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A new kind of wavelet-based method for spectrum deconvolution

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Abstract: To subtract the slit function from the measured spectrum, a wavelet-based deconvolution method is proposed to obtain a regularized solution of the problem. The method includes reconstructing the signal from the wavelet modulus maxima. For the purpose of maxima selection, the spatially selective noise filtration technique was used to distinguish modulus maxima produced by signal from the one created by noise. To test the method, sodium spectrum measured at a wide slit was deconvolved. He-Ne spectrum measured at the corresponding slit width was used as slit function. Sodium measured at a narrow slit was used as the reference spectrum. The deconvolution result shows that this method can enhance the resolution of the degraded spectrum greatly.

Key words: deconvolution; slit function; wavelet local maxima

In spectrum measurement, the resolution of a received signal is greatly affected by slit width of the monochromator[1]. In general, increasing the slit width will lead to a decrease of resolution. Therefore, a narrow slit is always preferred. On the other hand, when measuring a low intensity spectrum, a wide slit must be used not only to increase the received areas, but also to get a degraded spectrum. The influence of the slit width can be described as a slit function, and the received spectrum is a convolution of the ideal spectrum and the slit function. Thus, deconvolution is needed to restore the high resolution spectrum, especially in low intensity spectrum measurement.

The signal deconvolution problem is an inverse problem mathematically modeled by a Fredholm integral equation of the first kind. This equation is usually an ill-posed problem when it is considered in a Hilbert space framework, requiring regularization techniques to control arbitrary error amplifications and to get adequate solutions[2,3]. In this paper, we present, under a projection onto convex sets (POCS) framework, a wavelet-based method for obtaining a regularized solution to discrete Fredholm integral equations of the first kind corrupted by additive noise. Our method includes the multiscale edges reconstruction algorithm which was recently applied to spiky deconvolution[4]. In our case, we deal with a more general model, and our method has a wider application.

1 Theory

The convolution process has the following generic form:

$$\boldsymbol{h} * \boldsymbol{r} + \boldsymbol{n} = \boldsymbol{f} \tag{1}$$

where $\boldsymbol{h}$ is the slit function; $\boldsymbol{n}$ is noise; $\boldsymbol{r}$ is the ideal spectrum; $\boldsymbol{f}$ is the received spectrum. After discretization, Eq.(1) becomes

$$\boldsymbol{H}\boldsymbol{r} + \boldsymbol{n} = \boldsymbol{f} \tag{2}$$

where $\boldsymbol{H}$ is a matrix corresponding to the action of filter $\boldsymbol{h}$.

1.1 The POCS method

POCS[3] widely used in the recovery problem is an iterative method to find a common element for a series of given convex sets which are established according to the *a priori* known properties of the solution. If C_i ($i = 1, \cdots, m$) are the convex sets, and P_i ($i = 1, \cdots, m$) are the corresponding projection operators, assume that

$$C_0 = \bigcap_{i=1}^{m} C_i \neq \varnothing$$

Then the solution will be into C_0, and can be obtained by the iteration

$$\boldsymbol{r}_{k+1} = P_m P_{m-1} \cdots P_1 \boldsymbol{r}_k \qquad k = 0,1,2\cdots \tag{3}$$

where the initial value $\boldsymbol{r}_0$ is an arbitrary vector.

1.2 Signal reconstruction from multiscale edge

The multiscale edges reconstruction algorithm was proposed by S.Mallat, et al[5-7]. Here we review some important results of this method. First, we define the absolute maxima of the discrete wavelet transform as any sample $W(j,n)$ satisfying the conditions ① $|W(j,n)| \geqslant |W(j,n-1)|$ and ② $|W(j,n)| \geqslant |W(j,n+1)|$, with the following qualifiers. If ① is satisfied with equality, then ② should be satisfied with

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strict inequality along with the restriction that $|W(j,n-1)| \geqslant |W(j,n-2)|$. On the other hand, if ② is satisfied with equality, then ① should be satisfied with strict inequality along with the restriction that $|W(j,n+1)| \geqslant |W(j,n+2)|$[8].

