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One Computational Method of the Eigenvalues of the Horizontal Vibration Problem of Beam

Huang Bin*

(Department of Engineering, Jinling Institute of Technology, Nanjing 210038, China)

Abstract: This paper considers one computational method of the eigenvalues' approximate value of the horizontal vibration problem of beam. The proof of our main result is based on the variational formula. First of all, Cauchy inequality is used to obtain a basic inequality. Secondly, the functions of basis are made by Galerkin method, and the error estimates of eignevalues are obtained by Cauchy inequality. At last, the computational method of the approximate value of the eigenvalues turns out immediately, and accuracy of the $(n-1)$-th approximate value is estimated by the n-th approximate value. When n is increased, the accuracy of eigenvalue λ_k is increased. When n is appropriately selected, the accuracy of λ_k we need is obtained. This computational method is significant both in applications and in theory.

Key words: horizontal vibration problem of beam, eigenvalue, eigenfunction, Galerkin method

1 Main Result

Let $(a,b)\subset R$ be a bounded interval. We consider the approximate value of the eigenvalues of the horizontal vibration problem of beam

$$\left.\begin{aligned}&(p(x)y'')''-(q(x)y')'+r(x)y=\lambda s(x)y\\&y(a)=y(b)=y'(a)=y'(b)=0\end{aligned}\right\}\tag{1}$$

where $p(x)\in C^4([a,b])$; $q(x)\in C^2([a,b])$; $r(x), s(x)\in C([a,b])$, such that

$$\mu_{11}\leqslant p(x)\leqslant\mu_{12}\tag{2}$$

$$\mu_{21}\leqslant q(x)\leqslant\mu_{22}\tag{3}$$

$$\mu_{31}\leqslant r(x)\leqslant\mu_{32}\tag{4}$$

$$\mu_{41}\leqslant s(x)\leqslant\mu_{42}\tag{5}$$

where $0<\mu_{11}\leqslant\mu_{12}$, $0<\mu_{41}\leqslant\mu_{42}$, $0\leqslant\mu_{i1}\leqslant\mu_{i2}\ (i=2,3)$.

The estimates for the bound of the $(n+1)$-th eigenvalue of problem (1) are well known[1-5]. There is the work for the approximate value of the eigenvalue on problem (1)[6]. They discussed the approximate value of the first eigenvalue of problem (1). We now consider many approximate values of eigenvalues of problem (1), and computational method is simpler. This method is interesting and significant both in applications and in theory.

The proof of our main result is based on the variational formula. First of all, we use Cauchy inequality to obtain a basic inequality. Secondly, we make the functions of basis by Galerkin method, and give the error estimate of eignevalues by integral and Cauchy inequality. At last, the computational method of the approximate value of the eigenvalues turns out immediately.

Let $0<\lambda_1\leqslant\lambda_2\leqslant\cdots\leqslant\lambda_n\leqslant\cdots$ denote the successive eigenvalues for (1). Suppose that eigenfunctions $y_k\ (k=1,2,\cdots)$ of (1) corresponding to the eigenvalue λ_k is weighted normal to y_k such that

$$\int_a^b s(x)y_k^2\,\mathrm{d}x=1\qquad k=1,2,\cdots\tag{6}$$

By (6) and (1), we obtain

$$\lambda_k=\int_a^b p(x)(y''_k)^2\mathrm{d}x+\int_a^b q(x)(y'_k)^2\mathrm{d}x+\int_a^b r(x)y_k^2\,\mathrm{d}x\qquad k=1,2,\cdots\tag{7}$$

Using (2) to (4) and (7), we get

$$\int_a^b(y''_k)^2\mathrm{d}x\leqslant\frac{\lambda_k}{\mu_{11}}\qquad k=1,2,\cdots\tag{8}$$

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* Born in 1958, male, lecturer.

