

An Augmented Jacobian Method for Power Flow Analysis of Weakly Looped Distribution Systems with PV Buses

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Abstract: A power flow analysis method for weakly looped distribution systems with PV buses is proposed in this paper. The proposed method is computationally more efficient and more robust compared with the conventional compensation methods. The robustness is achieved by embedding the boundary conditions of loops and PV buses into the Jacobian matrix. The computational efficiency is achieved by the carefully designed factorization of Jacobian matrix. Test results on a 33-bus system are presented.

Key words: power flow, radial distribution systems, weakly loops, PV bus

The conventional Newton method was considered inappropriate for distribution systems due to the large r/x ratios of distribution lines as well as extremely small impedance, which caused the poor convergence characteristics^[1,2]. For this reason, a number of alternative methods were proposed for radial systems without PV nodes. One such method called the back/forward sweep method is described in Ref.[3].

For a system with PV nodes and a small number of loops (weakly looped), the system is converted to a radial system without PV buses. The boundary conditions of loops and PV buses are checked after the converted system is converged. If the boundary conditions are not satisfied, the Q injections at PV buses and the looping currents at loop breaking points are calculated to compensate for the differences^[3,4]. Then the converted system needs to be solved again. This process will be repeated until the boundary conditions are satisfied.

In the above approach, the load flow for the converted system has to be converged in order for the compensation to kick in. In cases, where the final looping currents and the Q generations of PV buses are far from their initial values, the above approach will not converge. The authors of Ref.[3] noticed this and suggested that breaking points should be selected at the parts where the power flows are low (assuming the initial looping currents are zeroes). Unfortunately, it is usually difficult to figure out the power flow distribution within a loop before solving the load flow. For PV cases, the situation is better since it is

relatively easy to come up with a good estimation of the initial Q generations of PV buses.

In this paper, the enhanced mathematical formulation from the one given in Ref. [5] to handle systems with loops and PV buses is presented. The Jacobian matrix is augmented to represent loops and PV buses. The test results show the augmented Jacobian method is far more efficient and robust than the conventional compensation method.

1 Mathematical Formulation for Loops

For a linear electrical system, we have

$$\mathbf{Y}\dot{\mathbf{V}}_n = \dot{\mathbf{I}}_n \quad (1)$$

where \mathbf{Y} is an admittance matrix; $\dot{\mathbf{V}}_n$ is a nodal voltage vector; and $\dot{\mathbf{I}}_n$ is a current injection vector. As shown in [5], for a radial system, \mathbf{Y} can be factorized into three square matrixes:

$$\mathbf{Y} = \mathbf{A}\mathbf{Y}_b\mathbf{A}^T \quad (2)$$

where \mathbf{Y}_b is a diagonal matrix with the line admittance's as diagonal elements; \mathbf{A} is a node to branch incidence matrix, and it becomes an upper triangular matrix if the nodes and branches are ordered appropriately. One way to achieve such an \mathbf{A} is to order branches by layers away from the root node (source bus) as suggested in [3]. This ordering scheme is adopted as follows. The direction of each branch is towards the root node. The node ordering is proceeded simultaneously with the branch ordering. The branch "from side" node number is the same as the branch number, as illustrated in Fig.1. Hence, system

voltages can be obtained from (1) by one backward and one forward substitution. In every iteration, the nodal injection currents are updated based on the newly obtained nodal voltages to account for the load nonlinearly. Note that all the shunt components including constant PQ loads, constant impedance loads, shunt capacitors, etc. will contribute to the injection currents.

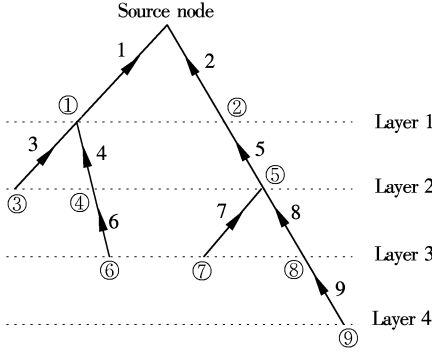


Fig. 1 A simple radial distribution system

A looped system can be converted into a radial system by breaking the loops at certain nodes as proposed in [3]. For example, as shown in Fig. 2, node i is split into i and i' , with \dot{I}_i as the virtual looping current.

