where $\xi_{\boldsymbol{k}} = \epsilon_{\boldsymbol{k}} - \epsilon_F$ and $\tau^{-1} = \tau_0^{-1} + \tau_i^{-1} + \tau_{so}^{-1}$, with τ_0, τ_i and τ_{so} being the elastic, inelastic and spin-orbit scattering times respectively. Using the Born approximation, we have $\tau_0^{-1} = 2\pi Nn_i u^2$ and $\tau_{so}^{-1} = \sum_{\mu}(\tau_{so}^{\mu})^{-1} = 2\pi Nn_i u^2\eta^2 \sum_{\mu}\langle(\boldsymbol{k} \times \boldsymbol{k}')_{\mu}^2\rangle_F$, where n_i is the concentration of impurities and $\langle(\boldsymbol{k} \times \boldsymbol{k}')_{\mu}^2\rangle_F$ represents the average over the Fermi surface[13]. The inelastic scattering time τ_i depends on the temperature due to electron-electron or electron-phonon interactions. In the weak – disorder regime, n_i is assumed to be so small that $\epsilon_F^{-1} \ll \tau_0 \ll \tau_{so}, \tau_i$.

The diffusion constant and the mean free path along μ direction are defined by $D_{\mu} = \langle v_{\mu}^2\rangle_F \tau$ and $l_{\mu} = (D_{\mu}\tau)^{1/2}$ respectively, where $\langle v_{\mu}^2\rangle_F$ represents the mean-square velocity on the Fermi surface. Making use of the dispersion relation (1), one can easily obtain $D_{\parallel} = v_F^2\tau/2$ and $D_z = t^2a^2\tau/2$. The Boltzmann DC conductivities can be easily calculated through the well-known Einstein relation $\sigma_{\mu} = 2e^2ND_{\mu}$, yielding $\sigma_{\parallel} = ne^2\tau/m$ and $\sigma_z = e^2mat^2\tau/2\pi$.

It is important to emphasize that both Boltzmann theory and weak-localization theory are correct within the region that the quasiclassical approximation is valid, therefore we must distinguish two different cases: ① $\tau^{-1} \ll t \ll \epsilon_F$, meaning $l_{\parallel} \gg \lambda_F$ (the Fermi wavelength) and $l_z \gg a$, in this case the quasiclassical method is valid for all directions; ② $t \lesssim \tau^{-1} \ll \epsilon_F$, meaning $l_{\parallel} \gg \lambda_F$ and $l_z \lesssim a$, in this case the quasiclassical method is valid only for the planar direction, with the wave functions of electrons being localized along z direction.

2 The Weak-Localization Corrections to the Conductivities

In a disordered electron system, the Cooperon responsible for weak-localization effects is the particle-particle propagator which can be diagrammatically represented in Fig.1. The dashed lines with crosses represent the impurity-averaged scattering amplitude. In the presence of spin-orbit scatterings, this scattering amplitude can be expressed by Refs.[2,13]

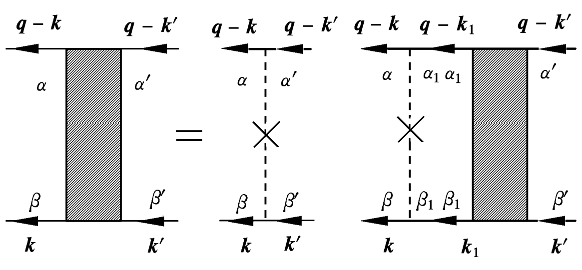

Fig.1 The diagram for the Cooperon

$$W_{\alpha\alpha', \beta\beta'} \} = (2\pi N\tau_0)^{-1}\left[\delta_{\alpha\alpha'}\delta_{\beta\beta'} - \sum_{\mu}(\tau_0/\tau_{so}^{\mu})\sigma_{\alpha\alpha'}^{\mu}\sigma_{\beta\beta'}^{\mu}\right] \tag{3}$$

where $\sigma^{\mu}(\mu = x, y, z)$ are the Pauli matrices. The Cooperon is decided by the following equation

$$C(\boldsymbol{q}, \omega)_{\alpha\alpha', \beta\beta'} = W_{\alpha\alpha', \beta\beta'} + K(\boldsymbol{q}, \omega)\sum_{\alpha_1, \beta_1} W_{\alpha\alpha_1, \beta\beta_1} C(\boldsymbol{q}, \omega)_{\alpha_1\alpha', \beta_1\beta'} \tag{4}$$

where the kernel $K(\boldsymbol{q}, \omega)$ is defined by

$$K(\boldsymbol{q},\omega) = \sum_{\boldsymbol{k}} G^R(\boldsymbol{q}-\boldsymbol{k},\omega)G^A(\boldsymbol{k},0) \tag{5}$$

