Journal of Southeast University (English Edition) Sept. 2002 Vol.18 No.3 ISSN 1003—7985

Decision-Making Method for Problem of Goal Type Based on Evaluation Criterion

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Abstract: The presently-existing decision-making method for problem of goal type, i.e. the goal-programming, is popular to some extent. In this paper we analyzed the features of the problem and the method, based on which we found some defects of the method and pointed out these defects. To overcome these defects we absorbed the spirit and exploited concepts of evaluation criterion and the fault-measure of evaluation criterion. We proposed and applied a method with an evaluation criterion, after which we also proposed a numerical example to illustrate applicability and efficiency of the method. Thus, we found a new type of method to tackle the decision-making problem of goal type.

Key words: multi-objective decision-making, evaluation criterion, fault-measure of evaluation criterion, pre-optimized objective set

The considered multi-objective decision-making problem (MDM) involves the multi-objective optimization problem (MOP),

$$\max \boldsymbol{F}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), \cdots, f_N(\boldsymbol{x}))$$
$$\text{s.t. } \boldsymbol{x} \in X$$

where $\boldsymbol{x}$ is an n-dimensional vector of decision variables; X is the decision space; $\boldsymbol{F}(\boldsymbol{x})$ is a vector of N real-valued functions. It is assumed that the objectives are in conflict and incommensurable.

Definition 1 (Efficient solution and non-inferior objective solution) A solution $\boldsymbol{x}^e \in X$ is said to be efficient if for any $\boldsymbol{x} \in X$ satisfying $f_i(\boldsymbol{x}) > f_i(\boldsymbol{x}^e)(\exists i)$, $f_j(\boldsymbol{x}) < f_j(\boldsymbol{x}^e)$ for at least one other index $j(\neq i)$. Correspondingly, $\boldsymbol{F}(\boldsymbol{x}^e)$ is said to be a non-inferior objective solution.

The solution to the MDM problem reduces to finding some of efficient solutions which can satisfy the decision-maker (DM). It is assumed that the DM has a real-valued value function v, defined on the values of objectives but it is not explicitly known. With this assumption, the MDM reduces to

$$\max v(\boldsymbol{F}(\boldsymbol{x}))$$
$$\text{s.t. } \boldsymbol{x} \in X$$

Within this mathematical framework, the primary objective of MDM solution methods is to find the best compromise solution.

Definition 2 (Best compromise solution and best compromise objective solution) The best compromise solution is an efficient solution that maximizes the DM's value function. If $\boldsymbol{x}^b$ is the best compromise solution, $\boldsymbol{F}(\boldsymbol{x}^b)$ is said to be the best compromise objective solution.

According to Shin and Ravindran[1], in the last two decades, most research has been concerned with developing solution methods based on different assumptions and approaches to measure or derive the preference function, and in some cases, part of the function. The solution methods developed in MDMs can be categorized by the basic assumptions made with respect to the preference function: ① When complete information of the preference function is available from the DM; ② When no information is available; ③ When partial information is obtainable progressively from the DM.

Among all the solution approaches, the interactive methods are becoming popular and are considered promising for MDMs[2,3] for in most cases partial information is obtainable.

In the process of the feasible region reduction methods, in the beginning, the DM is required to give ideal solution or the goal and the weights of objectives, then the goal and weights are adjusted by interactively obtained preference information to finally get the best compromise solution. However, in general, DM will find that it is very difficult to set a goal actually as it being very possible that there is no solution up to the level of expectation or of vast number, and the deciding and adjusting of the weights are also rather subjective

Received 2001-12-20.

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and unserious. According to this kind of model, the natural decision making heuristic is to concerntrate initially on improving what seems to be the most critical problem area (criterion), until it has been improved to some satisfactory level of performance. Thereafter, attention is shifted to the next most important issue and so on. Goal programming formalizes this heuristic, although we should note that Simon did not view this heuristic as necessarilly desirable, but it is a response to bounded rationality[4].

In Ref.[5], the authors proposed a class of interactive MDM method which did not make special assumption about the objective functions and the decision space. According to the experience of solving MDM problems of the same class and the feature of the MDM problem at hand, evaluation criterion from certain class is regarded as approximately a substitution for the value function. By actually collected preference information from the DM, the evaluation criterion is adjusted gradually to be one with smaller deviation, which is to be used in decision-making. Because of simplicity of the evaluation criterion as it is, the deviation of the criterion from the DM's preference structure often exists undoubtedly, hence we cannot obtain the best compromise objective solution directly just by means of the evaluation criterion, but a subset of $F(X)$ ($\{\boldsymbol{F}(\boldsymbol{x}) \mid \boldsymbol{x} \in X\}$ is denoted by $F(X)$) of small-range, a pre-optimized objective set, involving the best compromise objective solution. To obtain the best compromise objective solution, we should perform the same steps iteratively in the known pre-optimized objective set or use some MDM methods presently known[6-8] such as the GDF method.

