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Applied Mode Coordinate to Solve Dynamic Responses of Antenna Mechanic System

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Abstract: Based on the Lagrange's equation and the finite element method, this paper establishes the dynamic equation of a radar antenna mechanic system which is a high accuracy system and consists of two flexible bodies. Mode coordinates are used to reduce the orders of equation. Finally, the calculation method and engineering example are given when the rotational velocity of antenna is invariable and the wind velocity is 25?m/s. The error of antenna mechanic system can be estimated using the calculation results.

Key words: antenna, dynamics, mode coordinate

The measurement precision of radar is deeply influenced by the stiffness of antenna mechanic system, especially to the three dimensional radar which is a high accuracy system. In the typical calculation model of antenna mechanic system, the radar antenna and the antenna pedestal is calculated as an integrated structure, the rigid body degree between antenna and antenna pedestal is neglected. So it is not accurate enough to use the typical calculation model to calculate antenna mechanic error and antenna traction torque. In this paper, the dynamic equation of a radar antenna mechanic system which consists of two flexible bodies is established based on the Lagrange's equation and the finite element method, the rigid body degree between antenna and antenna pedestal has not been neglected. Mode coordinate is used to reduce the orders of equation. Finally, the calculation method and engineering example are given when the angular velocity of antenna is invariable.

1 The Dynamic Equation of Radar Antenna Mechanic System

Fig.1 is the antenna mechanic system model. In general, we assume that the deformation of antenna is small enough, the damp of antenna can be neglected, and the foundation of antenna is stiffness enough. The antenna and the antenna pedestal are jointed by hinge with one rigid body degree. The coordinate frame $X^0Y^0Z^0$ is fixed at antenna pedestal. It is an Earth-fixed frame, where the vertical direction is given in Z^0. The coordinate frame $X^1Y^1Z^1$ is fixed at antenna, the coordinate origin is in hinge axis. The antenna and the antenna pedestal can be calculated separately using finite element method (FEM), and the FEM node displacements of the antenna and the antenna pedestal are used as augmented coordinates. So we have augmented coordinates $\boldsymbol{q} = [\boldsymbol{\theta}\ \boldsymbol{a}^0\ \boldsymbol{a}^1]^{\mathrm{T}}$, where $\boldsymbol{\theta}$ is the rotation angle vector of antenna, $\boldsymbol{a}^0$ is the displacement matrix of antenna pedestal FEM nodes in $X^0Y^0Z^0$, $\boldsymbol{a}^1$ is the displacement matrix of antenna FEM nodes in $X^1Y^1Z^1$.

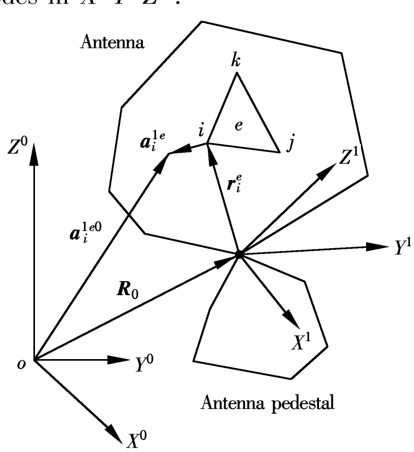

Fig.1 The model of radar antenna system

Using the augmented coordinates $\boldsymbol{q} = [\boldsymbol{\theta}\ \boldsymbol{a}^0\ \boldsymbol{a}^1]^{\mathrm{T}}$, the kinetic energy and the potential energy of antenna pedestal can be written as

$$V^0 = \frac{1}{2}\boldsymbol{q}^{\mathrm{T}}\overline{\boldsymbol{K}}^0\boldsymbol{q} - \boldsymbol{q}^{\mathrm{T}}\overline{\boldsymbol{Q}}^0 \tag{1}$$

$$T^0 = \frac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}}\overline{\boldsymbol{M}}^0\dot{\boldsymbol{q}} \tag{2}$$