Now let us evaluate the expression for the kernel $K(\boldsymbol{q},\omega)$ near the diffusion pole in a quasi-2D system. For small $q_\parallel$, substituting Eqs.(1) and (2) into Eq.(5), we get

$$K(\boldsymbol{q},\omega) = m\int_{-\infty}^{\infty}\frac{\mathrm{d}\xi_\parallel}{2\pi}\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\int_{-\pi/a}^{\pi/a}\frac{\mathrm{d}k_z}{2\pi}[\xi_\parallel - t\cos(k_z a) + i/2\tau]^{-1}\times$$
$$[\xi_\parallel + q_\parallel v_F\cos\theta - t\cos(k_z a + q_z a) - \omega - i/2\tau]^{-1} =$$
$$m\tau\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\int_{-\pi/a}^{\pi/a}\frac{\mathrm{d}k_z}{2\pi}\left[1 - i\omega\tau + iq_\parallel v_F\tau\cos\theta + i2t\tau\sin(\frac{1}{2}q_z a)\sin(k_z a + \frac{1}{2}q_z a)\right]^{-1}$$

In an anisotropic 3D system, the diffusion pole means $\omega\tau \ll 1$ and $q_\mu l_\mu \ll 1$[14]. However, for our quasi-2D model with the Fermi surface being a slightly corrugated cylinder, the diffusion pole is of some unique feature, which means $\omega\tau$, $q_\parallel l_\parallel$, $(2l_z/a)\left|\sin(q_z a/2)\right| \ll 1$. In this case, we have

$$K(\boldsymbol{q},\omega) \approx m\tau\int_0^{2\pi}\frac{\mathrm{d}\theta}{2\pi}\int_{-\pi/a}^{\pi/a}\frac{\mathrm{d}k_z}{2\pi}\left[1 + i\omega\tau - (q_\parallel v_F\tau\cos\theta)^2 - 4t^2\tau^2\sin^2(\frac{1}{2}q_z a)\sin^2(k_z a + \frac{1}{2}q_z a)\right] =$$
$$2\pi N\tau[1 + i\omega\tau - q_\parallel^2 l_\parallel^2 - (2l_z/a)^2\sin^2(q_z a/2)] \tag{6}$$

We expect that the expression for Cooperon has the similar structure with that of the scattering amplitude, assuming

$$C(\boldsymbol{q},\omega)_{\alpha\alpha',\beta\beta'} = (2\pi N\tau)^{-1}(A\delta_{\alpha\alpha'}\delta_{\beta\beta'} + \sum_\mu B_\mu\sigma^\mu_{\alpha\alpha'}\sigma^\mu_{\beta\beta'}) \tag{7}$$

where $B_x = B_y = B_\parallel$. Substituting Eqs.(3), (6) and (7) into Eq.(4), we can calculate the values of A and B_μ, and get

$$A = (1/2)F_1(\boldsymbol{q},\omega) + (1/4)F_2(\boldsymbol{q},\omega) + (1/4)F_3(\boldsymbol{q},\omega)$$
$$B_z = (1/2)F_1(\boldsymbol{q},\omega) - (1/4)F_2(\boldsymbol{q},\omega) - (1/4)F_3(\boldsymbol{q},\omega)$$
$$B_\parallel = (1/4)F_2(\boldsymbol{q},\omega) - (1/4)F_3(\boldsymbol{q},\omega)$$

