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Compound Genetics Annealing Optimal Algorithm for Realization of Locus Deduction of a Plane Link

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Abstract: A compound algorithm of genetic annealing is designed for optimizing the luffing mechanism locus of a plane link by means of random optimal algorithm, genetic and annealing algorithm. The computing experiment shows that the algorithm has much better steady convergence performance of optimal process and can hunt out the global optimal solution by biggish probability for objective function of multi-peak value.

Key words: genetic annealing algorithm, luffing mechanism, optimal algorithm

This is the luffing mechanism of a plane link that the luffing process is realized by means of a point locus of the plane link. A horizon line is required in some engineerings to satisfy the function and reduce power consumption. The optimal design of luffing mechanism of a plane link is a part of the synthetical optimal problem of machine track. These kinds of optimal problems have more non-linearity and exists multi partial optimal values. The traditional definite optimal algorithm, for example Lagrange method and penalty function method[1] and etc. is hard to find the global optimal solution.

The adoptive algorithm of genetics annealing belongs to random optimal algorithm. The acceptable rule of simulating annealing algorithm is introduced on the basis of traditional genetics algorithm. The traditional genetics algorithm can theoretically find the global optimal solution absolutely. But in practice, it always brings the problem of premature phenomena and poor local optimizing ability. Sometimes global optimal solution can be only neared and can't be found. There are a lot of generations and the optimal time of CPU[2] will be greatly increased if the global optimal solution is gotten. The simulating annealing algorithm has strong ability to find local optimal solution and break away from the trap of local optimal solution[3]. Therefore above problem will be solved when simulating annealing algorithm and traditional genetic algorithm are coupled.

1 Homotopy Algorithm to Solve Non-Linear Equation of Plane Link Curve

The movement diagram of luffing mechanism of a plane link is shown in Fig 1.

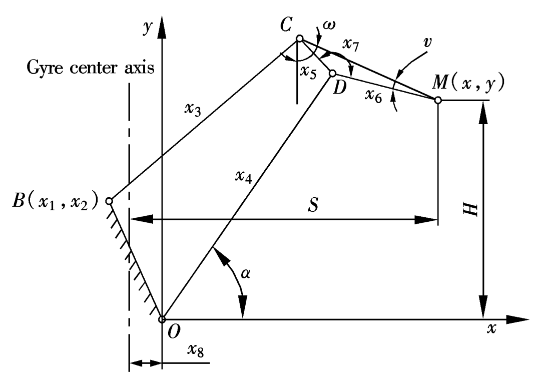

Fig.1 The movement diagram of the luffing mechanism of a plane link

In Fig.1, x_1 is the x coordinate of point B; x_2 is the y coordinate of point B; x_3 is the length of driven link; x_4 is the length of driving link; x_5 is the length of link back end; x_6 is the length of link fore end; x_7 is the angle between links; x_8 is the distance between gyre center to point O.

The locus of luffing mechanism of a plane link is decided by α, the angle of driving link x_4. α will be increased from $\alpha_{\min}$ to $\alpha_{\max}$ when the point M on the plane link moves from $x_{\max}$ to $x_{\min}$. The hunting range of driving link x_4 is $[\alpha_{\min}, \alpha_{\max}]$. The position M is the first position when the extent of driving link x_4 is maximum (corresponds to $x = x_{\max}$). We can get $\alpha_{\min}$

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by the geometry relation and $\alpha_{\max}$ is decided by solution of equation of plane link curve. The equation of the point M on the link is

$$F_1(x,y,\varphi) = x^2 + y^2 + 2b(x_1 - x)\sin\varphi + 2b(x_2 - y)\cos\varphi - 2x_1 x - 2x_2 y + x_1^{\,2} + x_2^{\,2} + b^2 - x_3^{\,2} = 0 \tag{1}$$

$$F_2(x,y,\varphi) = x^2 + 2x_6 x\sin(\varphi + \theta) + 2x_6 y\cos(\varphi + \theta) + x_6^{\,2} - x_4^{\,2} = 0 \tag{2}$$