Lemma Let $a < b$, $u''(x) \in L^2([a,b])$, $u(a) = u(b) = 0$, and $u'(a) = u'(b) = 0$. Then ① $\int_a^b (u'(x))^2 \mathrm{d}x \leqslant \frac{(b-a)^2}{4} \int_a^b (u''(x))^2 \mathrm{d}x$; ② $\int_a^b u^2(x) \mathrm{d}x \leqslant \frac{(b-a)^4}{16} \int_a^b (u''(x))^2 \mathrm{d}x$.

Proof At first we prove the inequality

$$\int_a^b u^2(x) \mathrm{d}x \leqslant \frac{(b-a)^2}{4} \int_a^b (u'(x))^2 \mathrm{d}x \tag{9}$$

Using $u(a) = 0$, for each $x \in [a,b]$, we have

$$u(x) = \int_a^x u'(t) \mathrm{d}t$$

By the Cauchy inequality, we obtain

$$u^2(x) = (\int_a^x u'(t) \mathrm{d}t)^2 \leqslant (x-a) \int_a^x (u'(t))^2 \mathrm{d}t \leqslant (x-a) \int_a^b (u'(x))^2 \mathrm{d}x$$

Therefore

$$\int_a^{\frac{a+b}{2}} u^2(x) \mathrm{d}x \leqslant \frac{(b-a)^2}{8} \int_a^b (u'(x))^2 \mathrm{d}x \tag{10}$$

Similarly, we have

$$\int_{\frac{a+b}{2}}^b u^2(x) \mathrm{d}x \leqslant \frac{(b-a)^2}{8} \int_a^b (u'(x))^2 \mathrm{d}x \tag{11}$$

Combining (10) with (11), we obtain (9). Replacing $u(x)$ in (9) by $u'(x)$, we see that lemma ① follows. Using (9) and lemma ①, we see that lemma ② follows. By lemma and (8), we get

$$y_k \in L^2([a,b]),\ y'_k \in L^2([a,b]),\ y''_k \in L^2([a,b]) \qquad k = 1,2,\cdots$$

Because $L^2([a,b])$ is separable space, we choose that $\varphi_i(x)(i = 1,2,\cdots)$ is the functions of basis in $L^2([a,b])$,

$$\varphi_i(a) = \varphi_i(b) = 0,\ \varphi'_i(a) = \varphi'_i(b) = 0 \qquad i = 1,2,\cdots$$

and $$y_k = \sum_{i=1}^{\infty} c_{ki}\varphi_i \qquad k = 1,2,\cdots$$

where $c_{ki}(k,i = 1,2,\cdots)$ is the constant.

We consider the following eigenvalue problem

$$(p(x)y''_{kn})'' - (q(x)y'_{kn})' + r(x)y_{kn} = \lambda_{kn}s(x)y_{kn} \tag{12}$$

where $y_{kn} = \sum_{i=1}^{n} c_{ki}\varphi_i$, $k = 1,2,\cdots n$.

$$y_k = y_{kn} + y_{kn}^* \tag{13}$$

where $y_{kn}^* = y_k - y_{kn} = \sum_{i=n+1}^{\infty} c_{ki}\varphi_i$.

By (1) and (13), we have

$$(p(x)y''_{kn})'' + (p(x)y_{kn}^{*''})'' - (q(x)y'_{kn})' - (q(x)y_{kn}^{*'})' + r(x)y_{kn} + r(x)y_{kn}^* = \\ \lambda_k s(x)y_{kn} + \lambda_k s(x)y_{kn}^* \tag{14}$$

Combining (12) with (14) yields

$$(\lambda_k - \lambda_{kn})s(x)y_{kn} = (p(x)y_{pn}^{*''})'' - (q(x)y_{pn}^{*'})' + r(x)y_{kn}^* - \lambda_k s(x)y_{kn}^* \tag{15}$$