In this paper, we shall apply this method to the problem of goal type to overcome the defects of feasible region reduction methods.

In section 1, we elicit the multiple objective decision-making method of goal type by a kind of evaluation criterion. In section 2, we give a numerical example to apply our new method. In section 3, we summarize the paper.

1 The Multiple Objective Decision-Making Method of Goal Type by a Kind of Evaluation Criterion

Definition 3 (Pre-optimized objective set (POO-set) A subset Y' of $F(X)$ is said to be a pre-optimized objective set if for any $\boldsymbol{y}_1 \in F(X)$, $\exists$ $\boldsymbol{y}'_1 \in Y', v(\boldsymbol{y}'_1) \geqslant v(\boldsymbol{y}_1)$.

By the definition, it is evident that if and only if the best compromise objective solution is involved in Y', *and* Y' is a POO-set.

Suppose that the evaluation criterion that the DM now using is $r(\boldsymbol{f})$ where $r(\boldsymbol{f})$ is a real-valued function defined on the values of objectives, and that $\forall \boldsymbol{f}^{(1)}$, $\boldsymbol{f}^{(2)} \in F(X)$, $\boldsymbol{f}^{(1)}$ is said to be better than $\boldsymbol{f}^{(2)}$ by $r(\boldsymbol{f})$ if $r(\boldsymbol{f}^{(1)}) > r(\boldsymbol{f}^{(2)})$. $\forall$ $\boldsymbol{A}, \boldsymbol{B} \in F(X)$ ($\boldsymbol{A} \neq \boldsymbol{B}$), the deviation of $r(\boldsymbol{f})$ from $v(\boldsymbol{f})$ in the case of $\boldsymbol{A}$ and $\boldsymbol{B}$ can be

$$\frac{|(r(\boldsymbol{A}) - r(\boldsymbol{B})) - (v(\boldsymbol{A}) - v(\boldsymbol{B}))|}{\|\boldsymbol{A} - \boldsymbol{B}\|}$$

This is only one kind of expression while the others can also be used according to the feature of the MDM problem at hand.

Definition 4 (Fault-measure of an evaluation criterion) On $\overline{\boldsymbol{Y}} \subseteq F(X)$, the fault-measure of the evaluation criterion $r(\boldsymbol{f})$ is

$$\mathrm{FM}_r(\overline{\boldsymbol{Y}}) \triangleq \max_{\substack{\boldsymbol{A}, \boldsymbol{B} \in \overline{\boldsymbol{Y}} \\ (\boldsymbol{A} \neq \boldsymbol{B})}} \frac{|(r(\boldsymbol{A}) - r(\boldsymbol{B})) - (v(\boldsymbol{A}) - v(\boldsymbol{B}))|}{\|\boldsymbol{A} - \boldsymbol{B}\|}$$

The effect of an evaluation criterion $r(\boldsymbol{f})$ can be expressed by this kind of fault-measure to some extent.

When the fault-measure of criterion $r(\boldsymbol{f})$ is not zero, by this evaluation criterion the best compromise objective solution cannot be derived by solving the one objective mathematical rogramming problem:

$$(r - \mathrm{MP}) \quad \max_{\boldsymbol{x} \in X} r(\boldsymbol{F}(\boldsymbol{x}))$$

But if the fault-measure of $r(\boldsymbol{f})$ is provided, the pre-optimized objective set can be separated from $F(X)$ on the basis of the optimal solution of the problem (r − MP).

Suppose there are $\boldsymbol{P} \in \boldsymbol{F}(\boldsymbol{x})$, $\boldsymbol{P}' \in F(X)$ where $r(\boldsymbol{P}') = \max_{\boldsymbol{P} \in F(X)} r(\boldsymbol{P})$ and $v(\boldsymbol{P}) = \max_{\boldsymbol{P} \in F(X)} v(\boldsymbol{P})$, then $r(\boldsymbol{P}) \leqslant r(\boldsymbol{P}')$, $v(\boldsymbol{P}) \geqslant v(\boldsymbol{P}')$.

