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Study on mutual impedance characteristics between probes in a circular waveguide

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Abstract: The expression of mutual impedance between two probes in a circular waveguide is derived by means of a vector potential function, reaction concept and reciprocity theorem. The waveguide is semi-infinite, and one end of the waveguide is terminated to a load with a reflection coefficient. The contribution to the mutual resistance is found to come from the dominant mode, while the contribution to the mutual reactance comes from the dominant mode and the higher order modes. The major contribution to the mutual reactance is from the dominant mode, since the higher modes decay rapidly with the increasing the probes' of separation distance. However, as the separation distance approaches zero, the higher modes become dominant, which results in a large value of the mutual reactance. The mutual impedance is dependent on the location and height of the probes, their separation distance and the location of the terminal plane.

Key words: probe; mutual impedance; reaction; reciprocity theorem

The problem of the single probe's self-impedance in a waveguide has been studied by many investigators[1-4]. The mutual impedance between two probes in a rectangular waveguide was discussed by Ittipiboon[5] and Wang[6]. The formulas of the mutual impedance between two probes in a circular waveguide were derived by Wang[7] using the dyadic Green's function.

In this paper, we mainly study the single probe's radiation field and mutual impedance between two probes in a circular waveguide. In the derivation, the vector potential function and reaction concept are used.

1 Theoretical Analysis

The probes to be considered are shown in Fig. 1. The two probes are radically located in the same cross section of a circular waveguide which propagates only the dominant mode. The waveguide is semi-infinite, and the reflection coefficient at the terminal plane ($z = -l$) is Γ. The first probe of height h_1 is located at $z = 0$, and the second probe of height h_2 at $z = d$. The radius of the circular waveguide is a.

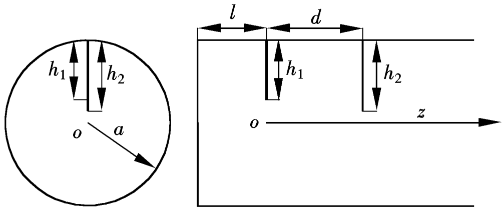

Fig. 1 Schematic diagram of two probes in a circular waveguide

The probe diameter is assumed to be very small, compared to the waveguide's dimensions, so that the surface current can be approximately represented by a sinusoid[1,2]. Assume that the current distributions of the first and second probe are given by

$$\boldsymbol{I}_1 = \begin{cases} -\hat{a}_r I_{10} \sin k_0 (r - a + h_1)\delta(\varphi - \varphi_0) & a - h_1 \leqslant r \leqslant a \\ 0 & r < a - h_1 \end{cases} \tag{1}$$

$$\boldsymbol{I}_2 = \begin{cases} -\hat{a}_r I_{20} \sin k_0 (r - a + h_2)\delta(\varphi - \varphi_0) & a - h_2 \leqslant r \leqslant a \\ 0 & r < a - h_2 \end{cases} \tag{2}$$

where $\delta(\varphi - \varphi_0)$ is the Dirac-delta function representing the probe location at φ_0 along the φ-axis.

1.1 Radiation field of the first probe

Since the probe current is in the r-direction, mainly the transverse electric modes are excited[1]. So the vector potential function $\boldsymbol{A}(= \hat{a}_z B)$ is introduced, where B satisfies the scalar Helmholtz equation: $\nabla^2 B + k_0^2 B = 0$,

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and the field components can be obtained from $\boldsymbol{A}$. After applying the boundary condition which requires the continuity of the tangential fields E_r and E_φ, and

$$H_{\varphi>} - H_{\varphi<} = I_{10}\sin k_0(r-a+h_1)\delta(\varphi-\varphi_0) \tag{3}$$

