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Dynamic Characteristics of Transmission System for the Internal Grinder*

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Abstract: The dynamics model of the transmission system of the internal grinder is established on the bases of Riccati transfer matrix. The dynamic characteristics of the internal grinder are obtained by analyzing the relationship between dynamic modal flexibility and modal flexibility, which is used to find out the dangerous model of the transmission system and its weak areas. Then design parameters of weak areas are modified, the new one from the old structure is put forward, and the dynamic characteristics of new structure are forecast through the calculations of the modal flexibility and elasticity distribution rate. This method is applied successfully in designing the transmission system of the internal grinder.

Key words: transmission system, dynamic characteristic, modal flexibility, elasticity distribution rate

When internal grinder works, the transmission system of the grinder endures a dynamic torque and creates dynamic response of rotation. If the performance of anti-torsional oscillation is low for the transmission system of internal grinder, the internal grinder can not work steadily, which results in low machining accuracy of work-piece machined and great noise. Therefore, dynamic performance of transmission system of an internal grinder will directly affect dynamic performance of an internal grinder. Since transmission system of an internal grinder is conventionally designed according to practical experience, analogy and static method, its anti-vibrancy can not be enhanced and machining accuracy of work-piece machined is often not obtained. For the purpose of acquiring higher machining accuracy of work-piece machined, it is useful that dynamic optimum design method is used to improve dynamic performance of an internal grinder and machining quality of work-piece machined in the design of transmission system of an internal grinder. Since the main reason for the vibration of transmission system of an internal grinder is torsional oscillation[1], the paper deals with characteristics of torsional oscillation of transmission system of an internal grinder, so that the existing design shortages of transmission system of original internal grinder are found out, improved and dynamic characteristics of transmission system of an internal grinder are increased.

1 Dynamic Optimum Design Principle for Transmission System of an Internal Grinder

On torsional oscillation of transmission system of an internal grinder, its dynamic characteristics, such as the natural frequency and natural mode can be obtained by Riccati transfer matrix method. For the problem of torsional oscillation of transmission system of internal grinder, it is solved by certain mathematical methods[2,3]. The obtained characteristics can be set up as an objective function, by some optimization methods, to get an ideal value. This paper takes the mode flexibility and elasticity distribution rate as cost function to make optimization for present system's dynamic characteristics and its design modification.

1.1 Dynamic model of transmission system of principal shaft in an internal grinder

Transmission system of principal shaft of an internal grinder is shown in Fig.1(a). Dynamic model is simplified in Fig.1(b). In Fig.1(b), I_{e1}, I_{e2}, I_{e3}, I_{e4} and I_{e5} are equivalent moments of inertia; k_{e1}, k_{e2}, k_{e3} and k_{e4} are equivalent torsional stiffness.

1.2 Dynamic optimum design principle of transmission system

In the transmission system of an internal grinder, the transfer function of grinding point p is given as[4]

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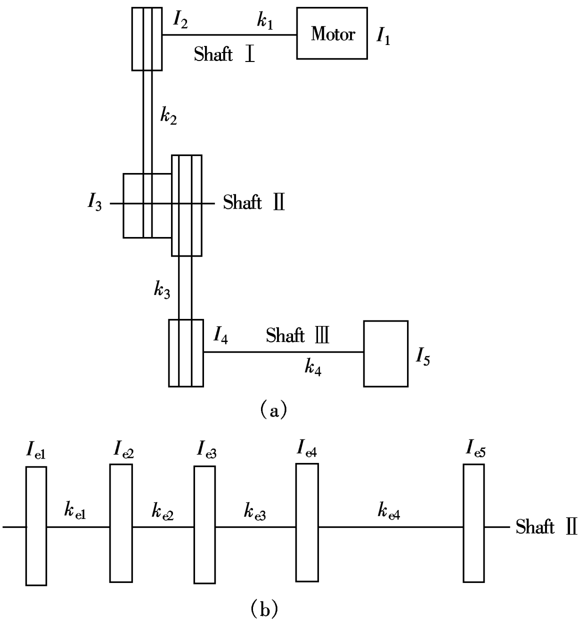