Theorem Let λ_k and λ_{kn} be the k-th eigenvalue of (1) and (12), y_k and y_{kn} be the eigenfunction of (1) and (12) correspond to the eigenvalue λ_k and λ_{kn}, respectively. Then

$$|\lambda_k - \lambda_{kn}| \leqslant \frac{1}{\int_a^b s(x)y_{kn}^2 \mathrm{d}x}\left[\mu_{12}\sqrt{\frac{\lambda_k}{\mu_{11}}} + \frac{(b-a)^2}{4}\left(\mu_{22}\sqrt{\frac{\lambda_k}{\mu_{11}}} + \frac{\mu_{32}}{\sqrt{\mu_{41}}} + \lambda_k\sqrt{\mu_{42}}\right)\right]\left(\int_a^b |y''_k - y''_{kn}|^2 \mathrm{d}x\right)^{\frac{1}{2}} \tag{16}$$

Proof Using (15) and integrating by parts, we find

$$(\lambda_k - \lambda_{kn})\int_a^b s(x)y_{kn}^2 \mathrm{d}x = \int_a^b y_{kn}(p(x)y_{kn}^{*''})'' \mathrm{d}x - \int_a^b y_{kn}(q(x)y_{kn}^{*'})' \mathrm{d}x + \int_a^b y_{kn}r(x)y_{kn}^* \mathrm{d}x - \lambda_k \int_a^b y_{kn}s(x)y_{kn}^* \mathrm{d}x =$$

$$\int_a^b p(x)y''_{kn}y^{*''}_{kn}\mathrm{d}x + \int_a^b q(x)y'_{kn}y^{*'}_{kn}\mathrm{d}x + \int_a^b r(x)y_{kn}y^{*}_{kn}\mathrm{d}x - \lambda_k\int_a^b s(x)y_{kn}y^{*}_{kn}\mathrm{d}x \tag{17}$$

By lemma, the Cauchy inequality, (2) – (5), (8), (13), and (17), we obtain

$$|\lambda_k - \lambda_{kn}|\int_a^b s(x)y_{kn}^2\mathrm{d}x \leqslant \mu_{12}\left(\int_a^b |y''_{kn}|^2\mathrm{d}x\right)^{\frac{1}{2}}\left(\int_a^b |y''_k - y''_{kn}|^2\mathrm{d}x\right)^{\frac{1}{2}} +$$
$$\mu_{22}\left(\int_a^b |y'_{kn}|^2\mathrm{d}x\right)^{\frac{1}{2}}\left(\int_a^b |y'_k - y'_{kn}|^2\mathrm{d}x\right)^{\frac{1}{2}} + \mu_{32}\left(\int_a^b y_{kn}^2\,\mathrm{d}x\right)^{\frac{1}{2}}\left(\int_a^b |y_k - y_{kn}|^2\mathrm{d}x\right)^{\frac{1}{2}} +$$
$$\lambda_k\left(\int_a^b s(x)y_{kn}^2\,\mathrm{d}x\right)^{\frac{1}{2}}\left(\int_a^b s(x)|y_k - y_{kn}|^2\mathrm{d}x\right)^{\frac{1}{2}} \leqslant$$
$$\left[\mu_{12}\sqrt{\frac{\lambda_k}{\mu_{11}}} + \frac{(b-a)^2}{4}\left(\mu_{22}\sqrt{\frac{\lambda_k}{\mu_{11}}} + \frac{\mu_{32}}{\sqrt{\mu_{41}}} + \lambda_k\sqrt{\mu_{42}}\right)\right]\left(\int_a^b |y''_k - y''_{kn}|^2\mathrm{d}x\right)^{\frac{1}{2}}$$

i.e.

$$|\lambda_k - \lambda_{kn}| \leqslant \frac{1}{\int_a^b s(x)y_{kn}^2\,\mathrm{d}x}\left[\mu_{12}\sqrt{\frac{\lambda_k}{\mu_{11}}} + \frac{(b-a)^2}{4}\left(\mu_{22}\sqrt{\frac{\lambda_k}{\mu_{11}}} + \frac{\mu_{32}}{\sqrt{\mu_{41}}} + \lambda_k\sqrt{\mu_{42}}\right)\right]\left(\int_a^b |y''_k - y''_{kn}|^2\mathrm{d}x\right)^{\frac{1}{2}}$$