When x value is known, (1) and (2) are non-linear equations about y and φ. y^* and φ^* of least position are gotten by solution of (1) and (2) when $x = x_{\min}(\alpha = \alpha_{\max})$. x_D^* and y_D^*, position of point D is decided at the same time. Where

$$x_D^* = x_{\min} - x_6\sin(\varphi^* + \theta)$$

$$y_D^* = y^* - x_6\cos(\varphi^* + \theta)$$

$$\theta = \sin^{-1}\frac{x_5\sin x_7}{b}$$

$$b = \sqrt{x_5^{\,2} + x_6^{\,2} - 2x_5 x_6\cos x_7}$$

$$\alpha_{\max} = \tan^{-1}\frac{y_D^*}{x_D^*}$$

The homotopy Eqs. (1) and (2) are introduced and the compound iteration solution of numerical continuation-Newton is applied to ensure constringency solution of non-linear Eqs. (1) and (2) at most region. For example, the vector format of (1) and (2) is shown as

$$F(\boldsymbol{X}) = 0 \tag{3}$$

where $\boldsymbol{X} = \{y,\varphi\}^{\mathrm{T}}$, the homotopy Eq. (3) is

$$H(\boldsymbol{X},t) = F(\boldsymbol{X}) + (t - 1)F(\boldsymbol{X}^0) = 0 \tag{4}$$

where $\boldsymbol{X}^0$ is the first given solution at discretion; t is a parameter between $[0,1]$. (4) will turn into the primary non-linear Eq. (3) when $t = 1$. In order to get the solution of Eq. (4) when $t = 1$, the t region $[0,1]$ is divided into n parts. That is

$$0 = t_0 < t_1 < t_2 \cdots < t_n = 1$$

$$H(\boldsymbol{X},t_i) = 0 \tag{5}$$

The solution $\boldsymbol{X}^i$ of Eq. (5) can be gotten by iteration because the solution $\boldsymbol{X}^{i-1}$ has been gotten in advance and $\boldsymbol{X}^{i-1}$ is regarded as the first solution approximately. It is proved that $\boldsymbol{X}^{i-1}$ is better near solution of $\boldsymbol{X}^i$ in Ref. [4]. Therefore, Eq. (5) can be solved by local convergent iteration. $\boldsymbol{X}^n$ has neared to the accurate solution of the equation $F(\boldsymbol{X}) = 0$ by n iteration steps. At the moment, the convergent solution of original equations, $\boldsymbol{X}^*$, can be gotten quickly after many iteration steps of $F(\boldsymbol{X}) = 0$ are carried on by Newton iteration according to first solution of $\boldsymbol{X}^n$. The format of numerical continuation-Newton compound iteration is

$$\left.\begin{aligned} &\boldsymbol{X}^{i+1} = \boldsymbol{X}^i - (F'(\boldsymbol{X}^i))^{-1}\left(F(\boldsymbol{X}^i) + \left(\frac{i}{n} - 1\right)F(\boldsymbol{X}^0)\right) \\ &\quad i = 1, 2, \cdots, n - 1 \\ &\boldsymbol{X}^{k+1} = \boldsymbol{X}^k - (F'(\boldsymbol{X}^k))^{-1}F(\boldsymbol{X}^k) \\ &\quad k = n,\ n + 1,\ n + 2, \cdots \end{aligned}\right\} \tag{6}$$

It was proved that iteration format (6) had more range astringency in Ref. [4]. Tab. 1 is the simulation numerical result when the parameters are: $x_1 = -4.5?$ m, $x_2 = 9.5?$m, $x_3 = 20.8?$m, $x_4 = 24.5?$m, $x_5 = 3.95?$m, $x_6 = 9.2?$m, $x_7 = 2.905?$rad, $x_8 = -2.8?$ m, $S_{\max} = 30?$m, $S_{\min} = 9?$m, $H = 14.95?$m. The result shows that the convergent solution $\{14.951?9, 5.3943\}^{\mathrm{T}}$ can be gotten by Eq. (6) when 10 groups are produced randomly if y^0 is in $[H - 20\Delta, H + 20\Delta]$ and φ^0 is in $[0,\pi/2]$. Where Δ is $0.015 \times (S_{\max} - S_{\min})$.