Fig. 1 Dynamic model of torsional oscillation transmission system in an internal grinder. (a) The principle of transmission system; (b) Dynamic model of torsional oscillation transmission system

$$H_p = \frac{\varphi_p}{M_p} = \sum_{r=1}^{n}\left(\frac{1}{1-\frac{\omega^2}{\omega_r^2}+i2\xi_r\frac{\omega}{\omega_r}}f_r\right) \tag{1}$$

where φ_p is the phase angle at grinding point p; M_p is the torsional moment at grinding point p; ξ_r is the model damping ratio; ω_r is the r-th order model circle frequency; $f_r = \varphi_{pr}^2/k_r$ is the r-th order modal flexibility in initial point of grinding point; φ_{pr} is the phase angle of r-th order modal at grinding point p, k_r is the stiffness of r-th order model.

In Refs.[5,6], during the r-th order modal vibration of transmission system, modal flexibility W_r in initial point of grinding point, modal flexibility f_r and modal damping ratio ξ_r have the following relationship,

$$W_r = \frac{f_r}{2\xi_r} \tag{2}$$

According to dynamic optimum design principle, the reduction of modal flexibility W_r is one of main methods in improving dynamic characteristics of transmission system. Furthermore, formula (2) is also used to decrease modal flexibility W_r from decreasing modal flexibility f_r and increasing modal damping ratio ξ_r. f_r is not relevant to damping ratio, and depends on the mass or moments of inertia, distributions and magnitudes of torsional stiffness, layout and direction in the transmission system[4]. Once the design of the transmission system is finished, modal flexibilities of every order can be determined exactly. But, modal flexibility f_r is sensitive to the modification of the transmission system, therefore the optimum design of the transmission system must be carried out.

According to mathematical model, orthogonal formula of torsional stiffness matrix is specified as

$$k_r = (\boldsymbol{\varphi}_r)^{\mathrm{T}}\boldsymbol{K}\boldsymbol{\varphi}_r \tag{3}$$

where k_r is the r-th order modal stiffness; $\boldsymbol{K}$ is stiffness matrix of the transmission system.

Maximum potential energy of the r-th order mode is given as

$$V_{\max}^r = \frac{1}{2}(\boldsymbol{\varphi}_r)^{\mathrm{T}}\boldsymbol{K}\boldsymbol{\varphi}_r \tag{4}$$

According to formulae (3) and (4), we obtain

$$k_r = 2V_{\max}^r$$

$$f_r = \frac{\varphi_{pr}^2}{2V_{\max}^r} \tag{5}$$

In formula (1), if $\omega = 0$, we obtain

$$H_p = \sum_{r=1}^{n} f_r = f_s \tag{6}$$

$$\left(\sum_{r=1}^{n} f_r\right)/f_s = 1 \tag{7}$$

where f_s is static modal flexibility in initial point of grinding point; f_r/f_s represents the r-th order mode influencing on static modal flexibility. Since large ratio of f_r/f_s determines dynamic characteristics of the transmission system, the transmission system is modified based on large ratio, and maximum ratio of f_r/f_s is main factor influencing on dynamic characteristics of the transmission system.

If the d-th order mode is high mode and normalization of mathematical model of mode on the grinding point is made, let $\varphi_{pd} = 1$, then according to formula (5), we obtain

$$f_d = 1/(2V_{\max}^d) = 1/2\left(\sum_{t=1}^{N} V_{dt}\right) \tag{8}$$

where V_{dt} is elastic energy of the d-th order in substructure t; f_d lies on elastic energy of every substructures in the system of torsional oscillation.

Let $V_{dt}/V_{\max}^d = \mu_{dt}$, we have $\sum_{t=1}^{N}\mu_{dt} = 1$, where μ_{dt} is elastic energy distributing rate of substructure t under the d-th order mode in the system of torsional oscillation.