Since $y_k \in L^2([a,b])$, $y'_k \in L^2([a,b])$, $y''_k \in L^2([a,b])(k = 1,2,\cdots)$ and (6), we get

$$\lim_{n\to\infty}\int_a^b s(x)y_{kn}^2\,\mathrm{d}x = \int_a^b s(x)y_k^2\,\mathrm{d}x = 1$$

and

$$\lim_{n\to\infty}\int_a^b |y''_k - y''_{kn}|^2\mathrm{d}x = 0 \qquad k = 1,2,\cdots$$

Therefore,

$$\lambda_k = \lim_{n\to\infty}\lambda_{kn} \qquad k = 1,2,\cdots$$

Namely, λ_{kn} is the approximate value of λ_k, the right-hand side of (16) is the error estimates of λ_{kn} and λ_k.

2 Computational Method

Let

$$y_n = \sum_{i=1}^n c_i\varphi_i \tag{18}$$

We consider the following eigenvalue problem

$$(p(x)y''_n)'' - (q(x)y'_n)' + r(x)y_n = \lambda s(x)y_n \tag{19}$$

Since $\varphi_i(a) = \varphi_i(b) = 0$, $\varphi'_i(a) = \varphi'_i(b) = 0(i = 1,2,\cdots,n)$, we have

$$y_n(a) = y_n(b) = 0,\ y'_n(a) = y'_n(b) = 0$$

Using (18), (19) and integrating by parts, we get

$$\sum_{i=1}^n c_i\left(\int_a^b p(x)\varphi''_i\varphi''_j\,\mathrm{d}x + \int_a^b q(x)\varphi'_i\varphi'_j\,\mathrm{d}x + \int_a^b r(x)\varphi_i\varphi_j\mathrm{d}x\right) = \lambda\sum_{i=1}^n c_i\int_a^b s(x)\varphi_i\varphi_j\mathrm{d}x \qquad j = 1,2,\cdots,n \tag{20}$$

Let

$$a_{ij} = \int_a^b p(x)\varphi''_i(x)\varphi''_j(x)\mathrm{d}x + \int_a^b q(x)\varphi'_i(x)\varphi'_j(x)\mathrm{d}x + \int_a^b r(x)\varphi_i(x)\varphi_j(x)\mathrm{d}x,\ a_{ij} = a_{ji}$$
$$i,j = 1,2,\cdots,n \tag{21}$$

$$b_{ij} = \int_a^b s(x)\varphi_i(x)\varphi_j(x)\mathrm{d}x,\ b_{ij} = b_{ji} \qquad i,j = 1,2,\cdots,n \tag{22}$$

Combining (20), (21), and (22) yields

$$\sum_{i=1}^n (a_{ij} - \lambda b_{ij})c_i = 0 \qquad j = 1,2,\cdots,n \tag{23}$$

The system of linear equations of (23) have non-zero solutions if and only if determinant is equal to zero. i.e.

$$f_n(\lambda)=\begin{vmatrix} a_{11}-\lambda b_{11} & a_{12}-\lambda b_{12} & \cdots & a_{1n}-\lambda b_{1n} \\ a_{21}-\lambda b_{21} & a_{22}-\lambda b_{22} & \cdots & a_{2n}-\lambda b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1}-\lambda b_{n1} & a_{n2}-\lambda b_{n2} & \cdots & a_{nn}-\lambda b_{nn} \end{vmatrix}=0 \tag{24}$$

By (24), we find out the eigenvalues $\lambda_{kn}(k=1,2,\cdots,n)$. Replacing λ in (23) by λ_{kn}, we obtain the eigenvector $(c_{k1},c_{k2},\cdots,c_{kn})$ of (19) corresponding to the eigenvalue λ_{kn}. Namely, we obtain that the eigenfunction y_{kn} of (19) corresponds to the eigenvalue $\lambda_{kn}(k=1,2,\cdots,n)$, and the $(n-1)$-th approximate value is estimated by n-th approximate value.