Tab. 1 Iteration result of numerical continuation-Newton

Computation method	y_o initial value/m	φ_o initial value/(°)	y value after iteration/m	φvalue after iteration/(°)	$\alpha_{\min}$/(°)	$\alpha_{\max}$/(°)
Handwork	—	—	—	—	43.5	79.5
Numerical continuation-Newton iteration	20.65	78.27	14.951?9	5.394?3	41.979?4	78.725?1
	14.01	73.88	14.951?9	5.394?3	41.979?4	78.725?1
	12.98	14.97	14.951?9	5.394?3	41.979?4	78.725?1
	14.46	59.42	14.951?9	5.394?3	41.979?4	78.725?1
	20.46	83.76	14.951?9	5.394?3	41.979?4	78.725?1
	16.73	34.10	14.951?9	5.394?3	41.979?4	78.725?1
	11.71	60.61	14.951?9	5.394?3	41.979?4	78.725?1
	12.10	49.41	14.951?9	5.394?3	41.979?4	78.725?1
	18.62	11.43	14.951?9	5.394?3	41.979?4	78.725?1
	15.00	52.47	14.951?9	5.394?3	41.979?4	78.725?1

2 Genetics Annealing Optimal Algorithm

2.1 The locus optimal model of luffing mechanism of a plane link

The optimal design variable of luffing mechanism of a plane link is $\boldsymbol{X} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}^{\mathrm{T}}$. It is defined as Fig. 1. The fall value of point M locus should be minimum when the luffing mechanism is moving. The target function is

$$\min f = \Delta y_{\max} - \Delta y_{\min}$$

where $\Delta y_{\max} = |y_{\max} - H|$; $\Delta y_{\min} = |y_{\min} - H|$; $y_{\max} = \max y_i$; $y_{\min} = \min y_i (i = 1,2,\cdots,n)$, y_i is y coordinate of point M in i position. We can get the y_i of point M corresponding to α_i by the geometry relation if the hunting range of driving link x_4, $[\alpha_{\min}, \alpha_{\max}]$, is divided into n points. The concrete computing process can refer to Ref. [1]. The constraint conditions of luffing mechanism of a plane link are

$$a_i \leqslant x_i \leqslant b_i \qquad i = 1,2,\cdots,8$$

$$M_1 \geqslant 0$$

$$M_2 \leqslant 0$$

$$M_{\mathrm{r}} \leqslant 0.08QS_{\max}$$
$$|M_{\max}| \leqslant 0.1QS_{\max}$$
$$y_{\min} - y_B \geqslant 0$$
$$S_{\min} \geqslant [S_{\min}]$$
$$S_{\max} \leqslant [S_{\max}]$$

where a_i and b_i are lower bound and upper bound of variable x_i, respectively; M_1 is the imbalance moment of freight of minimum amplitude; M_2 is the imbalance moment of freight of maximum amplitude; M_{r} is the root mean square of imbalance moment of freight; Q is elevating capacity; $S_{\min}$ is minimum amplitude; $S_{\max}$ is maximum amplitude; $M_{\max}$ is the maximum imbalance moment in the process of amplitude of variation; $y_{\min}$ is the lowest y coordinate in the process of amplitude; y_B is the y coordinate of lower knuckle point of big link; $[S_{\min}]$ is allowable minimum amplitude and the corresponding position is collinear x_4 and x_5, shown in Fig.2; $[S_{\max}]$ is allowable maximum amplitude and the corresponding position is collinear x_3 and CM, shown in Fig.3.

