

# A method based on mutual information and gradient information for medical image registration

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**Abstract:** Mutual information is widely used in medical image registration, because it does not require preprocessing the image. However, the local maximum problem in the registration is insurmountable. We combine mutual information and gradient information to solve this problem and apply it to the non-rigid deformation image registration. To improve the accuracy, we provide some implemental issues, for example, the Powell searching algorithm, gray interpolation and consideration of outlier points. The experimental results show the accuracy of the method and the feasibility in non-rigid medical image registration.

**Key words:** medical image registration; gradient information; mutual information; multi-modal images; non-rigid deformation

The application of mutual information in medical image registration has been proposed recently<sup>[1]</sup>. Mutual information is an intensity-based measure which does not require the definition of landmarks or features. Unlike measures based on correction of gray values or differences of gray values, mutual information does not assume a linear relation among the gray values in the image, it also does not require preprocessing or dividing the image. Therefore, mutual information is widely used in multi-modal images registration<sup>[2,3]</sup>.

The robustness of the measure is questionable, however. A possible reason for it is the absence of spatial information in the registration. The registration functions can be ill-defined containing local maxima. This can occur, for example, when the images are of low resolution, when images contain little information, when there is only a small region of overlap or as a result of interpolation methods. So improvement is necessary.

Pluim et al. proposed an improved method by combining mutual information and gradient information of images<sup>[4]</sup>. The method combines the advantage of mutual information in calculating images information and the advantage of gradient information in avoiding local maximam. But they only considered rigid transformation in their method. In this paper, we extend their algorithm to non-rigid image registration.

## 1 Method

### 1.1 Mutual information

Mutual information is a basic concept in infor-

mation theory<sup>[5]</sup>. It is used to describe the information relativity between two images and can be described by entropy. One interpretation of entropy is a measure of dispersion of a probability distribution.

The entropy of image  $A$  can be defined as

$$H(A) = - \sum_a p_A(a) \log p_A(a) \quad (1)$$

The definition of the joint entropy is

$$H(A, B) = - \sum_{a,b} P_{AB}(a, b) \log p_{AB}(a, b) \quad (2)$$

where  $P_A(a)$  and  $P_B(b)$  denote the marginal probability distributions of  $A$  and  $B$ , respectively;  $P_{AB}(a, b)$  is the joint probability distribution.

The mutual information of two images  $A$  and  $B$  is given by

$$I(A, B) = H(A) + H(B) - H(A, B) \quad (3)$$

When matching multi-modal images, although the sources of the two images are different, they are based on information from the same organ. Therefore, when the positions of two images are identical, the information shared between them must be maximized and the mutual information of these two images is the maximum. Basically, the mutual information can be estimated by the general distribution between the joint probability distribution  $P_{AB}(a, b)$  and marginal probability distributions  $p_A(a)$  and  $p_B(b)$ .

$$I(A, B) = \sum_{a,b} P_{AB}(a, b) \log \frac{p_{AB}(a, b)}{p_A(a) p_B(b)} \quad (4)$$

As for the discrete digital image, the joint probability distribution can be presented by the joint histogram.

$$P_{AB}(i, j) = \frac{h(i, j)}{\sum_{i,j} h(i, j)} \quad (5)$$

The distribution of marginal probability distribution is

$$p_A(i) = \sum_j p_{AB}(i, j) \quad (6)$$

$$p_B(j) = \sum_i p_{AB}(i, j) \quad (7)$$

Therefore, the mutual information can be written as

$$I(A, B) = \sum_{i,j} p_{AB}(i, j) \log \frac{p_{AB}(i, j)}{p_A(i) p_B(j)} \quad (8)$$

## 1.2 Gradient information

To compensate for the disadvantage of using mutual information only, some new algorithms were proposed<sup>[4,6]</sup>.

As a sample point in one image,  $A(x, y)$  has its corresponding point in another image,  $B(x', y')$ , which is found by affine transformation of  $A(x, y)$ . We can compute the gradient information of  $A(x, y)$  and  $B(x', y')$ . The angle between the gradients of the point  $A$  and  $B$  can be represented as

$$\alpha_{AB} = \arccos \frac{|\nabla x \cdot \nabla x'|}{|\nabla x| \cdot |\nabla x'|} \quad (9)$$

where  $\nabla x$  is the gradient vector of the point  $A$ , and  $\nabla x'$  is the gradient vector of the point  $B$ ;  $|\nabla x|$  and  $|\nabla x'|$  denote the magnitude of  $\nabla x$  and  $\nabla x'$ , respectively.