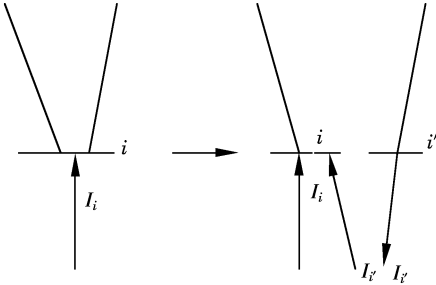


Fig. 2 Loop breaking representation

The following boundary condition must be satisfied by adjusting the virtual looping current:

$$\dot{V}_i = \dot{V}_{i'} \quad (3)$$

Eq. (1) is augmented to account for the boundary conditions:

$$\begin{bmatrix} 0 & J_L^T \\ J_L & AY_b A^T \end{bmatrix} \begin{bmatrix} \Delta \dot{I}_{LP} \\ \dot{V}_n \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{I}_n \end{bmatrix} \quad (4)$$

where $\Delta \dot{I}_{LP}$ is a looping current adjustment vector. For a simple system shown in Fig. 1 in which nodes 3 and 6 are a pair of breaking points, J_L is

$$[0 \ 0 \ 1 \ 0 \ 0 \ -1 \ 0 \ 0 \ 0]^T$$

The augmented matrix can be factorized into:

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{22} & \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \begin{bmatrix} C_{11}^T \\ C_{12}^T \\ C_{22}^T \end{bmatrix} \quad (5)$$

with

$$C_{22} D_2 C_{22}^T = AY_b A^T \quad (6)$$

$$C_{22} D_2 C_{12}^T = J_L \quad (7)$$

$$C_{11} D_1 C_{11}^T = -C_{12} D_2 C_{12}^T \quad (8)$$

Let $C_{22} = A$, $D_2 = Y_b$, and D_1 be a negative identity matrix (the explanation of C_{11} and C_{12} will be given later), Eq. (4) is converted to

$$\begin{bmatrix} C_{11} & C_{12} \\ & A \end{bmatrix} \begin{bmatrix} -I \\ Y_b A^T \end{bmatrix} \begin{bmatrix} C_{11}^T & C_{12}^T \\ A^{-T} C_{12}^T & I \end{bmatrix} \begin{bmatrix} \Delta \dot{I}_{LP} \\ \dot{V}_n \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{I}_n \end{bmatrix} \quad (9)$$

with conditions

$$AY_b C_{12}^T = J_L \quad (10)$$

$$C_{11} C_{11}^T = C_{12} Y_b C_{12}^T \quad (11)$$

The unknowns in Eq. (9) can be solved in three steps.

Step 1 Assume that

$$\begin{bmatrix} C_{11}^T & C_{12}^T \\ A^{-T} C_{12}^T & I \end{bmatrix} \begin{bmatrix} \Delta \dot{I}_{LP} \\ \dot{V}_n \end{bmatrix} = \begin{bmatrix} \dot{I}_{temp} \\ \dot{V}_{temp} \end{bmatrix}$$

then

$$\begin{bmatrix} C_{11} & C_{12} \\ & A \end{bmatrix} \begin{bmatrix} -I \\ Y_b A^T \end{bmatrix} \begin{bmatrix} \dot{I}_{temp} \\ \dot{V}_{temp} \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{I}_n \end{bmatrix} \quad (12)$$

A solution to \dot{V}_{temp} is obtained from

$$AY_b A^T \dot{V}_{temp} = \dot{I}_n \quad (13)$$

Note that, this step is equivalent to solving the voltages of a radial system.