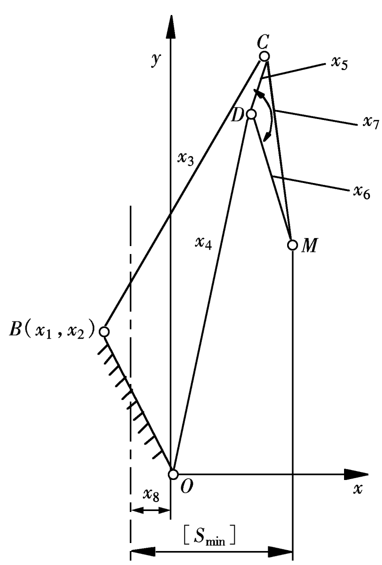

Fig.2 The plane link position of allowable minimum amplitude

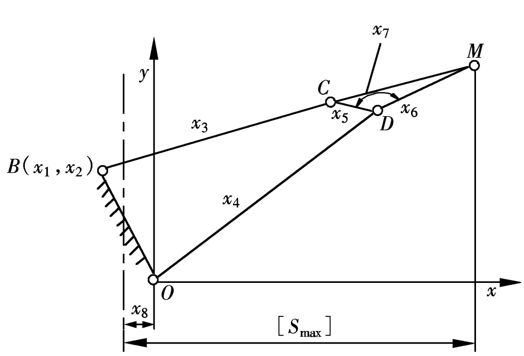

Fig.3 The plane link position of allowable maximum amplitude

The calculation formulae of M_1, M_2, M_{r}, $M_{\max}$ can refer to Ref.[1] in constraint condition.

After constraint conditions are normalized, the locus optimal model of luffing mechanism of a plane link is constituted as follows

$$\min f(\boldsymbol{X}) = |\Delta y_{\max} - \Delta y_{\min}|$$
$$\text{s.t.} \quad g_j(\boldsymbol{X}) - 1 \leqslant 0 \qquad j = 1, 2, \cdots, m$$

where m is the number of constraint condition.

2.2 The genetics annealing algorithm

The genetic annealing algorithm is composed of generating initial population, parents selection-procedure, crossover operator and mutation operator. The optical precision is very good because the design variables are all continuous variables when float-point coding is adopted.

2.2.1 Generate initial population

The initial population $\boldsymbol{X}_i^0 (i = 1, 2, \cdots, N)$ should be satisfied with close and bulgy condition because the luffing mechanism of a plane link that four links are closed and bulgy quadrangle is required. For example, the quadrangle($OBCD$) in maximum position (Fig.3) is defined as shape A and the quadrangle ($OBCD$) in minimum position (Fig.2) is defined as shape B ($OBCD$ degrades into a triangle at the moment). The arbitrary position of plane link mechanism is defined as shape C in the process of amplitude. It is proved that C is bulgy polygon when B and A are bulgy polygons in Ref.[5]. $\boldsymbol{X}_i^0$ is tolerable if it is ensured that the size $\boldsymbol{X}_i^0$ can form quadrangle ($OBCD$ shown in Fig.2 and Fig.3) in minimum and maximum positions.

2.2.2 Parents selection-procedure

The choice strategy of roulette is adopted. Fitness function of design vector $\boldsymbol{X}_i$ is defined as follows:

$$U_i = 1/F_i \qquad i = 1, 2, \cdots, N$$

where F_i is a nonrestraint penalty function of i vector $\boldsymbol{X}_i$.

Some design vectors are not in the feasible domain in the process of optimizing, but they are very near to the feasible domain and it is possible that they can become feasible better solution by genetic operator. Therefore fuzzy set is introduced to define penalty function F_i

$$F_i = f_i + C$$

where f_i is target function value of i vector; the definition of C is shown in Tab.2.

Tab.2 Genetic optimal result of great amplitude parameter

d	$[-\infty, 0.001]$	$(0.001, 0.01)$	$(0.01, 0.02)$	$(0.02, 0.05)$	$(0.05, 0.1)$	$(0.1, 0.4)$	$(0.4, 1)$	$(1, 2)$	$(2, 5)$	$(5, 15]$	$[15, +\infty]$
C	0	2	3	4	5	6	7	8	9	10	100

Note: C denotes the degree of disobeying constraint of design vector $\boldsymbol{X}_i$.