where the functions $F_l(\boldsymbol{q},\omega)$ ($l = 1,2,3$) are defined by

$$F_l(\boldsymbol{q},\omega) = [q_\parallel^2 l_\parallel^2 + (2l_z/a)^2\sin^2(q_z a/2) - i\omega\tau + \lambda_l]^{-1} \tag{8}$$

with $\lambda_1 = \tau/\tau_i + 2\tau/\tau_{so}^\parallel + 2\tau/\tau_{so}^z$, $\lambda_2 = \tau/\tau_i + 4\tau/\tau_{so}^\parallel$, $\lambda_3 = \tau/\tau_i$. Substituting the values of A and B_μ into Eq.(7), we obtain

$$\sum_{\alpha\beta} C(\boldsymbol{q},\omega)_{\alpha\beta,\beta\alpha} = (2\pi N\tau)^{-1}[2F_1(\boldsymbol{q},\omega) + F_2(\boldsymbol{q},\omega) - F_3(\boldsymbol{q},\omega)] \tag{9}$$

if $\left|q_z\right|a \ll 1$, Eq.(8) changes as the exact result in an anisotropic 3D system

$$F_l(\boldsymbol{q},\omega) = (q_\parallel^2 l_\parallel^2 + q_z^2 l_z^2 - i\omega\tau + \lambda_l)^{-1} \tag{10}$$

According to the Kubo formula, within the quasiclassical approximation, the weak-localization correction to the conductivity along μ direction is given by Ref.[2]

$$\delta\sigma_\mu = (e^2/2\pi)\sum_{\boldsymbol{kq}} v_\mu(\boldsymbol{k})v_\mu(\boldsymbol{q}-\boldsymbol{k})G^R(\boldsymbol{k},\omega)G^A(\boldsymbol{k},0)G^R(\boldsymbol{q}-\boldsymbol{k},\omega)G^A(\boldsymbol{q}-\boldsymbol{k},0)\sum_{\alpha\beta}C(\boldsymbol{q},\omega)_{\alpha\beta,\beta\alpha} \tag{11}$$

where ω is the frequency of the applied field, and $v_\mu(\boldsymbol{k}) = \partial\epsilon_{\boldsymbol{k}}/\partial k_\mu$ is the velocity for the electron along μ direction. In the case of $\omega\tau \ll 1$, the main contribution of Eq. (11) arises from $l_\parallel q_\parallel \ll 1$ and $(2l_z/a)\left|\sin(q_z a/2)\right| \ll 1$, therefore one can easily perform the integration over $\boldsymbol{k}$, obtaining

$$\delta\sigma_\mu/\sigma_\mu = -\tau^2\sum_{\boldsymbol{q}}\sum_{\alpha\beta}C(\boldsymbol{q},\omega)_{\alpha\beta,\beta\alpha} \tag{12}$$

In the following evaluations, we will consider only the DC conductivity, setting $\omega = 0$.

3 The Hall Conductivity and Its Quantum Correction

Let us consider an external magnetic field perpendicular to the layers, which is described by the vector potential $\boldsymbol{A}(\boldsymbol{r}) = \boldsymbol{A}e^{i\boldsymbol{p}\cdot\boldsymbol{r}}$ with $\boldsymbol{p}$ and $\boldsymbol{A}$ along x and y directions repectively. Then the magnetic field is given by $H = ip\boldsymbol{A}$. We assume that the field is weak enough so that $\omega_c\tau \ll 1$ with $\omega_c = eH/mc$ being the cyclotron frequency. We shall generalize the diagrammatic method in a 2D system[15] to the quasi-2D system with spin-orbit scatterings. The Hall current is the current response that is linear in the electric field and the magnetic field. The Hall conductivity can be calculated by the diagrams in Fig.2, where the inserted lines with arrows represent the magnetic field vertexes. In the quasiclassical approximation, the contributions of diagrams (a) and (b) to the Hall conductivity can be expressed by

$$\sigma_{yx}^{a} = \frac{e^3}{\pi\ cm^3}\sum_{\boldsymbol{k}}(k_y + \frac{1}{2}p_y)k_x\boldsymbol{k}\cdot\boldsymbol{A}G^R(\boldsymbol{k}+\boldsymbol{p})G^R(\boldsymbol{k})G^A(\boldsymbol{k}) \tag{13}$$

$$\sigma_{yx}^{b} = \frac{e^3}{\pi\ cm^3}\sum_{\boldsymbol{k}}(k_y - \frac{1}{2}p_y)k_x\boldsymbol{k}\cdot\boldsymbol{A}G^A(\boldsymbol{k}-\boldsymbol{p})G^R(\boldsymbol{k})G^A(\boldsymbol{k}) \tag{14}$$