Theorem 1[5] A pre-optimized objective set is $\mathrm{MS} \triangleq \{\boldsymbol{P} \mid r(\boldsymbol{P}) - r(\boldsymbol{P}') \geqslant \mathrm{FM}_r(F(X))d, \boldsymbol{P} \in F(X)\}$, where $d \triangleq \max\|\boldsymbol{P} - \boldsymbol{P}'\|$, $\boldsymbol{P} \in F(X)$ or noninferior objective frontier.

However, instead of $\mathrm{FM}_r(F(X))$ itself, we often get an upper boundary of $\mathrm{FM}_r(F(X))$, denoted as $\mathrm{UF}_r(F(X))$. Then a pre-optimized objective set can also be obtained, i.e. $\mathrm{MS} \triangleq \{\boldsymbol{P} \mid r(\boldsymbol{P}) - r(\boldsymbol{P}') \geqslant - \mathrm{UF}_r(F(X))d, \boldsymbol{P} \in F(X)\}$[5].

There have been many decision-making methods in feasible region reduction for problem of goal type[4], but all of which have basic defaults: ①No enough foundation to determine a rationalized goal; ② Unseriousness is assessment and modification of the weights of the objectives.

But, in the following part, these two defaults will be overcomed by using a kind of evaluation criterion, which can rationalize the goal and lead to the unnecessity for the DM to assess the weights of objectives. Here, also the preference information from the DM making contrast of feasible objective solutions is collected, by which the central point of the evaluation criterion (actually the goal) is to be adjusted, and the evaluation criterion as well. Using this adjusted evaluation criterion, we can obtain the best compromise objective solution or a pre-optimized objective set.

For a MDM problem of goal type we are discussing here, it roughly holds that there is a goal $\boldsymbol{H} = (h_1, h_2, \cdots, h_N)$ and a weight vector $(a_1, a_2, \cdots, a_{N-1}, a_N = 1 - \sum_{i=1}^{N-1} a_i)$ and a positive number p which is not explicitly known and that for $\boldsymbol{Q}_1 = (q_1^1, \cdots, q_1^N)$ and $\boldsymbol{Q}_2 = (q_2^1, \cdots, q_2^N)$, if and only if $\left(\sum_{i=1}^{N} a_i |q_1^i - h_i|^p\right)^{1/p} \geqslant \left(\sum_{i=1}^{N} a_i |q_2^i - h_i|^p\right)^{1/p}$, $v(\boldsymbol{Q}_1) \geqslant v(\boldsymbol{Q}_2)$. Then $r(\boldsymbol{Q}) \triangleq \left(\sum_{i=1}^{N} a_i |q^i - h_i|^p\right)^{1/p}$ is called a good evaluation criterion for the MDM problem.

Our strategy is to start from a rough evaluation criterion and to acquire preference information from the DM and with the help of the information to adjust the rough evaluation criterion to obtain a good evaluation criterion.

Firstly, the DM is required to give a rough goal vector $\boldsymbol{H} = (h_1, \cdots, h_N)$, and other rough weight vector $(a_1, \cdots, a_N)$ and a positive number p. Then we get a rough evaluation criterion i.e. $r(\boldsymbol{Q}) \triangleq \left(\sum_{i=1}^{N} a_i |q^i - h_i|^p\right)^{1/p}$.

Secondly, we shall get preference information from the DM.

Definition 5 (Equal level surface of an evaluation criterion) A set R is said to be an equal level surface of the evaluation criterion $r(\boldsymbol{Q})$, if $\forall$ $\boldsymbol{A}, \boldsymbol{B} \in R$, $r(\boldsymbol{A}) = r(\boldsymbol{B})$.

$\forall\, \boldsymbol{C} \in F(X)$, $\boldsymbol{D} \in R$, if $r(\boldsymbol{C}) = r(\boldsymbol{D})$, then $\boldsymbol{C} \in R$.

On an equal level surface of $r(\boldsymbol{f})$, $\boldsymbol{Q}_1'$ and $\boldsymbol{Q}_1$ are selected where the DM decides that $\boldsymbol{Q}_1'$ is evidently better than $\boldsymbol{Q}$, here, $r(\boldsymbol{Q}_1) = r_1$. We move from $\boldsymbol{Q}_1'$ along the direction that the value of $r(S)$ is either decreasing or increasing, but the value of the points is decreasing by DM. Suppose that we encounter $\boldsymbol{Q}_2$ whose value the DM decides is the same as point $\boldsymbol{Q}_1$, where $r(\boldsymbol{Q}_2) = r_2 (r_2 \neq r_1)$.