In a torsional oscillation system, to a substructure, its stiffness is lower, its elastic energy distribution rate is higher, then its f_d will be influenced greatly. Therefore μ_{dt} is used to estimate substructures of low stiffness in high modes and determine weakness area of the transmission system. The structure parameters are modified for aiming at weakness area of the transmission system, so that the eyeless modification of the structure parameters is avoided and dynamic characteristics of torsional oscillation system is improved effectively.

2 Dynamic Characteristics of Torsional Oscillation of Transmission System for the Internal Grinder

To a certain internal grinder (as shown in Fig. 1(b)), moments of inertia and torsional stiffness (dimensions of machined work-piece are 0.24?m in external diameter, 0.20?m in inner diameter and 0.20?m in length) are listed in Tab.1 and Tab.2.

Tab.1 Moments of inertia kg · m^2

I_{e2}	I_{e3}	I_{e4}	I_{e5}
0.000?4	0.000?2	0.008?9	0.021?6

Tab.2 Torsional stiffness kN · m/rad

k_{e1}	k_{e2}	k_{e3}	k_{e4}
28.782	8.748	2.099	96.920

2.1 Influence of work-piece dimensions on natural frequencies of torsional oscillation

Relationship between inner diameters of machined work-piece and natural frequencies is shown in Tab.3 for dimensions of a certain work-piece given 0.24?m in external diameter, 0.20 – 0.04?m in inner diameter and 0.20?m in length. From Tab.3, dimensions of machined work-piece obviously affect natural frequencies of torsional oscillation, and natural frequency is reduced with the decrease of inner diameter.

Tab.3 Relationship between inner diameters of machined work-piece and natural frequency

Inner diameter/m	First order natural frequency/Hz	Second order natural frequency/Hz
0.20	36.45	627.39
0.16	30.24	592.22
0.12	26.58	575.82
0.08	23.87	566.27
0.04	21.96	560.06

2.2 Dynamic characteristics and modification of transmission system for the original internal grinder

Dynamic characteristics of torsional oscillation system is discussed for dimensions of a certain work-piece given 0.24?m in external diameter, 0.20?m in inner diameter and 0.20?m in length. Riccati transfer function method is used to calculate the static modal flexibility f_s of original design, and the calculation results of original design are shown in Tab.4.

Tab.4 Calculation results of original design

Order	Mode frequency/Hz	$f_r/(\mathrm{rad}\cdot(\mathrm{N}\cdot\mathrm{m})^{-1})$	$f_r \cdot f_s^{-1}/\%$
1	36.45	6.252×10^{-4}	98.32
2	627.39	8.522×10^{-7}	0.134

Note: $f_s = 6.339\times10^{-4}\,\mathrm{rad}/(\mathrm{N}\cdot\mathrm{m})$.

Obviously, the first mode is high mode in the torsional oscillation system, modal flexibility accounts for 98.32% in the static flexibility, and calculation results of elastic energy distributing rate of every substructures under the first order mode are listed in Tab.5. As shown in Tab.5, weak substructures are transmission belts 6 and 4, and two transmission belts are weak in the whole transmission system and transmission belt 6 is the weakest. Therefore to enhance the stiffness of transmission belt, the stiffness of transmission belt 6 increases to 6 times that of the original value, the stiffness of transmission belt 4 increases to 2 times that of the original value, the stiffness of transmission shaft 2 will be 50% of the original one and the stiffness of transmission shaft 8 will be 50% of the original one. The static modal flexibility and modal flexibility is reduced by more than 60%, the first order modal frequency is increased by more than 30%, and comparison data of new transmission system with original transmission system are shown in Tab.6.