The radiation field distribution of the first probe, with the time variation $e^{j\omega t}$ suppressed throughout, can be written as

$$E_{r>} = -j\omega\mu_0\sum_{m=0}\sum_{n=1}C_{mn}\frac{m}{r}J_m\left(\frac{\xi_{mn}}{a}r\right)\cos m\varphi e^{-\gamma_{mn}z} \tag{4}$$

$$E_{\varphi>} = j\omega\mu_0\sum_{m=0}\sum_{n=1}C_{mn}\frac{d}{dr}\left[J_m\left(\frac{\xi_{mn}}{a}r\right)\right]\sin m\varphi e^{-\gamma_{mn}z} \tag{5}$$

$$H_{r>} = -\sum_{m=0}\sum_{n=1}C_{mn}\gamma_{mn}\frac{d}{dr}\left[J_m\left(\frac{\xi_{mn}}{a}r\right)\right]\sin m\varphi e^{-\gamma_{mn}z} \tag{6}$$

$$H_{\varphi>} = -\sum_{m=0}\sum_{n=1}C_{mn}\gamma_{mn}\frac{m}{r}J_m\left(\frac{\xi_{mn}}{a}r\right)\cos m\varphi e^{-\gamma_{mn}z} \tag{7}$$

$$H_{z>} = \sum_{m=0}\sum_{n=1}C_{mn}(k_0^2+\gamma_{mn}^2)J_m\left(\frac{\xi_{mn}}{a}r\right)\sin m\varphi e^{-\gamma_{mn}z} \tag{8}$$

when $z>0$, while when $z<0$,

$$E_{r<} = -j\omega\mu_0\sum_{m=0}\sum_{n=1}D_{mn}\frac{m}{r}J_m\left(\frac{\xi_{mn}}{a}r\right)\cos m\varphi\left[e^{\gamma_{mn}z}+\Gamma e^{-\gamma_{mn}(z+2l)}\right] \tag{9}$$

$$E_{\varphi<} = j\omega\mu_0\sum_{m=0}\sum_{n=1}D_{mn}\frac{d}{dr}\left[J_m\left(\frac{\xi_{mn}}{a}r\right)\right]\sin m\varphi\left[e^{\gamma_{mn}z}+\Gamma e^{-\gamma_{mn}(z+2l)}\right] \tag{10}$$

$$H_{r<} = \sum_{m=0}\sum_{n=1}D_{mn}\gamma_{mn}\frac{d}{dr}\left[J_m\left(\frac{\xi_{mn}}{a}r\right)\right]\sin m\varphi\left[e^{\gamma_{mn}z}-\Gamma e^{-\gamma_{mn}(z+2l)}\right] \tag{11}$$

$$H_{\varphi<} = \sum_{m=0}\sum_{n=1}D_{mn}\gamma_{mn}\frac{m}{r}J_m\left(\frac{\xi_{mn}}{a}r\right)\cos m\varphi\left[e^{\gamma_{mn}z}-\Gamma e^{-\gamma_{mn}(z+2l)}\right] \tag{12}$$

$$H_{z<} = \sum_{m=0}\sum_{n=1}D_{mn}(k_0^2+\gamma_{mn}^2)J_m\left(\frac{\xi_{mn}}{a}r\right)\sin m\varphi\left[e^{\gamma_{mn}z}+\Gamma e^{-\gamma_{mn}(z+2l)}\right] \tag{13}$$

where

$$C_{mn} = -\frac{I_{10}N_{mn}\cos m\varphi_0\left[1+\Gamma e^{-2\gamma_{mn}l}\right]}{2m\gamma_{mn}M_{mn}g_{0m}} \tag{14}$$

$$D_{mn} = -\frac{I_{10}N_{mn}\cos m\varphi_0}{2m\gamma_{mn}M_{mn}g_{0m}} \tag{15}$$

$$g_{0m} = \begin{cases}2\pi & m=0\\ \pi & m\geqslant 1\end{cases} \tag{16}$$

$$M_{mn} = \frac{a^2}{2}\left(1-\frac{m^2}{\xi_{mn}^2}\right)J_m^2(\xi_{mn}) \tag{17}$$

$$N_{mn} = \int_{a-h_1}^{a}\sin k_0(r-a+h_1)J_m\left(\frac{\xi_{mn}}{a}r\right)r^2\,dr \tag{18}$$

$$\gamma_{mn} = j\beta_{mn} \tag{19}$$

where ξ_{mn} is the n-th root of $J'_m(\xi)=0$.