In Earth-fixed frame $X^0Y^0Z^0$, the displacement

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vector of node i in element e of the antenna shown in Fig.1 is $\boldsymbol{a}_i^{1e0} = \boldsymbol{R}_0 + \boldsymbol{r}_i^e + \boldsymbol{a}_i^{1e}$, where $\boldsymbol{r}_i^e$ is the position vector of FEM node i in frame $X^1Y^1Z^1$, $\boldsymbol{a}_i^{1e}$ is the displacement vector of FEM node i in frame $X^1Y^1Z^1$. The velocity vector of FEM node i in frame $X^0Y^0Z^0$ is $\dot{\boldsymbol{a}}_i^{1e0} = \dot{\boldsymbol{R}}_0 + \dot{\boldsymbol{\theta}} \times \boldsymbol{r}_i^e + \dot{\boldsymbol{a}}_i^{1e}$. Using the augmented coordinates, the potential energy of antenna can be written as

$$V^1 = \sum_{e=1}^{L_1} V^{1e} = \frac{1}{2}\boldsymbol{q}^{\mathrm{T}}\overline{\boldsymbol{K}}^1\boldsymbol{q} - \sum_{e=1}^{L_1}(\boldsymbol{r}^e)^{\mathrm{T}}(\boldsymbol{A}^e)^{\mathrm{T}}\boldsymbol{Q}^{1e} - \boldsymbol{q}^{\mathrm{T}}\overline{\boldsymbol{Q}}^1 \tag{3}$$

where L_1 is the FEM element total number of antenna, the third term and the second term in Eq.(3) right hand side are the potential energy of external force (include external torque); $\boldsymbol{r}^e$ is the position vector matrix of element nodes; $\boldsymbol{A}^e$ is the transform matrix (it is the function of $\boldsymbol{\theta}$). It transforms the position and the displacement of FEM nodes in $X^1Y^1Z^1$ into the Earth-fixed frame $X^0Y^0Z^0$.

The kinetic energy of antenna in the Earth-fixed frame $X^0Y^0Z^0$ can be written as

$$T^{1e0} = \frac{1}{2}(\dot{\boldsymbol{a}}^{1e0})^{\mathrm{T}}\boldsymbol{M}^{1e}\dot{\boldsymbol{a}}^{1e0}$$

$$\boldsymbol{M}^{1e} = \int_{Ve}\rho\boldsymbol{N}^{\mathrm{T}}\boldsymbol{N}\mathrm{d}V$$

where $\boldsymbol{N}$ is the interpolation function. Using the augmented coordinates, the potential energy of antenna in the Earth-fixed frame $X^0Y^0Z^0$ can be written as

$$T^1 = \sum_{e=1}^{L_1} T^{1e0} = \frac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}}\overline{\boldsymbol{M}}^1\dot{\boldsymbol{q}} \tag{4}$$

Using Eq.(1) – Eq.(4), the total kinetic energy and the total potential energy of antenna and antenna pedestal can be written as $T = T^0 + T^1$, $V = V^0 + V^1$. According to Lagrange's equation $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial(T-V)}{\partial\dot{\boldsymbol{q}}_k}\right) - \frac{\partial(T-V)}{\partial\boldsymbol{q}_k} = 0$, we can deduce a $2n$ orders nonlinear differential equation

