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Multi-Parameter Design and Optimization for the Decomposition Projective Method and Its Applications

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Abstract: In order to solve the electromagnetic problems on the large multi-branch domains, the decomposition projective method(DPM) is generalized for multi-subspaces in this paper. Furthermore multi-parameters are designed for DPM, which is called the fast DPM(FDPM), and the convergence ratio of the above algorithm is greatly increased. The examples show that the iterative number of the FDPM with optimal parameters decreases much more, which is less than one third of the DPM iteration number. After studying the algorithm of FPDM, the representations of the optimal parameters are also given. Finally the applying efficiency of FPDM is raised.

Key words: decomposition projective method, projective operator, cavity filter

In the computation of electromagnetic fields, the problems on the large multi-branch domains are often met. Solving this kind of problem is very difficult because of the special patterns of domains which like a floor having a long passageway and many rooms or like the cavity filters. Due to the shorter contiguous borders between block areas, the present methods, such as DDM, are not available to the above problems whose variational equations don't satisfy the coercive conditions[1]. In order to solve this kind of problems, the decomposition projective methods(DPM) are presented in Ref.[2], which show that the convergence ratios of DPM are geometrical. With discretizing the differential equations on whole domain and solving them on different subdomains, the DPM avoids the requirements of the above coercive conditions or such like that. However DPM just has two projective spaces in Ref.[2], in the practical applications, three or more projective spaces are needed. So this paper generalizes the DPM for many projective spaces and designs an algorithm with many optimal parameters, in which the convergence was increased greatly. More importantly, since the optimal parameters are computed automatically, the algorithm is very available. On the basis of the cavity filter(see Fig.1), the algorithms are introduced as follows. Firstly a differential equation is discretized on the whole domain, and then the whole domain is divided into several areas, finally the discrete points are arrayed in conformity with the areas. According to the above process, a system of difference equation is got, and its coefficient matrix is as follows.

$$\boldsymbol{A} = \begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} & \cdots & A_{1,m} & 0 & 0 & \cdots & 0 \\ A_{2,1} & A_{2,2} & 0 & \cdots & 0 & A_{2,m+1} & 0 & \cdots & 0 \\ A_{3,1} & 0 & A_{3,3} & \cdots & 0 & 0 & A_{3,m+2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{m,1} & 0 & 0 & \cdots & A_{m,m} & 0 & 0 & \cdots & A_{m,2m-1} \\ 0 & A_{m+1,2} & 0 & \cdots & 0 & A_{m+1,m+1} & 0 & \cdots & 0 \\ 0 & 0 & A_{m+2,3} & \cdots & 0 & 0 & A_{m+2,m+2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & A_{2m-1,m} & \cdots & 0 & \cdots & A_{2m-1,2m-1} \end{bmatrix} \quad (1)$$

where $A_{i,i}(i = 1,\cdots,2m-1)$ are sparse strip matrices; $A_{1,1}$ corresponds to the main trunk area; $A_{i,i}(i = m+1,\cdots,2m-1)$ correspond to the cavities, $A_{i,i}(i = 2,\cdots,m)$ correspond to the connecting areas between the trunk and cavities. $A_{i,j}(i \neq j)$ have a few non-null elements to be distributed different rows and columns, and others are null matrices. When $A_{i,i}(i = 1,\cdots,2m-1)$ are all very large, solving the whole system of linear equations $\boldsymbol{Ax} = \boldsymbol{b}$ is very difficult. For that reason, this paper presents the multi-parameter fast decomposition projective method, and also gives the optimal parameter formulas. This algorithm decomposes the large problems into many small problems which almost are equivalent to the $A_{i,i}\tilde{x} = \tilde{b}(i = 1,\cdots,2m-1)$. Consequently the large problems can be solvable.

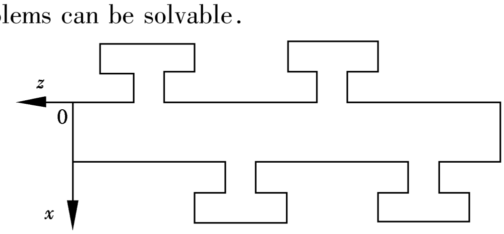

Fig.1 The cavity filter

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1 Fast Decomposition Projective Method