1.2 Mutual impedance between two probes

Using the reaction concept[2], the mutual impedance between the two probes is given by

$$Z_{12} = \frac{-1}{I_{1in}I_{2in}}\langle\boldsymbol{E}_1,\boldsymbol{I}_2\rangle = \frac{-1}{I_{1in}I_{2in}}\int_{a-h_2}^{a}\boldsymbol{E}_1\cdot\boldsymbol{I}_2\,dr \tag{20}$$

where I_{1in} and I_{2in} are the input currents at the first and second probes, respectively; $\boldsymbol{E}_1$ is the radiation field of the first probe at the location of the second probe in its absence; and $\boldsymbol{I}_2$ is the current distribution on the second probe.

By the reciprocity theorem we have

$$\langle\boldsymbol{E}_1,\boldsymbol{I}_2\rangle = \langle\boldsymbol{E}_2,\boldsymbol{I}_1\rangle \tag{21}$$

Hence

$$Z_{12} = Z_{21} \tag{22}$$

From (1) and (2), we have

$$I_{1\text{in}} = I_{10}\sin k_0 h_1 \tag{23}$$

$$I_{2\text{in}} = I_{20}\sin k_0 h_2 \tag{24}$$

Substituting $\boldsymbol{E}_1$ from (4) with $z = d$ and $\varphi = \varphi_0$, $\boldsymbol{I}_2$ from (2), $I_{1\text{in}}$ from (23), and $I_{2\text{in}}$ from (24) into (20), we have

$$Z_{12} = \frac{-1}{I_{1\text{in}}I_{2\text{in}}}\int_{a-h_2}^{a}\boldsymbol{E}_1\cdot\boldsymbol{I}_2\,\mathrm{d}r = \frac{1}{\sin k_0 h_1 \sin k_0 h_2}\cdot$$

$$\int_{a-h_2}^{a}\mathrm{j}\omega\mu_0\sum_{m=0}\sum_{n=1}\frac{N_{mn}\cos^2 m\varphi_0 \mathrm{e}^{-\gamma_{mn}d}[1+\Gamma\mathrm{e}^{-2\gamma_{mn}l}]}{2r\gamma_{mn}M_{mn}g_{0m}}\mathrm{J}_m\left(\frac{\xi_{mn}}{a}r\right)\sin k_0(r-a+h_2)\mathrm{d}r \tag{25}$$

Let $\varphi_0 = 0$ and $\Gamma = -1$, (25) can be simplified as

$$Z_{12} = \frac{\mathrm{j}\omega\mu_0}{2\sin k_0 h_1\sin k_0 h_2}\sum_{m=0}\sum_{n=1}\frac{N_{mn}Q_{mn}\mathrm{e}^{-\gamma_{mn}d}[1-\mathrm{e}^{-2\gamma_{mn}l}]}{\gamma_{mn}M_{mn}g_{0m}} \tag{26}$$

where

$$Q_{mn} = \int_{a-h_2}^{a}\sin k_0(r-a+h_2)\mathrm{J}_m\left(\frac{\xi_{mn}}{a}r\right)\frac{1}{r}\mathrm{d}r \tag{27}$$

It can be seen from (26) that the mutual resistance is mainly from the dominant mode and is given by

$$R_{12} = \frac{\omega\mu_0 N_{11}Q_{11}}{2\beta_{11}M_{11}g_{01}\sin k_0 h_1\sin k_0 h_2}(\cos\beta_{11}d + \sin\beta_{11}d\sin 2\beta_{11}l - \cos\beta_{11}d\cos 2\beta_{11}l) \tag{28}$$