We proceed to collect preference information from the DM from other indicative regions in $F(X)$, i.e. $\boldsymbol{Q}_1^{(i)}$ and $\boldsymbol{Q}_2^{(i)}$ where the value of $\boldsymbol{Q}_1^{(i)}$ is the same as $\boldsymbol{Q}_2^{(i)}$ while $r(\boldsymbol{Q}_2^{(i)}) \neq r(\boldsymbol{Q}_1^{(i)})$.

Thirdly, with the help of the preference information we can get a better evaluation criterion $r'(\boldsymbol{Q})$ than $r(\boldsymbol{Q})$. It is evident that a better evaluation criterion should have a smaller fault-measure, i.e.

$$\mathrm{FM}_r'(F(X)) = \min \mathrm{FM}_{\bar{r}}(F(X)) = \min \max \frac{|(\bar{r}(\boldsymbol{Q}_1) - \bar{r}(\boldsymbol{Q}_2)) - (v(\boldsymbol{Q}_1) - v(\boldsymbol{Q}_2))|}{\|\boldsymbol{Q}_1 - \boldsymbol{Q}_2\|}$$

Use $\max_i \dfrac{|(\bar{r}(\boldsymbol{Q}_1^{(i)}) - \bar{r}(\boldsymbol{Q}_2^{(i)})) - (v(\boldsymbol{Q}_1^{(i)}) - v(\boldsymbol{Q}_2^{(i)}))|}{\|\boldsymbol{Q}_1^{(i)} - \boldsymbol{Q}_2^{(i)}\|}$ to replace $\mathrm{FM}_{\bar{r}}(F(X))$, i.e.

$$\mathrm{FM}_r'(F(X)) = \min \max_i \frac{|(\bar{r}(\boldsymbol{Q}_1^{(i)}) - \bar{r}(\boldsymbol{Q}_2^{(i)})) - (v(\boldsymbol{Q}_1^{(i)}) - v(\boldsymbol{Q}_2^{(i)})|}{\|\boldsymbol{Q}_1^{(i)} - \boldsymbol{Q}_2^{(i)}\|} = \min \max_i \frac{|\bar{r}(\boldsymbol{Q}_1^{(i)}) - \bar{r}(\boldsymbol{Q}_2^{(i)})|}{\|\boldsymbol{Q}_1^{(i)} - \boldsymbol{Q}_2^{(i)}\|} \tag{1}$$

The problem now turns out to search for r' or the vector $\boldsymbol{H}$, the vector $(a_1, \cdots, a_N)$ and the positive number p to get the smallest $\mathrm{FM}_r'(F(X))$ possible.

Fourthly, use the good evaluation criterion we have acquired to obtain a POO-set(theorem 1).

We have acquired a good evaluation criterion r' and the fault-measure $\mathrm{FM}_r'(F(X))$, then we should solve $r'(\boldsymbol{Q}') \triangleq \max_{\boldsymbol{Q} \in F(X)} r'(\boldsymbol{Q})$ and furthermore $d \triangleq \max_{\boldsymbol{Q} \in F(X) \text{ or noninferior objective frontier}} \|\boldsymbol{Q} - \boldsymbol{Q}'\|$, so the POO-set turns out to be $\mathrm{MS} = \{\boldsymbol{Q} \mid 0 \geqslant r'(\boldsymbol{Q}) - r'(\boldsymbol{Q}') \geqslant -\mathrm{FM}_r'(F(X))d\}$.

Fifthly, within the POO-set MS, apply the GDF method or other interactive method we have known to obtain the best compromise objective solution or the best compromise solution.

2 A Numerical Example of Decision-Making Solved by the Method

There is an MDM problem of good type involving the following multiple objective programming problem:

$$\max \boldsymbol{F}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), f_3(\boldsymbol{x})) = \begin{bmatrix} 2 & 1 & -1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix} \boldsymbol{x}$$

subject to

$$x_1 + 12x_2 - x_3 - 32 \geqslant 0$$
$$11x_1 + 11x_2 + 4x_3 - 12 \geqslant 0$$
$$6x_1 + 7x_2 + 9x_3 - 12 \geqslant 0$$
$$2x_1 + x_2 - x_3 - 2 \geqslant 0$$
$$-x_1 + 2x_2 + x_3 - 2 \geqslant 0$$
$$x_1 - x_2 + 2x_3 - 4 \geqslant 0$$
$$(2x_1 + x_2 - x_3 - 5)^2 + (-x_1 + 2x_2 + x_3 - 6)^2 + (x_1 - x_2 + 2x_3 - 7)^2 \leqslant 625$$
$$x_i \geqslant 0 \qquad i = 1,2,3$$

Firstly, the DM is required to give rough parameters for the evaluation criterion.