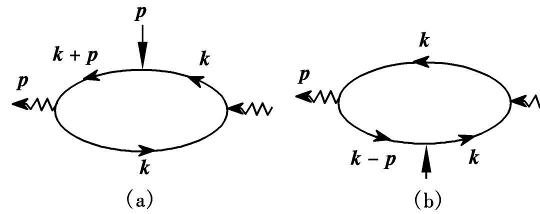

Fig.2 The diagrams for the conductivity

where $G^{RA}(\boldsymbol{k}) = G^{RA}(\boldsymbol{k},0)$. Summing Eqs.(13) and (14), and using the relation $G^{RA}(\boldsymbol{k}+\boldsymbol{p}) \approx G^{RA}(\boldsymbol{k}) + \boldsymbol{p}\cdot\boldsymbol{v}(\boldsymbol{k})G^{RA}(\boldsymbol{k})^2$ for small p, one obtains

$$\begin{aligned}\sigma &= i\frac{e^3H}{\pi\ cm^4}\sum_{\boldsymbol{k}}k_x^2k_y^2G^R(\boldsymbol{k})G^A(\boldsymbol{k})[G^A(\boldsymbol{k})^2 - G^R(\boldsymbol{k})^2] = \\ &\quad i\frac{e^3H}{8\pi\ cm^4}\sum_{\boldsymbol{k}}k_{\parallel}^2\ G^R(\boldsymbol{k})G^A(\boldsymbol{k})[G^A(\boldsymbol{k})^2 - G^R(\boldsymbol{k})^2]\end{aligned} \tag{15}$$

Substituting Eq.(2) into Eq.(15), and using the relation $k_{\parallel}^2 = k_F^2(1+\xi_{\parallel}/\epsilon_F)$, we can complete the intergration over $\boldsymbol{k}$ and obtain

$$\sigma_{yx} = \omega_c\tau\sigma_{\parallel} \tag{16}$$

The quantum correction to Hall conductivity can be evaluated by meaning of the diagrams shown in Fig.3. The contributions of these diagrams to the Hall conductivity can be expressed as

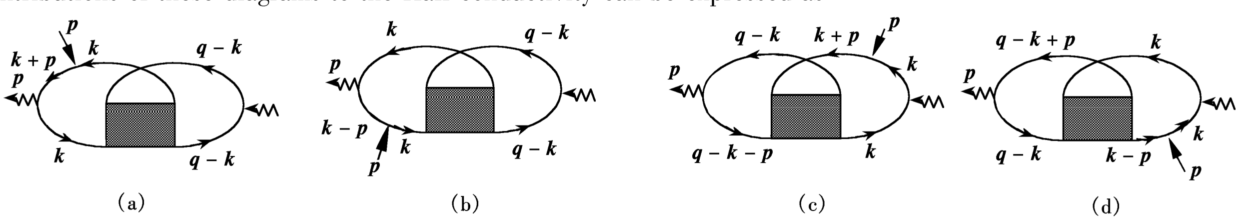

Fig.3 The diagrams for the quantum

$$\begin{aligned}\delta\sigma_{yx}^{a} &= \frac{e^3}{2\pi\ cm^3}\sum_{\boldsymbol{kq}}\sum_{\alpha\beta}(k_y + \frac{1}{2}p_y)(q_x - k_x)\boldsymbol{k}\cdot\boldsymbol{A}\times \\ &\quad G^R(\boldsymbol{k}+\boldsymbol{p})G^R(\boldsymbol{q}-\boldsymbol{k})G^R(\boldsymbol{k})G^A(\boldsymbol{q}-\boldsymbol{k})G^A(\boldsymbol{k})C(\boldsymbol{q},0)_{\alpha\beta,\beta\alpha}\end{aligned}$$