The mutual reactance, which consists of the dominant and the higher order modes, is given by

$$X_{12} = \frac{\omega\mu_0 N_{11}Q_{11}}{2\beta_{11}M_{11}g_{01}\sin k_0 h_1\sin k_0 h_2}(\sin\beta_{11}d\cos 2\beta_{11}l + \cos\beta_{11}d\sin 2\beta_{11}l - \sin\beta_{11}d) +$$

$$\frac{\omega\mu_0}{2\sin k_0 h_1\sin k_0 h_2}\sum_{\substack{m=0\\ mn\neq 11}}\sum_{n=1}\frac{N_{mn}Q_{mn}\mathrm{e}^{-\gamma_{mn}d}[1-\mathrm{e}^{-2\gamma_{mn}l}]}{\gamma_{mn}M_{mn}g_{0m}} \tag{29}$$

2 Numerical Results

It can be seen from (28) and (29) that the mutual impedance depends on the waveguide dimensions, operation frequency, the reflection coefficient at the terminal plane, probe heights, the location of the terminal plane, and the separation distance between the two probes as well.

When the waveguide radius is 48.935 mm and operation frequency is 2.45 GHz, only the dominant mode propagates whose guide wavelength $\lambda_g = 180.25$ mm, and other higher order modes are the evanescent modes. Some numerical results are given in Figs. 2–4.

In Fig. 2 and Fig. 3, the probe heights are used as a variable parameter, and we can find that the mutual impedance increases with the probe heights. That is caused by the radiation field distribution of the dominant mode. It can be seen from Fig. 4 that when $l = \lambda_g/2$, the mutual impedance caused by the dominant mode equals zero, since the phase difference between the incident and reflection wave of the dominant mode is π. So the mutual reactance in Fig. 4 comes from the higher order modes.

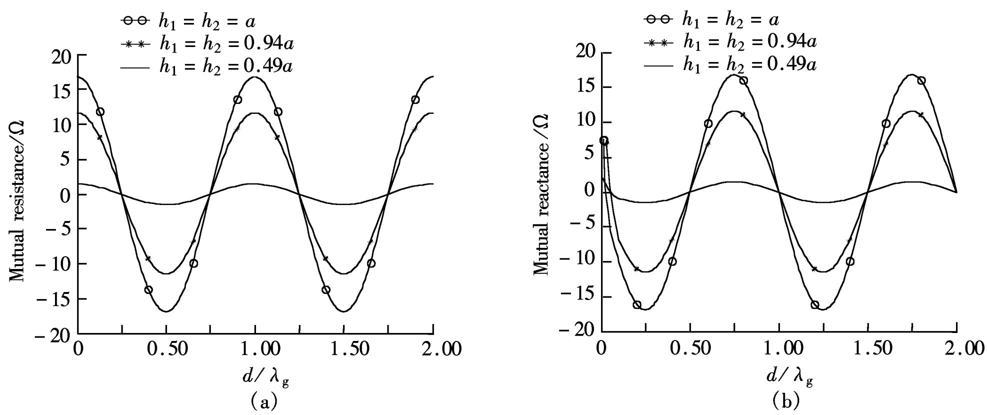

Fig. 2 The mutual impedance against probe separation distance for different probe heights with $l = \frac{\lambda_g}{4}$