The essence of this algorithm is to compute the approximation of $W(j,n)$, the dyadic wavelet transform of $f(x) \in L^2(\mathbf{R})$, using an alternate projection algorithm:

$$\left.\begin{aligned} &f_{k+1}(x) = P(S(J,n), W(j,n_0))_{1\leqslant j\leqslant J} \\ &P = \mathrm{DWT}^{-1}(P_Y P_\Gamma P_V) \end{aligned}\right\} \tag{4}$$

where $\{n_0\}$ is the positions of the local maxima of $\{|W(j,n)|\}$ at scale j; $\{S(J,n)\}$ is the low frequency signal; DWT^{-1} denotes the inverse dyadic wavelet transform operator; P_V is the projection operator onto V:

$$P_V = \mathrm{DWT}\cdot\mathrm{DWT}^{-1} \tag{5}$$

where V is the space of all dyadic wavelet transforms of functions in $L^2(\mathbf{R})$. P_Γ is the projection onto Γ, and Γ is the affine space of sequences of functions $g_j(x) \in K$ such that $g_j(x_{j,n_0}) = W(j,n_0)$ for any index j and all maxma positions (j,n_0) and K is the space of all sequence of functions $g_j(x)$ such that

$$|g_j(x)|^2 = \sum_j \left(\|g_j\|^2 + 2^{2j} \left\| \frac{\mathrm{d}g_j}{\mathrm{d}x} \right\|^2 \right) < +\infty$$

and the operator P_Γ transforms $g_j(x) \in K$ into $h_j(x) \in \Gamma$,

$$h_j(x) = P_\Gamma(g_j(x)) = g_j(x) + \alpha \mathrm{e}^{2^{-j}x} + \beta \mathrm{e}^{-2^{-j}x}$$

where the constants α and β are solutions of the following equations:

$$\left.\begin{aligned} &\alpha \mathrm{e}^{2^{-j}x_{j,0}} + \beta \mathrm{e}^{-2^{-j}x_{j,0}} = W(j,0) - g_j(x_{j,0}) \\ &\alpha \mathrm{e}^{2^{-j}x_{j,1}} + \beta \mathrm{e}^{-2^{-j}x_{j,1}} = W(j,1) - g_j(x_{j,1}) \end{aligned}\right\} \tag{6}$$

where $x_{j,0}$ and $x_{j,1}$ are the abscissa of two consecutive modulus maxima of $W(j,n)$. Finally, sign constraints can be imposed to suppress any spurious oscillation in the reconstructed wavelet transform. Thus, P_Y is the projection operator onto convex set Y, and let Y be the set of sequence $(h_j(x)) \in K$ such that for any pair of consecutive maxima positions $(x_{j,n}, x_{j,n+1})$ and $x \in [x_{j,n},\ x_{j,n+1}]$.

$$\left.\begin{aligned} &\mathrm{sgn}(g_j(x)) = \mathrm{sgn}(x_{j,n}) \\ &\qquad \text{if } \mathrm{sgn}(x_{j,n}) = \mathrm{sgn}(x_{j,n+1}) \\ &\mathrm{sgn}\left(\frac{\mathrm{d}g_j(x)}{\mathrm{d}x}\right) = \mathrm{sgn}(x_{j,n+1} - x_{j,n}) \\ &\qquad \text{if } \mathrm{sgn}(x_{j,n}) \neq \mathrm{sgn}(x_{j,n+1}) \end{aligned}\right\} \tag{7}$$

1.3 Wavelet-based deconvolution

Here, we present our nonlinear adaptive POCS method for the deconvolution problem. First we suppose *a priori* known the noise variance (i.e., we consider an additive, white, zero-mean Gaussian noise). And we formulate the following convex set

$$C = \{\boldsymbol{r} : \|\boldsymbol{f} - \boldsymbol{H}\boldsymbol{r}\|_2^2 \leqslant \delta^2\} \tag{8}$$

where δ^2 can be calculated from the known variance. The projection operator onto C can be obtained by solving the following problem:

$$\left.\begin{aligned} &\min \|\boldsymbol{r}_{\mathrm{p}} - \boldsymbol{r}\|_2^2 \\ &\text{s.t. } \|\boldsymbol{f} - \boldsymbol{H}\boldsymbol{r}_{\mathrm{p}}\|_2^2 = \delta^2 \end{aligned}\right\} \tag{9}$$

where $\boldsymbol{r}_{\mathrm{p}}$ is the projection onto C of $\boldsymbol{r}$. Using the Lagrange multipliers method, we obtain