For the same tissue, the different imaging techniques lead to different images that have different gray values. So the gradient points correspond to different directions. However, for the images of the same tissue, the directions of most pixels are the same or opposite. Considering the problems above, a weighting function is used as

$$\omega(\alpha) = \frac{\cos(2\alpha) + 1}{2} \quad (10)$$

Combining mutual information and gradient information, the registration function can be defined as

$$I_{\text{new}}(A, B) = G(A, B) \cdot I(A, B) \quad (11)$$

where

$$G(A, B) = \sum_{(x, x') \in (A \cap B)} \omega(\alpha_{x, x'}) \min(|\nabla x|, |\nabla x'|) \quad (12)$$

## 1.3 Optimization

The Powell optimization algorithm is used in this paper. The reason for selecting it is that it does not need to calculate the gradient while searching the direction. In the Powell algorithm, all the parameters are initialized as zero, and the initial searching direction is a conjugated unit vector.

Four parameters are considered in this paper: translation in two directions, rotation around the center and scaling factor.

## 1.4 Precision

Pluims et al.<sup>[4]</sup> only considered the translation and rotation parameter in their algorithm. To apply the measure to the non-rigid deformation image registration, we introduce the scaling parameter. We use the following process to improve precision.

### 1) Selection of sample points

There are two ways to select the sample points. One is random selection and the other is regular selection. The random selection is faster, but the error is larger than the regular one. Therefore, we adopt the regular selection. In the background area, we sample once during several other points to improve speed. In the information area, we sample each point. We calculate the gray values of all sampling points to compute the mutual information and gradient information. In this way, the errors can be reduced.

### 2) Interpolation

In the registration process, the sample point  $a$  in the image  $F$  for registration will map the point  $b$  in image  $R$ . Usually, the coordinate of  $b$  will be a non-integer value. Many scholars gave their own interpolation algorithm to solve this problem. Luo<sup>[2]</sup> used a trilinear PV interpolation to contribute the gray values of the pixels around  $b$  to the joint probability distribution. That is

$$\forall i: h(f, r(i)) = \omega_i \quad (13)$$

This interpolation algorithm is effective in reducing the error when computing mutual information. However, it cannot improve the accuracy of the gradient-calculation, because it only focuses on the joint probability distribution but not the gray value of each pixel.

Thus, we give a new interpolation algorithm for this problem. By redistributing the weighting factor, we contribute the gray value of the pixel  $b$  to the gray values of the pixels around  $b$ . That is

$$R(i) = R(a) \omega_i \quad (14)$$

where  $R(a)$  is the gray value of the pixel  $b$ ;  $R(i)$  is the gray value of the pixel around  $b$ ;  $\omega_i$  is the weighting factor satisfying

$$\sum_i \omega_i = 1 \quad i = 1, 2, 3, 4 \quad (15)$$

Without introducing new interpolation points, the algorithm can improve accuracy when computing gradient information. Though the algorithm cannot

improve precision when computing mutual information, the result of the experiment shows that, when considering the mutual information and gradient information together, the algorithm will bring smaller error than the algorithm by using trilinear PV interpolation.

### 3) Consideration of outline point

After transformation, the corresponding point  $b$  of the sample point  $a$  in image  $F$  is out of the referenced image, we call the point  $a$  an outside point.

There are two ways to deal with this problem: neglecting the outside point or regarding its gray value as zero. The results of the experiment show that there are adverse influences on the registration accuracy in these two ways. An algorithm is presented in this paper by expanding the background of the referenced image to reduce the number of outside points and reduce the error.

## 1.5 Estimation of medical image registration

The estimation of medical image registration, especially in the multi-modal image, is very tough. Because the multi-modal image for registration is

obtained at different times and conditions, there is no absolutely optimal registration, but relatively optimal registration under certain rules.

In this paper, we estimate the algorithm in two ways (see Fig.1). ① Give a referenced image and make testing images with the given transformation parameters. Match the referenced image to testing image and estimate the results. ② Match the medical image and estimate the results.