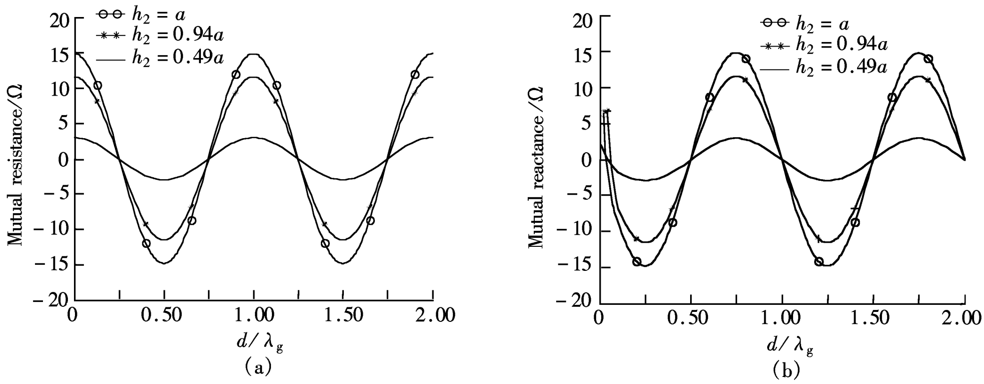

Fig.3 The mutual impedance against probe separation distance for different probe heights of the second with $h_1 = 0.94a$ and $l = \frac{\lambda_g}{4}$

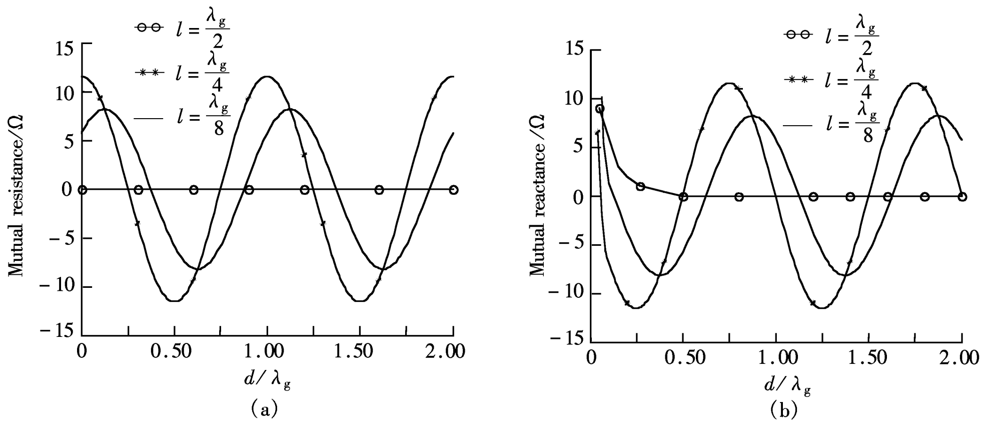

Fig.4 The mutual impedance against probe separation distance for different l with $h_1 = h_2 = 0.94a$

As seen from (29), the higher order modes decay exponentially with the increasing of the separation distance between the two probes, so the contribution of the higher order modes to the mutual reactance is usually negligible for large separation distances. However, as the separation distance approaches zero, the higher order modes become dominant, which results in a large value of the mutual reactance. The cases whose curves have a violent variety at the zero point are shown in Figs.2 - 4.

3 Conclusion

The mutual impedance characteristics between two probes in a circular waveguide are discussed in detail by means of the reaction concept and reciprocity theorem. We find that the contribution to the mutual resistance comes from the dominant mode, while the contribution to the mutual reactance comes from the dominant mode and the higher order modes. The mutual impedance is dependent on the waveguide dimensions, operation frequency, the reflection coefficient at the terminal plane, probe heights, the location of the terminal plane, and the separation distance between the two probes as well.

The conclusions and numerical results we've got in this paper are very consistent with those of Refs.[5 - 7].

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圆波导中探针互阻抗特性的研究

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摘　要　应用矢量位函数、反应的概念和互易原理,推导出圆波导中双探针的互阻抗表达式,圆波导是半无限长的,终端接有一定反射系数的负载.发现了探针的互电阻是由主模产生的,而互电抗则是由主模和高次模共同引起的;由于高次模随着探针之间距离的增加急剧衰减,所以对互电抗的主要贡献来自于主模.然而当探针之间的距离趋近于零时,高次模起主导作用,产生很大值的互电抗.互阻抗取决于探针的位置、高度和相互之间的距离,以及终端面的位置.

关键词　探针；互阻抗；反应；互易原理

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