Step 2 A solution for $\Delta \dot{I}_{LP}$ is obtained from (14):

$$C_{11} C_{11}^T \Delta \dot{I}_{LP} = C_{11} \dot{I}_{temp} = C_{12} Y_b A^T \dot{V}_{temp} = J_L^T \dot{V}_{temp} \quad (14)$$

Note that $J_L^T \dot{V}_{temp}$ is the voltage drop at every pair of loop breaking points and hence $C_{11} C_{11}^T$ is the loop impedance matrix, which represents the sensitivity of breaking point voltage drops to the virtual looping currents. This sensitivity matrix can be formed by summing up all the branch impedance of a loop as the corresponding diagonal element and the coupling branch impedance of two loops as the corresponding off diagonal element. It also can be obtained one column a time by introducing looping current change one loop a time and solving for voltage drop differences of all loops as recommended in [3].

Step 3 A solution to \dot{V}_n is obtained from

$$A^{-T} C_{12}^T \Delta \dot{I}_{LP} + \dot{V}_n = \dot{V}_{temp} \quad (15)$$

Eq. (15) is then rearranged as

$$AY_b A^T (\dot{V}_{temp} - \dot{V}_n) = J_L \Delta \dot{I}_{LP} \quad (16)$$

This step is required to correct nodal voltages due to the looping currents. This step is not actually implemented because it can be combined with the step

1 of next iteration. Note that there is no need to know C_{12} because of the relationship shown in (10).

The proposed method is a new form of the compensation method. The difference between this method and the compensation method suggested in Ref.[3] is that this method compensates the loop breaking point voltage drops at every iteration instead of compensating that after the radial power flow converges. This change in compensation order introduces a significant performance improvement in terms of the computational efficiency and robustness of convergence.

2 Mathematical Formulations for PV Buses

For systems with PV buses, the boundary condition is

$$\dot{V}_i = \dot{V}_{\text{controlled}} \tag{17}$$

The Jacobian matrix can be augmented easily to accommodate this boundary condition if PQ based formulation is adopted.

For example, assume that (18) is used to solve the load flow of a radial system without PV nodes iteratively:

$$\mathbf{J}\Delta\mathbf{x} = \Delta\mathbf{b} \tag{18}$$

For every PV node i , one equation, $\Delta V_i = 0$, is added into (18) and ΔV_i is treated as a variable and moved from the right hand side of (18) to the left hand side. In this way, the dimension of the Jacobian matrix is augmented by the number of PV nodes:

$$\begin{bmatrix} 0 & \mathbf{J}_v^T \\ \mathbf{J}_v & \mathbf{J} \end{bmatrix} \begin{bmatrix} -\Delta\mathbf{Q}_v \\ \Delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 \\ \Delta\mathbf{b}_m \end{bmatrix} \tag{19}$$

where $\Delta\mathbf{Q}_v$ is used to update the Q injection of PV nodes; $\Delta\mathbf{b}_m$ is the same as $\Delta\mathbf{b}$ except that the entries corresponding to Q mismatch of PV nodes are zero; and \mathbf{J}_v is the PV node index matrix to indicate where the PV nodes are. Then the unknowns in (19) can be solved exactly in the same way as proposed in section 2.

However, for the current based formulation, it is difficult to form the augmented Jacobian matrix. To overcome this difficulty, we only adopt the idea here, which is to compensate the boundary conditions at every iteration.

3 Test Results for Systems with Loops

The proposed method was tested in the following system:

Number of buses: 33
Voltage level: 10 kV

Loading level: 149.5 kW + j57 kvar
Largest r/x ratio: 9
Smallest r/x ratio: 0.166 7
Convergence criterion: 0.001 kW, 0.001 kvar
Max. No. of iterations: 20

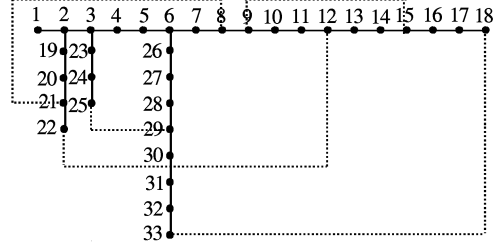


Fig.3 Test system

The “Old Method” below refers to the conventional compensation method, and the “New Method” refers to the augmented Jacobian method proposed in this paper. Fig.4 shows that the “New Method” is much less sensitive to the number of loops which implies that the “New Method” is more efficient and robust.