Suppose we want to seek the solution of the following problem

$$\boldsymbol{Ax} = \boldsymbol{b} \tag{2}$$

where the coefficient matrix $\boldsymbol{A}$ is the same as (1). Let

$$\boldsymbol{A} = [\boldsymbol{A}_1 \quad \boldsymbol{A}_2 \quad \boldsymbol{A}_3] \tag{3}$$

where $\boldsymbol{A}_1$ includes the first column block matrices of $\boldsymbol{A}$, $\boldsymbol{A}_2$ includes column 2 to column m of $\boldsymbol{A}$, $\boldsymbol{A}_3$ includes the other columns of $\boldsymbol{A}$. $V = \text{span}[\boldsymbol{A}]$ and $V_i = \text{span}[\boldsymbol{A}_i]$ $(i = 1,2,3)$ separately denote the linear spaces spanned by the column vectors of $\boldsymbol{A}$ and $A_i(i = 1,2,3)$. Distinctly $V = V_1 + V_2 + V_3$. Let $\boldsymbol{x} = [\boldsymbol{x}_1^{\mathrm{T}} \quad \boldsymbol{x}_2^{\mathrm{T}} \quad \boldsymbol{x}_3^{\mathrm{T}}]^{\mathrm{T}}$ be decomposition corresponding to (3). The algorithm of FDPM is as follows.

Algorithm(FDPM)

Step 0 Give $m:= 0$, $\boldsymbol{x}_3^0 = 0$, $\boldsymbol{x}_i^* = 0$, $i = 1,2,3$. $\boldsymbol{b}^0 = \boldsymbol{b}$, small $\varepsilon > 0$.

Step 1 Let $\boldsymbol{b}^{3m+1} = \boldsymbol{b}^{3m} - \boldsymbol{A}_3\boldsymbol{x}_3^m$, seek the least square solution of the problem $\boldsymbol{A}_1\boldsymbol{x}_1^{m+1} = \boldsymbol{b}^{3m+1}$, and choose an appropriate θ_1, then seek the sum $\boldsymbol{x}_1^* := \boldsymbol{x}_1^* + \theta_1\boldsymbol{x}_1^{m+1}$.

Step 2 Let $\boldsymbol{b}^{3m+2} = \boldsymbol{b}^{3m+1} - \theta_1\boldsymbol{A}_1\boldsymbol{x}_1^{m+1}$, seek the least square solution of the problem $\boldsymbol{A}_2\boldsymbol{x}_2^{m+1} = \boldsymbol{b}^{3m+2}$, and choose an appropriate θ_2, then seek the sum $\boldsymbol{x}_2^* := \boldsymbol{x}_2^* + \theta_2\boldsymbol{x}_2^{m+1}$.

Step 3 Let $\boldsymbol{b}^{3m+3} = \boldsymbol{b}^{3m+2} - \theta_2\boldsymbol{A}_2\boldsymbol{x}_2^{m+1}$, seek the least square solution of the problem $\boldsymbol{A}_3\boldsymbol{x}_3^{m+1} = \boldsymbol{b}^{3m+3}$, and seek the sum $\boldsymbol{x}_3^* := \boldsymbol{x}_3^* + \boldsymbol{x}_3^{m+1}$.

Step 4 If $\|\boldsymbol{b}^{3m+3} - \boldsymbol{A}_3\boldsymbol{x}_3^{m+1}\| < \varepsilon$, stop iteration, else let $m:= m + 1$, then go to step 1.

In step 4, the norm is the square norm. After infinite iteration, the limit of $\boldsymbol{x}^* = [\boldsymbol{x}_1^{*\mathrm{T}} \quad \boldsymbol{x}_2^{*\mathrm{T}} \quad \boldsymbol{x}_3^{*\mathrm{T}}]^{\mathrm{T}}$ is just the solution of the problem (2). The above algorithm is actually a cyclic projective procedure, which decomposes $\boldsymbol{b}$ as $\boldsymbol{b} = \boldsymbol{A}_1\boldsymbol{x}_1^* + \boldsymbol{A}_2\boldsymbol{x}_2^* + \boldsymbol{A}_3\boldsymbol{x}_3^*$. Furthermore, when the matrix $\boldsymbol{A}$ is divided into more than three parts, the algorithm can be generalized similarly.

2 Optimization of Parameters in FDPM

The parameters θ_1, θ_2 in the algorithm must be given two appropriate values in each iteration. The following analysis gets the representations of θ_1, θ_2, which are very suitable for calculation.