$$\begin{bmatrix} \boldsymbol{M}_{\theta\theta} & \boldsymbol{M}_{\theta0} & \boldsymbol{M}_{\theta1} \\ \boldsymbol{M}_{\theta0}^{\mathrm{T}} & \boldsymbol{M}_{00} & \boldsymbol{M}_{01} \\ \boldsymbol{M}_{\theta1}^{\mathrm{T}} & \boldsymbol{M}_{01}^{\mathrm{T}} & \boldsymbol{M}_{11} \end{bmatrix}\begin{bmatrix} \ddot{\boldsymbol{\theta}} \\ \ddot{\boldsymbol{a}}^0 \\ \ddot{\boldsymbol{a}}^1 \end{bmatrix} + \begin{bmatrix} \frac{\partial\boldsymbol{M}_{\theta\theta}}{\partial t} & \frac{\partial\boldsymbol{M}_{\theta0}}{\partial t} & \frac{\partial\boldsymbol{M}_{\theta1}}{\partial t} \\ \frac{\partial\boldsymbol{M}_{\theta0}^{\mathrm{T}}}{\partial t} & \frac{\partial\boldsymbol{M}_{00}}{\partial t} & \frac{\partial\boldsymbol{M}_{01}}{\partial t} \\ \frac{\partial\boldsymbol{M}_{\theta1}^{\mathrm{T}}}{\partial t} & \frac{\partial\boldsymbol{M}_{01}^{\mathrm{T}}}{\partial t} & \frac{\partial\boldsymbol{M}_{11}}{\partial t} \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{a}}^0 \\ \dot{\boldsymbol{a}}^1 \end{bmatrix} - \begin{bmatrix} \frac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}}\frac{\partial\boldsymbol{M}}{\partial\boldsymbol{\theta}}\dot{\boldsymbol{q}} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \boldsymbol{K}^0 & 0 \\ 0 & 0 & \boldsymbol{K}^1 \end{bmatrix}\begin{bmatrix} \boldsymbol{\theta} \\ \boldsymbol{a}^0 \\ \boldsymbol{a}^1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{Q}_\theta + \sum_{e=1}^{L_1}[(\boldsymbol{r}^e)^{\mathrm{T}} + (\boldsymbol{a}^{1e})^{\mathrm{T}}]\left(\frac{\partial\boldsymbol{A}^e}{\partial\boldsymbol{\theta}}\right)^{\mathrm{T}}\boldsymbol{Q}^{1e} \\ \boldsymbol{Q}^0 \\ \boldsymbol{Q}^1 \end{bmatrix} \tag{5}$$

Compared with $\boldsymbol{r}^e$, the deformation displacement $\boldsymbol{a}^{1e}$ is very small and can be neglected. So the right side of Eq.(5) can be written as $\boldsymbol{Q}_\theta + \sum_{e=1}^{L_1}(\boldsymbol{r}^e)^{\mathrm{T}}\left(\frac{\partial\boldsymbol{A}^e}{\partial\boldsymbol{\theta}}\right)^{\mathrm{T}}\boldsymbol{Q}^{1e}$, where $\boldsymbol{Q}_\theta$ is antenna traction torque.

2 Modal Reduction and the Calculation Method

In general, the rotational velocity of antenna is required invariable. It means $\ddot{\boldsymbol{\theta}} = 0$, Eq.(5) can be written as

$$\begin{bmatrix} \boldsymbol{M}_{00} & \boldsymbol{M}_{01} \\ \boldsymbol{M}_{01}^{\mathrm{T}} & \boldsymbol{M}_{11} \end{bmatrix}\begin{bmatrix} \ddot{\boldsymbol{a}}^0 \\ \ddot{\boldsymbol{a}}^1 \end{bmatrix} + \begin{bmatrix} \frac{\partial\boldsymbol{M}_{00}}{\partial t} & \frac{\partial\boldsymbol{M}_{01}}{\partial t} \\ \frac{\partial\boldsymbol{M}_{01}^{\mathrm{T}}}{\partial t} & \frac{\partial\boldsymbol{M}_{11}}{\partial t} \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{a}}^0 \\ \dot{\boldsymbol{a}}^1 \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}^0 & 0 \\ 0 & \boldsymbol{K}^1 \end{bmatrix}\begin{bmatrix} \boldsymbol{a}^0 \\ \boldsymbol{a}^1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{Q}_0 \\ \boldsymbol{Q}_1 \end{bmatrix} - \dot{\boldsymbol{\theta}}\begin{bmatrix} \frac{\partial\boldsymbol{M}_{\theta0}^{\mathrm{T}}}{\partial t} \\ \frac{\partial\boldsymbol{M}_{\theta1}^{\mathrm{T}}}{\partial t} \end{bmatrix} \tag{6a}$$