Remark By taking $n=1$ in (24), we obtain the first approximate value of the eigenvalue λ_1. By taking $n=2$ in (24), we obtain the second approximate value of the eigenvalue λ_1, the first approximate value of the eigenvalue λ_2. Similarly, by taking $n=k$ in (24), we obtain the k-th approximate value of the eigenvalue λ_1, $(k-1)$-th approximate value of the eigenvalue $\lambda_2,\cdots$, the first approximate value of the eigenvalue λ_k.

Example 1

$$\begin{cases} -((2x+1)y'')''=\lambda(x+1)y & x\in(0,1) \\ y(0)=y(1)=y'(0)=y'(1)=0 \end{cases}$$

Let $p(x)=2x+1$, $q(x)=r(x)=0$, $s(x)=x+1$, $1\leqslant p(x)\leqslant 3$, $1\leqslant s(x)\leqslant 2$.

Choose $\varphi_1(x)=\sin^2\pi x$, $\varphi_2(x)=\sin^2 2\pi x$, $\varphi_3(x)=\sin^2 3\pi x$, $\varphi_4(x)=\sin^2 4\pi x$, such that satisfy boundary condition, we obtain

$$a_{11}=\int_0^1 p(x)(\varphi''_1(x))^2\mathrm{d}x=\int_0^1(2x+1)4\pi^4\cos^2 2\pi x\mathrm{d}x=389.636$$

$$a_{12}=a_{21}=\int_0^1 p(x)\varphi''_1(x)\varphi'''_2(x)\mathrm{d}x=\int_0^1(2x+1)16\pi^4\cos 2\pi x\cos 4\pi x\mathrm{d}x=0$$

Similarly, we have

$$a_{13}=a_{31}=0,\ a_{14}=a_{41}=0,\ a_{22}=6\,234.18,\ a_{23}=a_{32}=0,\ a_{24}=a_{42}=0,\ a_{33}=31\,560.5$$

$$a_{34}=a_{43}=0,\ a_{44}=99\,746.9,\ b_{11}=b_{22}=b_{33}=b_{44}=0.562\,5$$

$$b_{12}=b_{21}=b_{13}=b_{31}=b_{14}=b_{41}=b_{23}=b_{32}=b_{24}=b_{42}=b_{34}=b_{43}=0.375$$

Therefore,

$$f_4(\lambda)=\begin{vmatrix} 389.636-0.562\,5\lambda & -0.375\lambda & -0.375\lambda & -0.375\lambda \\ -0.375\lambda & 6234.18-0.562\,5\lambda & -0.375\lambda & -0.375\lambda \\ -0.375\lambda & -0.375\lambda & 31560.5-0.562\,5\lambda & -0.375\lambda \\ -0.375\lambda & -0.375\lambda & -0.375\lambda & 99\,746.9-0.562\,5\lambda \end{vmatrix}=0 \tag{25}$$

By (25), $389.636-0.562\,5\lambda=0$, we obtain the first approximate value of λ_1, i.e.

$$\lambda_{11}=692.686$$

Solving the equation

$$\begin{vmatrix} 389.636-0.562\,5\lambda & -0.375\lambda \\ -0.375\lambda & 6\,234.18-0.562\,5\lambda \end{vmatrix}=0$$

we obtain the second approximate value of λ_1 and the first approximate value of λ_2, respectively, i.e.

$$\lambda_{12}=673.328,\ \lambda_{22}=20\,522.9$$

Solving the following equation

$$\begin{vmatrix} 389.636-0.562\,5\lambda & -0.375\lambda & -0.375\lambda \\ -0.375\lambda & 6\,234.18-0.562\,5\lambda & -0.375\lambda \\ -0.375\lambda & -0.375\lambda & 31\,560.5-0.562\,5\lambda \end{vmatrix}=0$$

we obtain the third approximate value of λ_1, the second approximate value of λ_2, and the first approximate value of λ_3, respectively, i.e.

$$\lambda_{13}=669.637,\ \lambda_{23}=19\,859.9,\ \lambda_{33}=124\,936$$

Solving Eq. (25), we get the 4th approximate value of λ_1, the third approximate value of λ_2, and the second approximate value of λ_3, and the first approximate value of λ_4, respectively, i.e.

$$\lambda_{14}=668.476 \tag{26}$$

$$\lambda_{24} = 19\,664.1 \tag{27}$$

$$\lambda_{34} = 121\,539 \tag{28}$$

$$\lambda_{44} = 430\,288$$

By (26), (27), and (28), λ_1, λ_2 and λ_3 have two effective digits, respectively.