Suppose that P_{V_i} is the projective operator of $V \to V_i$, $V_i^{\perp}$ is the orthogonal complement space of V_i, $P_{V_i^{\perp}}$ is projective operator of $V \to V_i^{\perp}$, $(i = 1,2,3)$. According to step 2, after $\boldsymbol{b}^{3m+2}$ has been projected to the space V_2, we obtain $\boldsymbol{b}^{3m+2} = P_{V_2}\boldsymbol{b}^{3m+2} + P_{V_2^{\perp}}(\boldsymbol{I} - \theta_1 P_{V_1})\boldsymbol{b}^{3m+1}$. Obviously, when the norm of the complementary vector $P_{V_2^{\perp}}(\boldsymbol{I} - \theta_1 P_{V_1})\boldsymbol{b}^{3m+1}$ reaches the least, the value of θ_1 is the most optimal. Choose the function as

$$f(\theta) = \left\| P_{V_2^{\perp}}(I - \theta P_{V_1})\boldsymbol{b}^{3m+1} \right\|^2$$

By a series of calculation, the above formula can be simplified as

$$f(\theta) = \theta^2 \left\| P_{V_2^{\perp}} P_{V_1}\boldsymbol{b}^{3m+1} \right\|^2 - 2\theta(P_{V_2^{\perp}}\boldsymbol{b}^{3m+1}, P_{V_1}\boldsymbol{b}^{3m+1}) + \left\| P_{V_2^{\perp}}\boldsymbol{b}^{3m+1} \right\|^2$$

Let $f'(\theta) = 0$, its null point is

$$\theta = \frac{(\boldsymbol{b}^{3m+1}, P_{V_2^{\perp}} P_{V_1}\boldsymbol{b}^{3m+1})}{\left\| P_{V_2^{\perp}} P_{V_1}\boldsymbol{b}^{3m+1} \right\|^2}$$

This is the optimal value of parameter θ_1 in the above algorithm. With the same reason, we can obtain the optimal value of θ_2. Finally, we have the theorem.

Theorem In the algorithm of FDPM, the optimal values of parameter θ_1, θ_2 are

$$\theta_1 = \frac{(\boldsymbol{b}^{3m+1}, P_{V_2^{\perp}} P_{V_1}\boldsymbol{b}^{3m+1})}{\left\| P_{V_2^{\perp}} P_{V_1}\boldsymbol{b}^{3m+1} \right\|^2}$$

$$\theta_2 = \frac{(\boldsymbol{b}^{3m+2}, P_{V_3^{\perp}} P_{V_2}\boldsymbol{b}^{3m+2})}{\left\| P_{V_3^{\perp}} P_{V_2}\boldsymbol{b}^{3m+2} \right\|^2}$$

With the more subspaces, the parameters can be added correspondingly. Generally, in order to get more computation efficiency, the optimal parameters are calculated mainly for the high dimension spaces, and chosen fix values for the low dimension spaces.

3 Applications and Comparisons

As factual applications, the algorithm of FDPM with optimal parameter is used to calculate the scattering parameter $|s_{11}|$ of cavity filters (Fig.1), in which the structure of cavities is fixed, and the length of the guide is also fixed. Choosing $\varepsilon = 10^{-4}$, the iteration is stopped when the norm of $\boldsymbol{b}^m$ is less than ε. The results refer to Tab.1, in which $\theta_1 = \theta_2 = 1.0$ means to use DPM, and θ_2 isn't optimized because of the calculating efficiency. According to Tab.1, the iterative number of FDPM with the optimal parameters deduces greatly, which is less than one third of the DPM iteration number.

Tab.1 The comparisons of the iterative number

The number of cavities	θ_1	θ_2	The number of iteration	$\|s_{11}\|$ /dB
10	1.0	1.00	841	-29.24
10	Optimal values	1.94	263	-29.23
12	1.0	1.00	906	-27.97
12	Optimal values	1.94	285	-27.97
14	1.0	1.00	981	-27.81
14	Optimal values	1.94	285	-27.80

4 Conclusion

In order to solve the electromagnetic problems on the large multi-branch domains, the paper generalizes the DPM, and presents the FDPM. With the optimal parameter analysis, their representations are given. Because the optimal parameters can be computed automatically, the algorithm is very available. The examples show that the convergence rate of FPDM increases greatly. Furthermore, when the contiguous borders between sub-areas are shorter, the algorithm efficiency is higher.

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投影分解法的多参数设计与优化及应用

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摘　要　本文针对大型多枝区域上的电磁场问题,将投影分解法推广到向多个空间投影的情形,并设计了带有多个参数的快速算法,大大提高了算法收敛速度,实际算例表明迭代次数不到原来的三分之一.通过对所取参数的分析,给出了其最优值表达式,使得该算法具有很好的实用性.

关键词　投影分解法, 投影算子, 腔体滤波器

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