In Tab.2, $d = \max d_j (j = 1, 2, \cdots, m)$, where m is the number of constraint conditions,

$$d_j = \begin{cases} 0 & g_j(\boldsymbol{X}_i) \leqslant 0 \\ g_j(\boldsymbol{X}_i) & g_j(\boldsymbol{X}_i) > 0 \end{cases}$$

2.2.3 Crossover operator

Besides mating operator can affect the chromosome in binary system coding, it can also directly affect the chromosome in floating-point coding. The mating operator can be defined as the linear combination of two vectors $\boldsymbol{X}_i^k$ and $\boldsymbol{X}_j^k$ of parent population. The vectors of offspring is

$$\boldsymbol{X}_i^{k+1} = r\boldsymbol{X}_i^k + (1 - r)\boldsymbol{X}_j^k$$
$$\boldsymbol{X}_j^{k+1} = r\boldsymbol{X}_j^k + (1 - r)\boldsymbol{X}_i^k$$

2.2.4 Mutation operator

The consistent mutation operator is adopted. Every vector $\boldsymbol{X}_p^i (p = 1, 2, \cdots, n)$ of the vector $\boldsymbol{X}_i$ in floating-point coding mutates with the same probability. $(\boldsymbol{X}_p^i)'$ denotes the vector after mutating. $(\boldsymbol{X}_p^i)'$ is random generation in closed interval $[a_p, b_p]$. a_p and b_p are the lower bound and upper bound of p part vector domain.

2.2.5 Acceptant criterion

The traditional genetic algorithm is: $k + 1$ generation design vector $\boldsymbol{X}^{k+1}$ evolves from k generation design vector $\boldsymbol{X}^k$ and $\boldsymbol{X}^{k+1}$ replaces $\boldsymbol{X}^k$ unconditionally and the new generation design vector is formed after selection, crossover and mutation. Such simplistic genetic algorithm can lead to lose the chromosome that has better fitness in last generation. In addition, the algorithm can't break away from it when the genetic algorithm hunts out local optimal trap. On the other side simulating annealing algorithm has the ability to break away from local optimal trap. The above problem is solved better when Metropolis acceptant criterion is adopted and genetics annealing algorithm is structured.

Suppose that i design vector is $\boldsymbol{X}_i^k$ and the corresponding value of nonrestraint penalty function is F_i^k, $\boldsymbol{X}_i'$ evolves from $\boldsymbol{X}_i^k$ after genetics annealing algorithm. The acceptant criterion is

$$\boldsymbol{X}_i^{k+1} = \begin{cases} \boldsymbol{X}_i' & F_i' < F_i^k \\ \boldsymbol{X}_i^k & P_i > r,\ F_i' > F_i^k \\ \boldsymbol{X}_i' & P_i \leqslant r,\ F_i' > F_i^k \end{cases}$$

where $P_i = \exp(\Delta F / T)$, $\Delta F = F_i^k - F_i'$, T is annealing temperature; P_i is acceptant probability; r is random number in $[0, 1]$.

The block diagram of genetic annealing algorithm is shown in Fig.4. The convergent criterion in the diagram is that the optimal solution of continuous 40 generations, $\boldsymbol{X}^*$, is invariable.

Input parameter: $S_{\max}, S_{\min}, H, Q, T_{\max}, \alpha$
Generate initial design vector of N population randomly $\boldsymbol{X}_i (i = 1, 2, \cdots, N)$
$T = T_{\max}$, $K = 1$
Compute the value of fitness of N groups design vector $\boldsymbol{U}_i$
Operation of roulette choice
Operation of crossover operator
Operation of mutation operator
Accept the design vector according to Metropolis acceptant criterion
Form design vector $\boldsymbol{X}_i^{K+1} (i = 1, 2, \cdots, N)$ of $K + 1$ population
$T = \alpha T$, $K = K + 1$
Judge astringent criterion
Output optimal solution
Stop

Fig.4 The block diagram of genetic annealing algorithm

3 Case

The amplitude parameters of portal hoisting machine in dockyard are: maximum amplitude $S_{\max} = 45?$m, minimum amplitude $S_{\min} = 11?$m, height of maximum amplitude $H = 12.8?$m, hoisting capacity $Q = 80?$T, hoisting scale factor is 1. The optimal solutions of 10 times are shown in Tab.3. The curve of fall value is shown in Fig.5.