Three single objective programming problems $\max_{x \in X} f_i(\boldsymbol{x})$ $(i = 1,2,3)$ are solved, and the optimal objective solutions for $f_1(\boldsymbol{x})$, $f_2(\boldsymbol{x})$, $f_3(\boldsymbol{x})$ are 30, 31 and 31.516, respectively. Then the DM sets $\boldsymbol{H} = (30, 31, 31.516)$.

Furthermore, the DM sets $p = 1$, and after weighing $f_1(\boldsymbol{x})$, $f_2(\boldsymbol{x})$ and $f_3(\boldsymbol{x})$ the DM gives a rough weight vector $(a_1, a_2, a_3) = (0.4, 0.3, 0.3)$. So, the rough evaluation criterion turns out to be $r(\boldsymbol{f}) = (0.4|f_1(\boldsymbol{x}) - 30| + 0.3|f_2(\boldsymbol{x}) - 31| + 0.3|f_3(\boldsymbol{x}) - 31.516|)$.

Secondly, with the help of the rough evaluation criterion, we obtain the couples of point, each of which has the same value as the other,

$\boldsymbol{Q}_1^{(1)} = (17.702, 18.702, 19.218)$ and

$\boldsymbol{Q}_2^{(1)} = (23.202, 14.013, 19.218)$

$\boldsymbol{Q}_1^{(2)} = (19.702, 18.702, 17.502)$ and

$\boldsymbol{Q}_2^{(2)} = (19.702, 22.402, 13.058)$

$\boldsymbol{Q}_1^{(3)} = (16.912, 17.495, 21.515)$ and

$\boldsymbol{Q}_2^{(3)} = (20.412, 17.495, 14.148)$

Thirdly, with the preference information, we proceed to obtain a good evaluation criterion. We solve the optimization problem (1), during which we search for the parameters to get the smallest $\mathrm{FM}'_r(F(X))$ possible. Then we get $\boldsymbol{H}' = (34.510, 32.637, 25.369)$, $(a'_1, a'_2, a'_3) = (0.33, 0.33, 0.34)$, and $P' = 2$. The $\mathrm{FM}'_r(F(X)) = 0.003\,45$.

Then to get the POO-set, we solve $r'(\boldsymbol{Q}') = \max_{Q \in F(X)} r'(\boldsymbol{Q})$ to get $\boldsymbol{Q}' = (21.850, 21.210, 17.489)$. And we solve $d \triangleq \max_{Q} \|\boldsymbol{Q} - \boldsymbol{Q}'\| = 30.971$. So, the POO-set $\mathrm{MS} = \{\boldsymbol{Q} \mid 0 \geqslant r'(\boldsymbol{Q}) - r'(\boldsymbol{Q}') \geqslant -30.971 \times 0.003\,45 = -0.106\,8\}$.

Finally, using the GDF method, within MS we obtain the best compromise objective solution $\boldsymbol{Q}^{(*)} = (21.904, 20.986, 17.709)$. The best compromise solution is $\boldsymbol{x}^{(*)} = (10.119, 10.924, 9.257)$.

3 Conclusion

In the beginning, we have surveyed the two defects of old methods for problems of goal type. Then in section 1, we applied other type of decision-making method for this kind of problems. The numerical example in section 2 illustrates the method having avoided the defects and the practical essence of the method.

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基于评价准则的目标理想点类型多目标决策方法

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摘　要　现有的求解目标理想点类型决策问题的方法——目标规划法,在某种程度上是较为常用的方法.本文分析了该问题及方法的特点.在此基础上发现了该方法的缺陷并加以指出.为了克服这些缺陷,利用了评价准则及其偏差测度的概念,提出并运用了基于评价准则的方法,给出了一个数值运算的例子,展示了该方法的可用性及有效性.我们发现了一种新类型的方法来求解目标理想点类型决策问题.

关键词　多目标决策,评价准则,偏差测度,准最优目标集

中图分类号　F202