$$\boldsymbol{r}_{\mathrm{p}} = P_C\boldsymbol{r} = \boldsymbol{r} + (\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H} + (1/\lambda)\boldsymbol{I})^{-1}\boldsymbol{H}^{\mathrm{T}}(\boldsymbol{f} - \boldsymbol{H}\boldsymbol{r}) \tag{10}$$

where λ is the Lagrange multiplier which must satisfy the constraint of (9). Charalambous[9] suggested an effective method to find it by solving a nonlinear equation:

$$\phi(\lambda) = \frac{1}{N}\sum_{\mu=0}^{N-1} \frac{|\boldsymbol{F}(\mu)|^2}{(\lambda|\boldsymbol{H}(\mu)|^2 + 1)^2} - \delta^2 = 0 \tag{11}$$

where $\boldsymbol{F}(\mu)$ and $\boldsymbol{H}(\mu)$ are DFT's of vector $\boldsymbol{f}$ and matrix $\boldsymbol{H}$, respectively, N is the length of $\boldsymbol{F}(\mu)$. Noticing (11) monotonically decreases if $\lambda > 0$, one dimensional range-search algorithm can be used to find the optimum value of λ.

As for the selection of maxima, we use the spatially selective noise filtration (SSNF) technique[10,11] which is based on the fact that sharp edges have large signal over many wavelet scales and noise dies out swiftly with scale. Thus, given a signal approximation $\boldsymbol{f}_k$ at each iteration, the procedure of wavelet modulus maxima selection is as follow.

① Locate the absolute modulus maxima at each scale j, and form the set

$$S_j = \{n_i : |W(j,n_i)| \text{ is local maxima}\}$$

② Compute the correlation $\mathrm{corr}_l(j,n)$ at scale j, where

$$\mathrm{corr}_l(j,n) = \prod_{m=0}^{l-1} W(j+m,n) \tag{12}$$

where $l < M - j + 1$, M is the total number of scale. Usually, we select $l = 2$.

③ Choose the locations

$$n_S \in S_j \qquad \left|\frac{\mathrm{corr}_l(j,n_S)}{\sqrt{\mathrm{Pcorr}(j)}}\right| \geqslant \left|\frac{W(j,n_S)}{\mathrm{PW}(j)}\right| \tag{13}$$

$$\mathrm{Pcorr}(j) = \sum_n \mathrm{corr}_2(j,n)^2, \mathrm{PW}(j) = \sum_n W(j,n)^2$$

Then save the location and values of $W(j,n_S)$, reset the corresponding $\mathrm{corr}_2(j,n_S)$ and $W(j,n_S)$ to zero, and remove n_S from S_j.

④ At the finest scale, where noise is dominating, $|W(1,n_S)|$ is multiplied with a weight $\lambda \geqslant 1$ to suppress noise.

⑤ Repeat steps ② and ③ until the power $W(j,n)$ is near equal to a pre-decided threshold or S_j is empty.

We consider these n_S as locations of modulus maxima of $\{|W(j,n)|\}$ created by the signal. These values, joint with the low frequency $S(J,n)$, can serve as the input of the multiscale edge reconstruction algorithm described in section 1.2.

Finally, we summarize our algorithm:

$$\left.\begin{aligned} \boldsymbol{r}_{k+1} &= PP_C\,\boldsymbol{r}_k \qquad k = 0,1,2,\cdots \\ \boldsymbol{r}_0 &= \boldsymbol{f} \end{aligned}\right\} \tag{14}$$

2 Experiment

An ARC-spectrapro-300I(Acton Research Corporation,USA) served as a spectrograph with the resolution equalling 0.1 nm and the spectrasense CCD (Acton Research Corporation,USA) was used to record the spectrum. To get the slit function, He-Ne source (5 mA,1.7 mW) was used. It has a very narrow half-width. Sodium spectrum with two lines at 589.0 nm and 589.6 nm, respectively, were used as the input signal. We first measured the sodium spectrum at a relatively large slit width (e.g. 220 μm), and recorded the He-Ne spectrum at this slit width to use as the slit function, then measured the sodium spectrum at a narrow width (e.g. 50 μm) to contrast with the result of deconvolution.

From Fig.1, it is clear that the resolution is greatly influenced by the slit width: two peaks can be distinguished clearly at the narrow slit, but when enlarging the slit, they can't be separated any more.