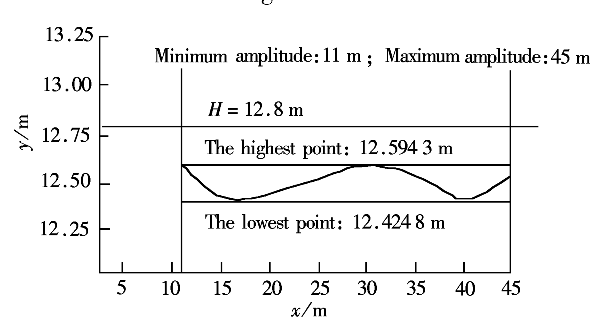

Fig.5 The curve of fall value

4 Conclusions

The following results have been gotten by numerical simulation:

1) The structural genetics-annealing algorithm can

Tab.3 The genetic optimal result of great amplitude parameter

No.	x_1/m	x_2/m	x_3/m	x_4/m	x_5/m	x_6/m	x_7/rad	x_8/m	Fall value/ m	Maximum elevation angle/(°)	Minimum elevation angle/(°)
1	–7.732?3	9.323?3	34.151	34.938	5.073?6	20.963	3.047?3	–1.626?5	0.167?8	74.869	43.012
2	–7.732?1	9.323?1	34.150	34.936	5.073?8	20.964	3.047?3	–1.626?5	0.167?8	74.870	43.012
3	–7.732?7	9.323?2	34.152	34.937	5.073?6	20.965	3.047?3	–1.626?5	0.167?8	74.869	43.012
4	–7.732?3	9.323?8	34.151	34.935	5.073?6	20.965	3.047?3	–1.626?5	0.167?8	74.870	43.012
5	–7.732?6	9.323?6	34.150	34.936	5.073?6	20.967	3.047?3	–1.626?5	0.167?9	74.870	43.012
6	–7.732?8	9.323?7	34.150	34.936	5.073?6	20.966	3.047?3	–1.626?5	0.167?8	74.869	43.012
7	–7.732?4	9.323?2	34.153	34.939	5.073?7	20.968	3.047?3	–1.626?5	0.167?9	74.868	43.013
8	–7.732?3	9.323?0	34.150	34.937	5.073?6	20.965	3.047?3	–1.626?5	0.167?9	74.868	43.012
9	–7.732?4	9.323?0	34.151	34.936	5.073?6	20.965	3.0473	–1.626?5	0.167?8	74.869	43.012
10	–7.732?5	9.323?9	34.150	34.936	5.073?5	20.965	3.047?3	–1.626?5	0.167?8	74.86?9	43.012

solve synthetical optimal problem of luffing mechanism locus of a plane rigidity link.

2) The algorithm has strong robustness. Show itself from the optimal result in Tab.3. The optimal result can all steadily be constringed the same optimal solution by 10 times simulating experiment. It is considered that the optimal solution of Tab.3 is the global optimal solution of the case by a lot of results of testing function.

3) It has the astringency of big extent that numerical continuation-Newton compound iteration is used to solve the curve equations of nonlinear plane link.

4) The algorithmic ability of breaking away from local optimal solution is enhanced and the probability of hunting out global optimal solution is improved greatly after Metropolis acceptant criterion is adopted.

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实现平面连杆变幅机构轨迹复演的遗传退火混合优化算法的研究

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摘 要 利用随机优化算法、遗传法和模拟退火法的优点设计了一套遗传退火混合优化算法,并将其应用于平面连杆变幅机构轨迹复演的优化.计算实践表明,该算法的稳定收敛性良好,对多峰值的目标函数能以较大的概率搜索到全局最优解.

关键词 遗传退火算法，变幅机构，优化算法

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