$$\begin{aligned}\delta\sigma_{yx}^{b} &= \frac{e^3}{2\pi cm^3}\sum_{\boldsymbol{kq}}\sum_{\alpha\beta}(k_y - \frac{1}{2}p_y)(q_x - k_x)\boldsymbol{k}\cdot\boldsymbol{A}\times \\ &\quad G^A(\boldsymbol{k}-\boldsymbol{p})G^A(\boldsymbol{q}-\boldsymbol{k})G^A(\boldsymbol{k})G^R(\boldsymbol{q}-\boldsymbol{k})G^R(\boldsymbol{k})C(\boldsymbol{q},0)_{\alpha\beta,\beta\alpha}\end{aligned}$$

$$\begin{aligned}\delta\sigma_{yx}^{c} &= \frac{e^3}{2\pi\ cm^3}\sum_{\boldsymbol{kq}}\sum_{\alpha\beta}(p_y - k_y - \frac{1}{2}p_y)k_x\boldsymbol{k}\cdot\boldsymbol{A}\times \\ &\quad G^R(\boldsymbol{k}+\boldsymbol{p})G^R(\boldsymbol{q}-\boldsymbol{k})G^R(\boldsymbol{k})G^A(\boldsymbol{q}-\boldsymbol{k}-\boldsymbol{p})G^A(\boldsymbol{k})C(\boldsymbol{q},0)_{\alpha\beta,\beta\alpha}\end{aligned}$$

$$\begin{aligned}\delta\sigma_{yx}^{d} &= \frac{e^3}{2\pi\ cm^3}\sum_{\boldsymbol{kq}}\sum_{\alpha\beta}(q_y - k_y + \frac{1}{2}p_y)k_x\boldsymbol{k}\cdot\boldsymbol{A}\times \\ &\quad G^A(\boldsymbol{k}-\boldsymbol{p})G^A(\boldsymbol{q}-\boldsymbol{k})G^A(\boldsymbol{k})G^R(\boldsymbol{q}-\boldsymbol{k}+\boldsymbol{p})G^R(\boldsymbol{k})C(\boldsymbol{q},0)_{\alpha\beta,\beta\alpha}\end{aligned}$$

Summing the above equations for small $\boldsymbol{p}$, and noting that the main contribution arises from the diffusion pole, we get

$$\delta\sigma_{yx} = -i\frac{e^3H}{\pi\ cm^4}\sum_{\boldsymbol{k}}\sum_{\alpha\beta}k_x^2k_y^2G^R(\boldsymbol{k})^2[G^A(\boldsymbol{k})^2G^A(\boldsymbol{k})^2 - G^R(\boldsymbol{k})^2]C(\boldsymbol{q},0)_{\alpha\beta,\beta\alpha} \tag{17}$$

Substituting Eq.(2) into Eq.(17), we can complete the integration over $\boldsymbol{k}$ and obtain

$$\delta\sigma_{yx} = -2\omega_c\tau^3\sigma_{\parallel}\sum_{\boldsymbol{q}}\sum_{\alpha\beta}C(\boldsymbol{q},0)_{\alpha\beta,\beta\alpha} \tag{18}$$

It is interesting to discuss the dimensional crossover behavior in the Hall conductivity. In the case of $\tau^{-1} \ll t \ll \epsilon_F$, the main contribution of Eq.(18) arises from $q_{\parallel}l_{\parallel} \ll 1$ and $l_zq_z \ll 1$. Substituting Eqs.(9) and (10) into Eq.(18), and replacing the summation over $\boldsymbol{q}$ by the integral

$$(2\pi)^{-3}\int_0^{l_\parallel^{-1}} 2\pi\ q_\parallel\, \mathrm{d}q_\parallel \int_{-l_z^{-1}}^{l_z^{-1}} \mathrm{d}q_z$$

we obtain

$$\frac{\delta\sigma_{yx}}{\sigma_\parallel} = -\frac{2\sqrt{2}\omega_c}{\pi^2 \epsilon_F t\tau}\left[1 - \frac{\pi}{4}\left(2\sqrt{\lambda_1} + \sqrt{\lambda_2} - \sqrt{\lambda_3}\right)\right] \tag{19}$$