Example 2

$$\begin{cases} ((2x+1)y'')'' - (x^2y')' + xy = \lambda(x+1)y \quad x \in (0,1) \\ y(0) = y(1) = y'(0) = y'(1) = 0 \end{cases}$$

Let $p(x) = 2x+1$, $q(x) = x^2$, $r(x) = x$, $s(x) = x+1$, $1 \leqslant p(x) \leqslant 3$, $0 \leqslant q(x) \leqslant 1$, $0 \leqslant r(x) \leqslant 1$, $1 \leqslant s(x) \leqslant 2$. Choose $\varphi_1(x) = \sin^2 \pi x$, $\varphi_2(x) = \sin^2 2\pi x$, $\varphi_3(x) = \sin^2 3\pi x$, $\varphi_4(x) = \sin^2 4\pi x$, such that satisfy boundary condition, we obtain

$$a_{11} = \int_0^1 p(x)(\varphi''_1(x))^2 \mathrm{d}x + \int_0^1 q(x)(\varphi'_1(x))^2 \mathrm{d}x + \int_0^1 r(x)(\varphi_1(x))^2 \mathrm{d}x =$$

$$\int_0^1 (2x+1)4\pi^4 \cos^2 2\pi x \mathrm{d}x + \int_0^1 x^2\pi^2 \sin^2 2\pi x \mathrm{d}x + \int_0^1 x \sin^4 \pi x \mathrm{d}x = 391.406$$

$$a_{12} = a_{21} = \int_0^1 p(x)\varphi''_1(x)\varphi''_2(x)\mathrm{d}x + \int_0^1 q(x)\varphi'_1(x)\varphi'_2(x)\mathrm{d}x + \int_0^1 r(x)\varphi_1(x)\varphi_2(x)\mathrm{d}x =$$

$$\int_0^1 (2x+1)16\pi^2 \cos 2\pi x \cos 4\pi x \mathrm{d}x + \int_0^1 2\pi^2 x^2 \sin 2\pi x \sin 4\pi x \mathrm{d}x + \int_0^1 x \sin^2 \pi x \sin^2 2\pi x \mathrm{d}x = 0.569\,444$$

Similarly, we have

$$a_{13} = a_{31} = 0.265\,625,\ a_{14} = a_{41} = 0.196\,111,\ a_{22} = 6\,240.88,\ a_{23} = a_{32} = 1.565$$

$$a_{24} = a_{42} = 0.569\,444,\ a_{33} = 31\,575.4,\ a_{34} = a_{43} = 3.063\,78,\ a_{44} = 99\,773.3$$

$b_{ij}(i,j = 1,2,3,4)$ is the same in example 1.

Therefore,

$$f_4(\lambda) = \begin{vmatrix} 391.406 - 0.562\,5\lambda & 0.569\,444 - 0.375\lambda & 0.265\,625 - 0.375\lambda & 0.196\,111 - 0.375\lambda \\ 0.569\,444 - 0.375\lambda & 6\,240.88 - 0.562\,5\lambda & 1.565 - 0.375\lambda & 0.569\,444 - 0.375\lambda \\ 0.265\,625 - 0.375\lambda & 1.565 - 0.375\lambda & 31\,575.4 - 0.562\,5\lambda & 3.063\,78 - 0.375\lambda \\ 0.196\,111 - 0.375\lambda & 0.569\,444 - 0.375\lambda & 3.063\,78 - 0.375\lambda & 99\,773.3 - 0.562\,5\lambda \end{vmatrix} = 0 \tag{29}$$