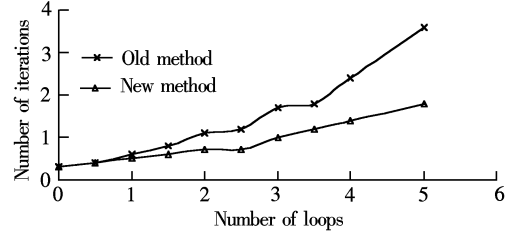


Fig.4 Iteration number vs. number of loops

Fig.5 to Fig.8 show that the “New Method” is much less sensitive to system loading levels as well as line impedance.

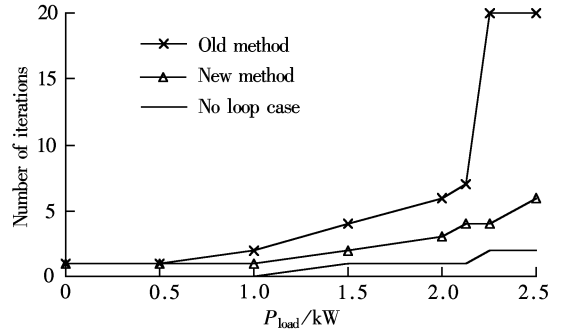


Fig.5 Iteration number vs. active loading level

Fig.9 shows a typical convergence pattern of both “New” and “Old” methods. As shown in the figure, the mismatches of “Old Method” jump at few iterations as compensation is performed, whereas the mismatches of “New Method” decrease steadily.

Fig.10 shows a typical divergence pattern of the “Old Method”. Usually, the “Old Method” diverges