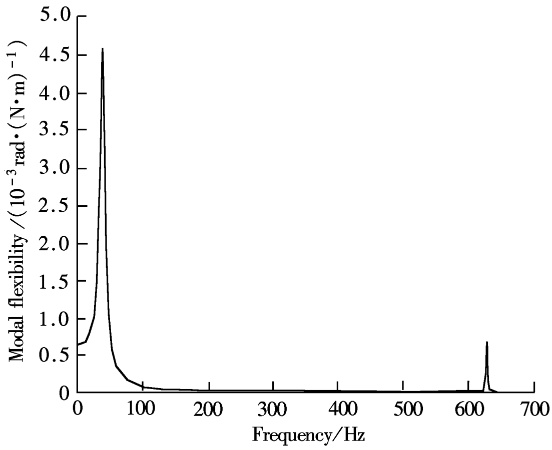
Tab.5 Elastic energy distributing rate of every substructures under the first order mode

Transmission shaft or belt	Elastic energy distributing rate μ_{1j}	Notes
2	0.055?3	
4	0.181?6	Transmission belt
6	0.754?9	Transmission belt
8	0.008?3	
Total	1.00	

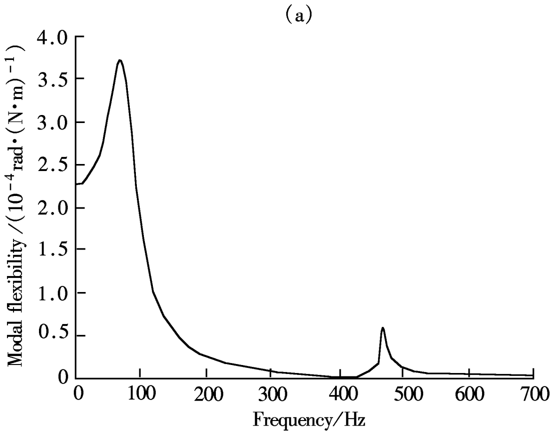
Tab.6 Comparison data of new transmission system with original transmission system

Items	First order modal frequency/Hz	Static flexibility $/(\mathrm{rad}\cdot(\mathrm{N}\cdot\mathrm{m})^{-1})$	First modal flexibility $/(\mathrm{rad}\cdot(\mathrm{N}\cdot\mathrm{m})^{-1})$
Original transmission system	36.45	6.359×10^{-4}	6.252×10^{-4}
New transmission system	61.91	2.267×10^{-4}	2.244×10^{-4}
Variance ratio	Increased 69.8%	Decreased 64.3%	Decreased 64.1%

As shown in Fig.2(a) and Fig.2(b), Riccati transfer matrix method is used to obtain dynamic flexibilities of original point in new transmission system and original transmission system. From Fig.2(a) and Fig. 2(b), maximum dynamic flexibility of original point of new transmission system is obviously less than that of original point of original transmission system, and the first order natural frequency is increased in evidence. The calculation results show that dynamic stiffness of the transmission system is obviously enhanced in the new transmission system.



(a)



(b)

Fig.2 Amplitude frequency response characteristics of original point of grinding point. (a) Original transmission system; (b) New transmission system

3 Conclusion

Relationship between dynamic modal flexibility and modal flexibility is used to find out the high mode of the transmission system and its weak areas, then design parameters of weak areas are modified, modal flexibility is reduced, thus dynamic modal flexibility is decreased and dynamic characteristics of the whole transmission system are enhanced. The method of finding out high modal flexibility and modifying design parameters avoids parameter modifications blindly and enhances the quickness of optimum modifications. It is feasible and effective that the method is applied to the design of the transmission system in internal grinder.

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内圆磨床传动系统的动力特性分析

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摘　要　基于 Riccati 传递矩阵法建立了内圆磨床传动系统的动力学模型.由模态柔度和弹性分布率的计算,分析了原内圆磨床传动系统的动态特性,应用动柔度与模态柔度的关系,找出传动系统的危险模态及其该模态上的薄弱环节,提出了结构修改方案,避免了参数调整的盲目性,并预测了其动态性能,在内圆磨床传动系统设计中得到了有效的应用.

关键词　传动系统, 动力特性, 模态柔度, 弹性分布率

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