It can be abbreviated as

$$\boldsymbol{M}\ddot{\boldsymbol{A}} + \boldsymbol{C}\dot{\boldsymbol{A}} + \boldsymbol{K}\boldsymbol{A} = \boldsymbol{F} \tag{6b}$$

where $\boldsymbol{K}$ is constant matrix. The antenna traction torque can be written as

$$Q_\theta = [\boldsymbol{M}_{\theta0} \quad \boldsymbol{M}_{\theta1}]\begin{bmatrix} \ddot{\boldsymbol{a}}^0 \\ \ddot{\boldsymbol{a}}^1 \end{bmatrix} + \left[\frac{\partial\boldsymbol{M}_{\theta\theta}}{\partial t} \quad \frac{\partial\boldsymbol{M}_{\theta0}}{\partial t} \quad \frac{\partial\boldsymbol{M}_{\theta1}}{\partial t}\right]\cdot\begin{bmatrix} \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{a}}^0 \\ \dot{\boldsymbol{a}}^1 \end{bmatrix} - \frac{1}{2}[\dot{\boldsymbol{\theta}} \quad \dot{\boldsymbol{a}}^0 \quad \dot{\boldsymbol{a}}^1]\frac{\partial\boldsymbol{M}}{\partial\boldsymbol{\theta}}\begin{bmatrix} \dot{\boldsymbol{\theta}} \\ \dot{\boldsymbol{a}}^0 \\ \dot{\boldsymbol{a}}^1 \end{bmatrix} - \sum_{e=1}^{L_1}(\boldsymbol{r}^e)^{\mathrm{T}}\left(\frac{\partial\boldsymbol{A}^e}{\partial\boldsymbol{\theta}}\right)^{\mathrm{T}}\boldsymbol{Q}^{1e} \tag{7}$$

The Eq.(5) is established using FEM, so the order of Eq.(6b) is very large. It is too much calculation work to solve Eq.(6b) directly. But the modal function of antenna and antenna pedestal can be calculated firstly using FEM, then the order of Eq.(6b) can be reduced greatly using mode coordinate. $\begin{bmatrix} \boldsymbol{a}^0 \\ \boldsymbol{a}^1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}^0 & 0 \\ 0 & \boldsymbol{\Phi}^1 \end{bmatrix}\begin{bmatrix} \boldsymbol{X}^0 \\ \boldsymbol{X}^1 \end{bmatrix} = \boldsymbol{\Phi}\boldsymbol{X}$, where $\boldsymbol{X}$ is modal coordinate, Eq.(6b) can be written as

$$\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{M}\boldsymbol{\phi}\ddot{\boldsymbol{X}}+\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{\phi}\dot{\boldsymbol{X}}+\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{K}\boldsymbol{\phi}\boldsymbol{X}=\boldsymbol{\phi}^{\mathrm{T}}\boldsymbol{F} \tag{8}$$

The order of Eq.(8) is small enough to be calculated.

One period of antenna rotation can be divided into n intervals and every interval is very equal. In every interval, the change of $\boldsymbol{\theta}$ is very small and can be neglected, $\boldsymbol{\theta}$ can be treated as a constant. In arbitrary mth interval, the derivative

$$\frac{\partial \boldsymbol{M}}{\partial t}=\frac{[\boldsymbol{M}(m\Delta\boldsymbol{\theta})-\boldsymbol{M}((m-1)\Delta\boldsymbol{\theta})]}{\Delta t}$$

$$\frac{\partial \boldsymbol{M}}{\partial \boldsymbol{\theta}}=\frac{[\boldsymbol{M}(m\Delta\boldsymbol{\theta})-\boldsymbol{M}((m-1)\Delta\boldsymbol{\theta})]}{\Delta\boldsymbol{\theta}}$$

The external force $\boldsymbol{F}$ can also be treated using this method in arbitrary mth interval. Thus Eq.(6b) is a linear differential equation in every interval, it can be solved easily. The process of solution is as follows.