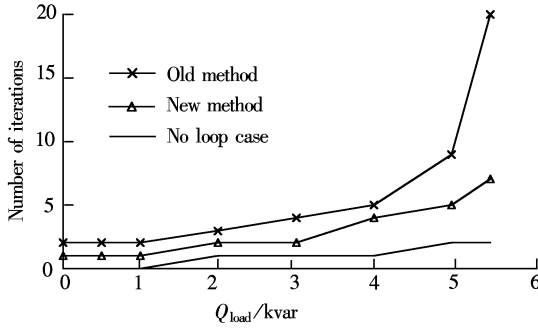


Fig. 6 Iteration number vs. reactive loading level

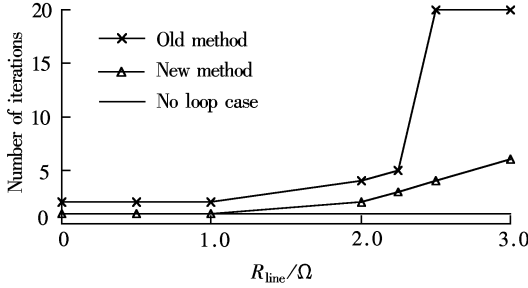


Fig. 7 Iteration number vs. line resistance

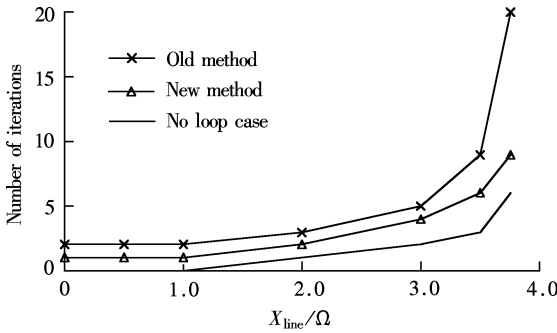


Fig. 8 Iteration number vs. line reactance

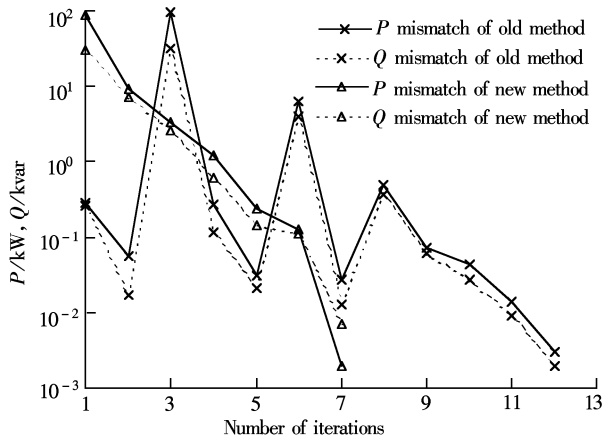


Fig. 9 Convergence pattern (5 Loops)

because it may never get compensated.

4 Test Results for Systems with PV Buses

Tests on systems with PV buses are not as straightforward as the tests on systems with loops because they are also related to the specified voltages.

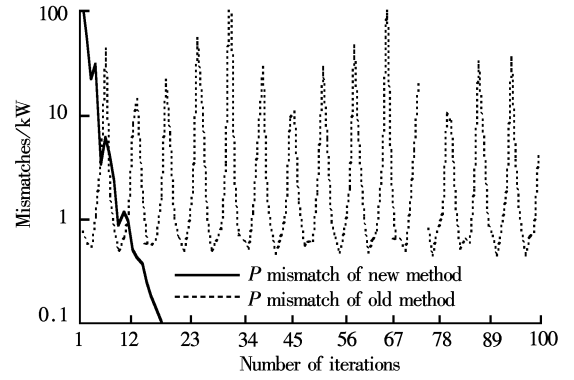


Fig. 10 Divergence pattern (5 Loops)

The curves in Fig. 11 are obtained from system with single PV bus. First, the power flow of the system without PV bus is solved. Then a bus is selected and converted into PV bus, which voltage is taken as base voltage. The minimal point in Fig. 11 corresponds to this base voltage. As shown in the figure, the larger the specified voltage division from the base voltage is, the more iterations the “Old Method” requires.

Fig. 12 illustrates that the “New Method” is less sensitive to the number of PV buses, where the voltage of PV bus is 0.01 pu (per unit) higher than the bus voltage before it is converted.

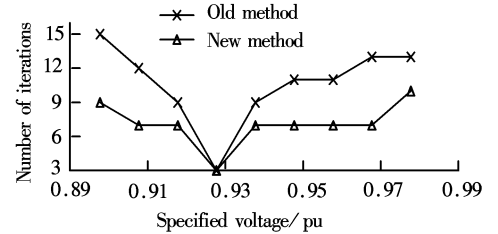


Fig. 11 Iteration number vs. controlled voltage

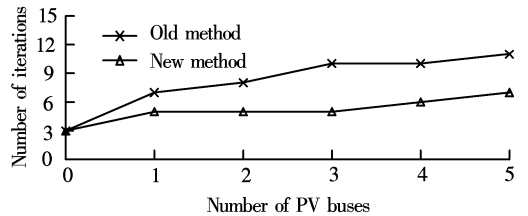


Fig. 12 Iteration number vs. number of PV buses

5 Conclusion

The test results show that the augmented Jacobian method is more efficient and robust than the conventional compensation method in handling loops and PV buses for the power flow analysis of distribution system.

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一种适用于具有弱环网和 PV 节点的配电网的增广雅可比矩阵潮流法

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摘 要 针对带弱环网和 PV 节点的配电网提出了一种增广雅可比矩阵潮流计算方法.将弱环网和 PV 节点的边界条件嵌入到雅可比矩阵中,并对雅可比矩阵的因子分解进行了详尽的设计.仿真结果表明了该算法的鲁棒性和计算效率.

关键词 功率潮流, 辐射型配电网, 弱环网, PV 节点

中图分类号 TM32