In this experiment, the wavelet is the quadratic spline of compact support and one vanishing moment[5]. This class of wavelet is especially efficient in edge detection of a signal.

Fig.2 shows the result of deconvolution. The resolution is greatly enhanced, but the half-width of the deconvolved signal is still larger than the one measured at narrow slit. This is because in the procedure of the selection of modulus maxima at the finest scale where noise is dominating, a large weight $\lambda = 1.2$ is used to remove most of the noise, with the result of reducing the resolution. And the asymmetry of the system made the measured slit function a little different from the real one, which also was contributive to the degraded resolution of the deconvolution result.

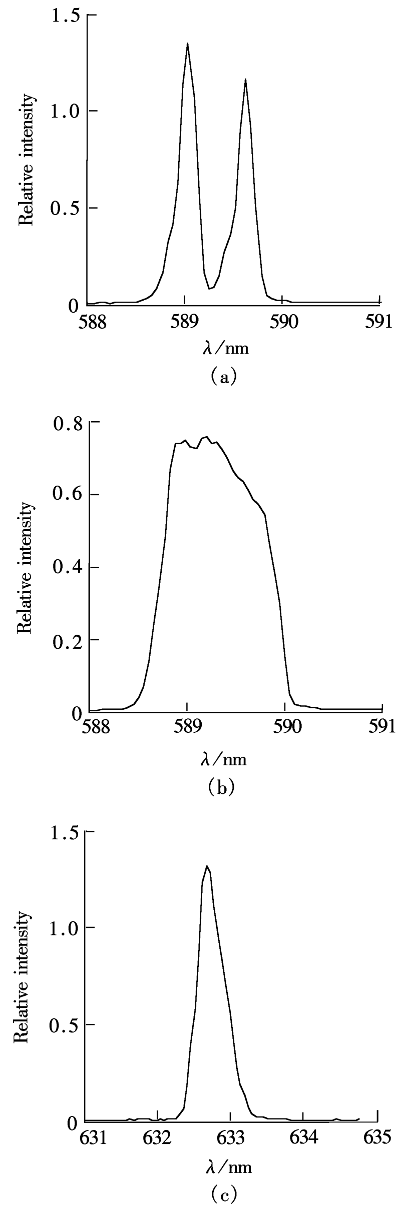

Fig.1 Degraded spectrum and slit function. (a) Sodium spectrum at slit width equals 50 μm; (b) Sodium spectrum at slit width equals 220 μm; (c) He-Ne spectrum at slit width equals 220 μm

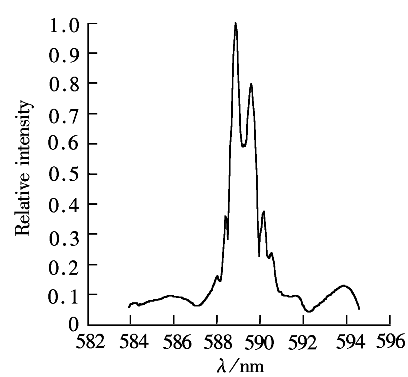

Fig.2 Deconvolution result

3 Conclusion

In this paper, we use a nonlinear POCS method to solve the spectrum deconvolution problem. The method includes restoring signal from wavelet modulus maxima. For the purpose of selecting modulus maxima, we use the spatially selective noise filtration method which is more straightforward, easier to implement and more robust. Sodium spectrum with two lines at 589.0 nm and 589.6 nm is deconvolved and a satisfactory result is achieved. The method allows the inclusion of any kind of constraint in other practical problems.

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一种基于小波的光谱反卷积方法

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摘　要　为消除测量光谱中狭缝宽度的影响,提出了一种基于小波的反卷积算法以获得问题的规整化解,该方法使用一维多尺度边缘重构技术.空域选择滤波算法被用来区分由信号产生的极值与由噪声产生的极值.实验中,对宽狭缝时测得的 Na 黄光 2 条特征谱线进行反卷积处理,以相应狭缝宽度时测得的 He-Ne 光谱作为狭缝函数,并与较窄狭缝时测得的 Na 光谱作了比较.反卷积结果显示本方法可以极大地提高测量光谱的分辨率.

关键词　反卷积; 狭缝函数; 小波局部极大值

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