First, $\boldsymbol{X}_0$ and $\dot{\boldsymbol{X}}_0$ are set to zero in 0 degree of antenna rotation angle, they are the initial value of interval 1th. Matrix $\boldsymbol{M},\boldsymbol{C},\boldsymbol{K},\boldsymbol{F}$ can be established using FEM, the value $\boldsymbol{X}_1,\dot{\boldsymbol{X}}_1,\ddot{\boldsymbol{X}}_1$ can be calculated in interval 1th.

Second, $\boldsymbol{X}_1$ and $\dot{\boldsymbol{X}}_1$ are set to the initial value of interval 2th. Matrix $\boldsymbol{M},\boldsymbol{C},\boldsymbol{F}$ should be built over again and the value $\boldsymbol{X}_2,\dot{\boldsymbol{X}}_2,\ddot{\boldsymbol{X}}_2$ can be calculated in interval 2th.

Third, $\boldsymbol{X}_2$ and $\dot{\boldsymbol{X}}_2$ are set to the initial value of interval 3th. Matrix $\boldsymbol{M},\boldsymbol{C},\boldsymbol{F}$ should be built over again and the value $\boldsymbol{X}_3,\dot{\boldsymbol{X}}_3,\ddot{\boldsymbol{X}}_3$ can be calculated in interval 2th.

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The last step, $\boldsymbol{X}_{n-1}$ and $\dot{\boldsymbol{X}}_{n-1}$ are set to the initial value of interval nth. Matrix $\boldsymbol{M},\boldsymbol{C},\boldsymbol{F}$ should be built over again and the value $\boldsymbol{X}_n,\dot{\boldsymbol{X}}_n,\ddot{\boldsymbol{X}}_n$ can be calculated in interval nth. $\boldsymbol{X}_n,\dot{\boldsymbol{X}}_n,\ddot{\boldsymbol{X}}_n$ are on the 0 degree of antenna rotation angle.

Compare $\boldsymbol{X}_0,\dot{\boldsymbol{X}}_0$ with $\boldsymbol{X}_n,\dot{\boldsymbol{X}}_n$, if the error between $\boldsymbol{X}_0,\dot{\boldsymbol{X}}_0$ and $\boldsymbol{X}_n,\dot{\boldsymbol{X}}_n$ is too big, substitute $\boldsymbol{X}_n,\dot{\boldsymbol{X}}_n$ for $\boldsymbol{X}_0,\dot{\boldsymbol{X}}_0$, come back to the first step, calculate again, till the error is small enough.

According to $\boldsymbol{X}_n,\dot{\boldsymbol{X}}_n$, the antenna traction torque can be calculated using Eq.(7).

3 Example

Fig.2 is a radar antenna system. Its rotation per minute is six circles. We assume that wind speed is 20?m/s, and the wind direction is invariable. Now we calculate the antenna traction torque and the displacement of point A in antenna when the antenna rotation per minute is kept on six circles.

Firstly one period of antenna rotation is divided into 360 intervals and every interval is equal. In every interval, the mass matrix, stiffness matrix and relevant matrix are formed. Secondly a seven orders equation can be built in every interval when the first three modals of antenna and antenna pedestal are used. Thirdly, the equation has been solved using abovementioned method.