The square-root behavior in above equation is the typical feature of a 3D system[1,2]. In the case of $t \lesssim \tau^{-1} \ll \epsilon_F$, the main contribution of Eq.(18) arises from $l_\parallel q_\parallel \ll 1$ and $|q_z| < \pi/a$. Substituting Eqs. (8) and (9) into Eq.(18), and replacing the summation over $\boldsymbol{q}$ by the integral

$$(2\pi)^{-3}\int_0^{l_\parallel^{-1}} 2\pi q_\parallel\, \mathrm{d}q_\parallel \int_{-\pi/a}^{\pi/a} \mathrm{d}q_z$$

one gets

$$\frac{\delta\sigma_{yx}}{\sigma_\parallel} = \frac{\omega_c}{\pi\epsilon_F}\ln\left(\eta_1\sqrt{\frac{\eta_2}{\eta_3}}\frac{t^2\tau^2}{2}\right) \tag{20}$$

where $\eta_l = 1 + \lambda_l/t^2\tau^2 + \sqrt{(1+\lambda_l/t^2\tau^2)^2 - 1}$ $(l = 1,2,3)$. If the interlayer coupling is small enough so that $t \ll \sqrt{\lambda_l}/\tau$, meaning $\eta_1 \approx 2\lambda_l/t^2\tau^2$, Eq.(20) changes as

$$\frac{\delta\sigma_{yx}}{\sigma_\parallel} = \frac{\omega_c}{\pi\epsilon_F}\ln\left(\lambda_1\sqrt{\frac{\lambda_2}{\lambda_3}}\right) \tag{21}$$

The logarithmic behavior in above equation is the typical feature of a 2D system. Therefore, Eqs.(19) – (21) show apparently a dimensional crossover from 3D to 2D with decreasing the interlayer hopping energy t.

The Hall coefficient is defined by $R = -\sigma_{yx}/(\sigma_\parallel^2 H)$, then its quantum correction can be calculated as

$$\frac{\delta R}{R} = \frac{\delta\sigma_{yx}}{\sigma_{yx}} - 2\frac{\delta\sigma_\parallel}{\sigma_\parallel} \tag{22}$$

Substituting Eqs.(12), (16) and (18) into Eq.(22), one obtains $\delta R = 0$. Therefore the quantum interference effects have a vanishing correction to the Hall coefficient. This result is consistent with that of a purely 2D system[15].

4 Conclusion

In this paper, making use of the diagrammatic techniques in the perturbation theory, we have calculated the weak-localization correction to the Hall conductivity in a layered quasi-2D disordered electron system with spin-orbit scatterings. We show that the spin-orbit scattering time (included in a parameters λ_1, λ_2 and λ_3) has very important influences on the dimensional crossover in the Hall conductivity. The condition $t \gg \tau^{-1}$ corresponds to a 3D situation. In the case of $t \ll \sqrt{\lambda_l}/\tau$ $(l = 1,2,3)$, the Hall conductivity is exactly the same as in an isotropic 2D system. It is apparent that the relevant dimensional crossover occurs at the region of $\sqrt{\lambda_l}/\tau \lesssim t \lesssim \tau^{-1}$ $(l = 1,2,3)$. However, the Hall coefficient has a vanishing quantum correction provided that the condition $\tau^{-1} \ll \epsilon_F$ is satisfied.

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无序电子系统中自旋轨道散射对 Hall 电导率的影响

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摘　要　利用微扰论中的图形技术讨论了层状准二维无序电子系统中自旋轨道散射效应的有关问题.得到作为层间耦合函数的 Cooperon(粒子-粒子通道的传播函数)的表达式,以及对 Hall 电导率量子相干修正的解析表达式,它是弹性、非弹性和自旋轨道散射时间的函数.强耦合和弱耦合分别对应于三维和二维的情况.Hall 系数变为零.此外还讨论了随层间耦合的减小由三维到二维的维度跨越行为和维度跨越的条件.该理论可以应用于解决隧道超晶格中的电子输运问题.

关键词　层状系统,弱局域化，Hall 电导率，自旋轨道散射

中图分类号　O482.4