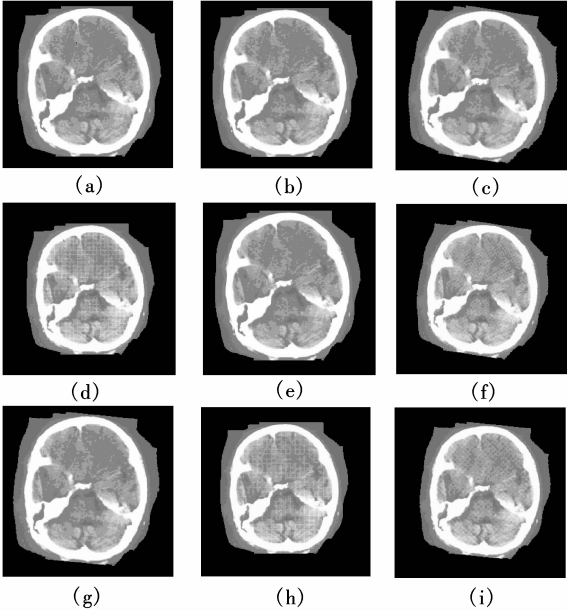
## 2 Experimental Results

We match the testing image to the referenced image with combined method and the method by using mutual information only. The comparison results are shown in Tab.1. Here, method 1 is the method by using mutual information only. Method 2 is the method combining mutual information and gradient information. Images from testing image 1 to testing image 8 are images before registration. The corresponding parameters are the variations of the given parameters.

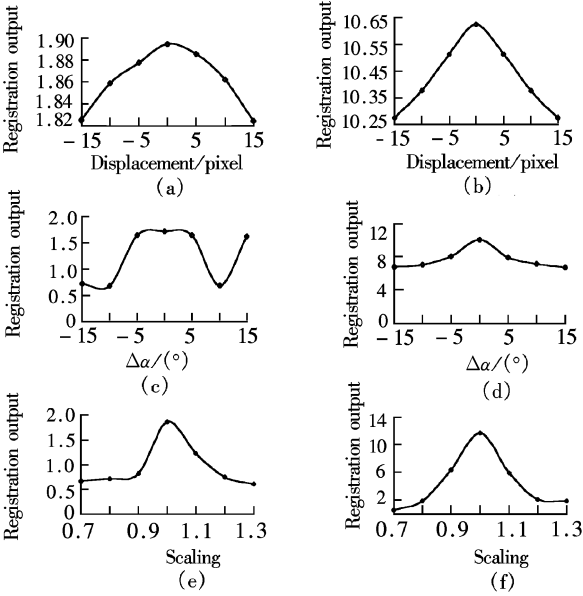
**Tab.1** Comparison of registration results with two methods

Transformation parameters	Translation $\Delta X/\text{pixel}$	Translation $\Delta Y/\text{pixel}$	Rotation $\Delta\alpha/(^{\circ})$	Scaling $\Delta L$
Testing image 1	5.0	0.0	0.0	1.0
Result of method 1	5.0	- 0.0	0.0	1.0
Result of method 2	4.995 599	- 0.005 44	0.000 024	0.999 981
Testing image 2	0.0	0.0	0.15	1.0
Result of method 1	0.046 142 8	0.073 243	0.149 77	1.000 014
Result of method 2	- 0.020 80	0.051 394	0.150 498	1.000 356
Testing image 3	0.0	0.0	0.0	1.15
Result of method 1	0.001 759	- 0.000 505	- 0.000 013	1.15
Result of method 2	11.471 120	4.055 308	0.951 322	1.164 709
Testing image 4	5	- 6	0.0	1.0
Result of method 1	4.999 980	- 5.999 849	0.0	1.0
Result of method 2	4.995 794	- 6.009 192	0.000 016	0.999 977
Testing image 5	0.0	0.0	0.15	1.12
Result of method 1	- 0.005 61	0.005 806	0.150 180	1.119 730
Result of method 2	5.428 34	1.157 346	0.981 586	1.114 652
Testing image 6	5.0	- 5.0	0.1	1.0
Result of method 1	5.445 207	- 4.516 719	0.099 724	1.000 019
Result of method 2	5.416 314	- 4.529 941	0.099 933	1.000 056
Testing image 7	5.0	- 7.0	0.0	1.12
Result of method 1	4.460 922	- 6.250 075	- 0.000 008	1.119 976
Result of method 2	6.209 679	- 5.500 974	0.996 073	1.174 603
Testing image 8	- 8.0	- 5.0	0.1	1.12
Result of method 1	- 6.670 41	- 5.161 281	0.100 259	1.120 424
Result of method 2	0.125 239	0.378 674	0.960 920	1.021 249
The average error of using method 1	0.295 925 6	0.184 274	0.000 165 5	0.000 109 37
The average error of using method 2	1.363 089	1.483 648	0.446 161	0.021 925
The maximal error of using method 1	1.329 588	0.749 925	0.000 276	0.000 424
The maximal error of using method 2	11.471 120	4.055 308	0.951 322	0.098 761