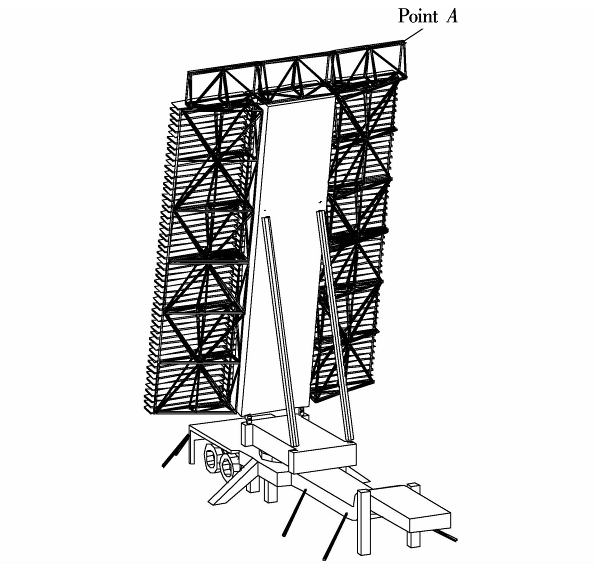

Fig.2 The radar antenna system in calculation example

The calculation results of first twenty seconds have been drawn in Fig.3 and Fig.4. Fig.3 is the displacement of point A in $X^0Y^0Z^0$. Fig.4 is the antenna traction torque. Because the rotation radius of point A is 3.5?m, the deformation displacement of point A is too small to be observed versus the rigid displacement of point A, so the deformation displacement of point A is amplified 500 times in Fig.3. And in Fig.4, the velocity and the acceleration of point A in Eq.(7) are also amplified 500 times.

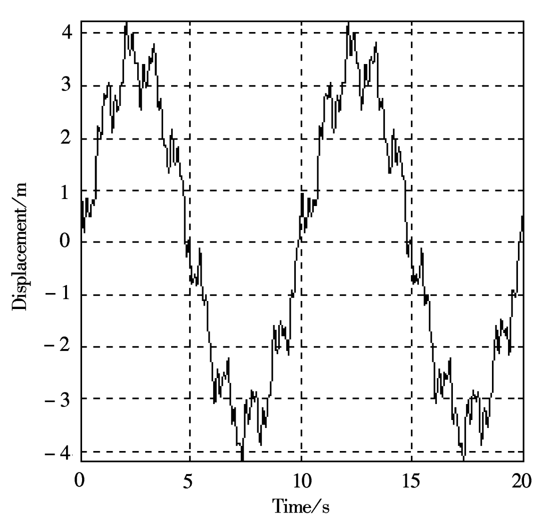

Fig.3 The displacement of point A

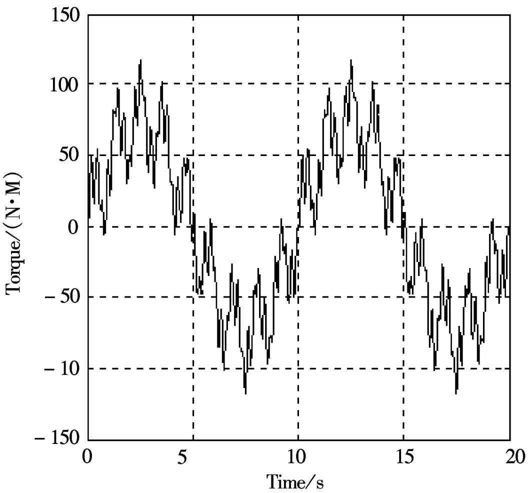

Fig.4 The antenna traction torque

4 Conclusion

In this paper, the dynamic equation of a radar antenna mechanic system which consists of two flexible bodies is established based on the Lagrange's equation and the finite element method, mode coordinate is used to reduce the orders of equation. Finally, the calculation method and engineering example are given when the angular velocity of antenna is invariable. The equations in this paper are built on the small deformation assumption. Large deformation is not allowed for radar antenna, so the assumption is rational.

Because mode coordinate is used to reduce the orders of equation, the solution efficiency is very higher than the direct solution.

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应用模态坐标求解天线机械系统动力学响应

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摘 要 应用 Lagrange 方程和有限元法建立了由 2 个弹性体组成的有高精度要求的某型雷达天线机械系统动力学方程,并应用模态坐标进行了降阶,大大降低了方程自由度,减轻了计算量,给出了天线匀速旋转时的方程解法,得到了天线机械系统在 25? m/s 风速下保持匀速运转时的位移响应和所需的驱动力矩.为天线机械系统误差确定和驱动系统设计提供了依据.

关键词 天线, 动力学, 模态坐标

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