By (29), $391.406 - 0.562\,5\lambda = 0$, we obtain the first approximate value of λ_1, i.e.

$$\lambda_{11} = 695.833$$

Solving the equation

$$\begin{vmatrix} 391.406 - 0.562\,5\lambda & 0.569\,444 - 0.375\lambda \\ 0.569\,444 - 0.375\lambda & 6\,240.88 - 0.562\,5\lambda \end{vmatrix} = 0$$

we obtain the second approximate value of λ_1 and the first approximate value of λ_2, respectively. i.e.

$$\lambda_{12} = 676.403,\ \lambda_{22} = 20\,544.5$$

Solving the following equation

$$\begin{vmatrix} 391.406 - 0.562\,5\lambda & 0.569\,444 - 0.375\lambda & 0.265\,625 - 0.375\lambda \\ 0.569\,444 - 0.375\lambda & 6\,240.88 - 0.562\,5\lambda & 1.565 - 0.375\lambda \\ 0.265\,625 - 0.375\lambda & 1.565 - 0.375\lambda & 31\,575.4 - 0.562\,5\lambda \end{vmatrix} = 0$$

we obtain the third approximate value of λ_1, the second approximate value of λ_2, and first approximate value of λ_3, respectively, i.e.

$$\lambda_{13} = 672.688,\ \lambda_{23} = 19\,880.2,\ \lambda_{33} = 124\,993$$

Solving Eq.(29), we obtain the 4th approximate value of λ_1, the third approximate value of λ_2, and the second approximate value of λ_3, and the first approximate value of λ_4, respectively, i.e.,

$$\lambda_{14} = 671.519 \tag{30}$$

$$\lambda_{24} = 19\,685.2 \tag{31}$$

$$\lambda_{34} = 121\,596 \tag{32}$$

$$\lambda_{44} = 430\,398$$

By (30) to (32), λ_1, λ_2 and λ_3 have two effective digits, respectively. When n is increased, the accuracy of λ_k is increased. We appropriately select n, using (24) we can get the accuracy of λ_k we need.

References

[1] Hile G N, Protter M H. Inequalities for eigenvalue of the Laplacian[J]. *Indiana Univ Math*, 1980, **29**:523 - 538.
[2] Hile G N, Yeh R Z. Inequalities for eigenvalue of the biharmonic operator[J]. *Pacific J Math*, 1984, **112**:115 - 133.
[3] Chen Z C, Qian C L. Esttimates for discrete spectrum of Laplacian operator with any order[J]. *China Univ Sci Tech*, 1990, **20**: 259 - 265.
[4] Protter M H. Can one hear the shape of a drum? [J]. *SIAM Rev*, 1987, **29**:185 - 197.
[5] Chen Z C, Qian C L. On the upper bound of eigenvalue for elliptic equations with higher orders[J]. *Math Anal Appl*, 1994, **186**:821 - 834.
[6] Gu Yu, Qian Chunlin. The computation method of first eigenvalue of differential equation[J]. *Chinese Electronic Education*, 1996, (7): 7 - 10. (in Chinese)

梁横向振动问题的特征值的一种算法

黄 滨

(金陵科技学院工程系,南京 210038)

摘 要 本文考虑计算梁横向振动问题的特征值的近似值的一种算法. 主要结果的证明运用变分公式. 首先利用 Cauchy 不等式证明了一个基本不等式;其次采用 Galerkin 方法来构造适当的基函数, 并利用 Cauchy 不等式给出了其特征值计算的误差估计式;最后得到计算梁横向振动问题的特征值的近似值的算法, 而且可以用第 n 次近似值来估计第 $n-1$ 次的近似值的精确度. 随着 n 的增大,特征值 λ_k 的精确度逐步提高,只要适当选取 n,就可以求得所要精确度的特征值的近似值. 这个算法具有广泛的实用价值和理论价值.

关键词 梁横向振动问题, 特征值, 特征函数, Galerkin 方法
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