Fig.2 shows the results of registration by method 1 and method 2. In Fig.2, (a), (c) and (e) are the registration functions using mutual information only. (b), (d) and (f) are the registration functions using the combined method. In Fig.2 (a) and (b), the transformation is a displacement along an in-plane axis; in (c) and (d) the transformation is a rotation around the center; in (e) and (f) the transformation is a zoom around the center.



**Fig.1** Testing images and referenced image. (a) Referenced image; (b) Testing image 1; (c) Testing image 2; (d) Testing image 3; (e) Testing image 4; (f) Testing image 5; (g) Testing image 6; (h) Testing image 7; (i) Testing image 8



**Fig.2** Registration functions

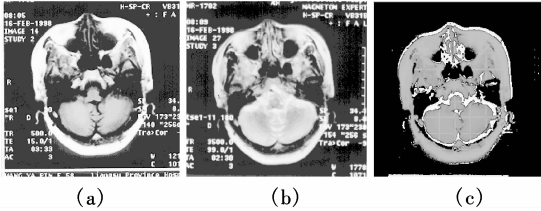
From Tab.1 and Fig.2, we can see that when using the mutual information method only, if there are rotation or scaling parameters in the registration, the graph of

the registration function is not smooth, the error is large and unstable. For example, when  $\Delta\alpha = 8(^{\circ})$ , and the other parameters remain the same; when  $\Delta\alpha = 8(^{\circ})$ ,  $\Delta L = 1.12$ , and the other parameters remain the same; and another situation is when  $\Delta X = -8$  pixel,  $\Delta Y = -5$  pixel,  $\Delta\alpha = 6(^{\circ})$ ,  $\Delta L = 1.12$ , the registration result is not good. But when using the combined method, there are fewer errors.

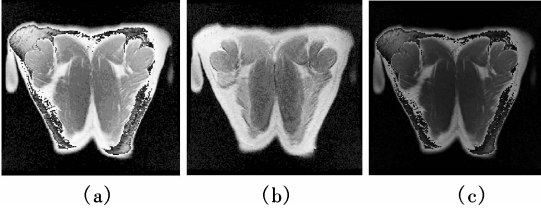
The experimental results show that the method presented in this paper is efficient in improving the accuracy at the situation of larger transformation of scaling and rotation angles.

We apply the combined method in practical multi-modal medical image. Fig.3 and Fig.4 are the registration results of the MR-T1 and MR-T2 images of human brain and lung.

The results show this method can apply to the registration of multi-modal image of brain and to the registration of the non-rigid deformation image of lung.



**Fig.3** Result of matching MR-T1 and MR-T2 in brain. (a) MR-T1 image in brain before registration; (b) MR-T2 image in brain before registration; (c) Overlap the MR-T1 (after registration) to MR-T2 (the deep gray describe MR-T1)



**Fig.4** Result of matching MR-T1 and MR-T2 in lung. (a) MR-T1 image in lung before registration; (b) MR-T2 image in lung before registration; (c) Overlap the MR-T1 (after registration) to MR-T2 (the deep gray describe MR-T1)

### 3 Conclusion

A medical image registration method combining mutual information and gradient information is introduced. The method is applied to the medical image registration of a non-rigid transformation image. The results show that the effect is very good whether in rigid or in non-rigid transformation medical image registration.

We have only considered the two-dimensional registration. Our further work is to apply the method to three-dimensional registration.

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# 一种基于互信息和梯度信息的医学图像配准算法

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**摘 要** 由于互信息不需要对图像进行预处理,因此被广泛地应用于医学图像配准中.但是,配准过程中的局部极大值难以克服.本文引进了梯度信息,用于解决局部极值问题.并将这种方法应用到人体的非刚性形变的医学图像配准中.同时,给出了一些用于改进精度的方法,如:Powell 搜索算法、灰度插值和出界点问题,提高了匹配精度.采用此方法对脑部和肺部的多模图像进行配准,实验结果表明该方法对非刚体医学图像的配准有很大的可行性.

**关键词** 医学图像配准; 梯度信息; 互信息; 多模态图像; 非刚性变换

**中图分类号** R445