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Composite grid method for analysis of electromagnetic field

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Abstract: In this paper, a composite grid method (CGM) for finite element (FE) analysis of an electromagnetic field with strong local interest is proposed. The method is based on the regular finite element method in conjunction with three basic steps, i.e. global analysis, local analysis, and modified global analysis. In the first two steps, a coarse finite element mesh is used to analyze the global model to obtain the nodal potentials which are subsequently used as artificial boundary conditions for local regions of interest. These local regions with the prescribed boundary conditions are then analyzed with refined meshes to obtain more accurate potential and density distribution. In the third step, a modified global analysis is performed to obtain more improved solution for potential and density distribution. And iteratively, successively improved solutions can be obtained until the desired accuracy is achieved. Various numerical experiments show that CGM yields accurate solutions with significant savings in computing time compared with the regular finite element method.

Key words: composite grid; global analysis; local analysis

Singularity, large variations or discontinuities of material coefficients, potential or density in limited parts of the domain which are common in most electromagnetic field models such as electric machines and transformers increase the difficulty of accurately predicting detailed local potential and density distributions. For this kind of problem, the usual approach of engineers is using unstructured meshing with local refinement of large size-transition. However, this treatment often leads to problems: bad conditions of stiffness matrix; very expensive computation. Especially for 3-D models, unstructured meshing with the above requirement is still not mature. In this paper, we propose a composite grid method (CGM) which is not computationally expensive yet still an accurate method for FE analysis of such problems.

CGM has a lot in common with the fast adaptive composite grid method[1], local defect correction (LDC)[2], and global-local method[3,4]. While CGM possesses all the advantages of the above three methods, it also has its own characteristics. With CGM, the FE analysis is performed in several iterative steps. It begins with an FE analysis using a coarse mesh of the global model. According to this global analysis, local regions requiring more detailed analysis are subsequently identified. Along the boundaries of every local region, the electric potentials interpolated from the global analysis are used as the enforcing boundary conditions. Consequently, each local region is posed as an independent problem. Fine meshes are used over these small local regions for local analysis to obtain accurate local field distribution. After the local analysis, the resulting potentials of local regions are utilized to perform a refined global analysis which obtains more accurate potential and density. Then results of the refined global analysis are fed back to the local analysis. Thus, we can iteratively seek successively improved solutions until the desired accuracy is obtained.

In the following, section 1 describes the algorithm of the CGM. Section 2 presents three numerical examples to advance the discussions. First, we use CGM to solve a Possion equation to demonstrate the behavior of the method, such as convergence rate and accuracy of the solution; then, we apply CGM to an electric machines' FE analysis which has strong local interest in the air gap area; at last, we begin to use CGM to deal with Team Problem-21. These three examples demonstrate that CGM can obtain solutions of the same quality as those from the regular finite element with a global fine mesh. In addition, it is demonstrated that the computing time of CGM is much shorter than that of the regular finite element analysis with the same degree of accuracy. Hence, CGM is a general-purpose analysis tool for electromagnetic FE problems which need local detailed analysis. In section 3, we present the conclusions and future work.

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1 Composite Grid Method

For simplicity, we consider the following boundary problem:

$$Lu = f \quad \text{in } \Omega$$
$$u = g \quad \text{on } \partial\Omega$$

where Ω is a domain in Fig.1 and $\partial\Omega$ is the boundary of Ω; L is an elliptic linear operator; f and g are functions on Ω. And u has comparatively large variations or strong local behavior in the domain Ω_{cr} which is a subdomain of Ω and $\Omega_f (\Omega_f \subset \Omega)$.

First we recall the concept of composite grids and introduce some notations. In Fig.2, the mesh of Ω is called the global coarse mesh; the mesh of Ω_f is called the local fine mesh. We denote them by G^* and g^*, respectively. The mesh $G := G^* \cup g^*$ is called the composite grid (Here we use the traditional term, not "composite mesh".) For later uses, we have the following additional notations:

1) We call Ω_{cr} the critical region, Ω_f the local region and Ω the global area.

2) We denote $\Gamma := \partial\Omega_f - \partial\Omega$.

3) We denote $G_i := \{p \mid p \text{ is a node of } G^* \text{ and } p \text{ is in } \Omega_{cr}\}$.

4) We denote the following variational form:

$$(Lu, v) = \int_\Omega Lu \cdot v \mathrm{d}\Omega$$

Now we come to the outline of the algorithm of CGM:

1) Perform the initial global analysis;

2) Perform the local analysis;

3) Perform the modified global analysis. If they converge, go to end; else, go to 2).

Below, the three steps are discussed in detail.

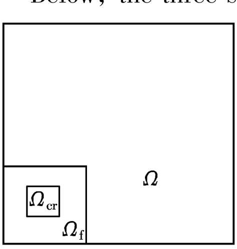

Fig.1 A domain Ω

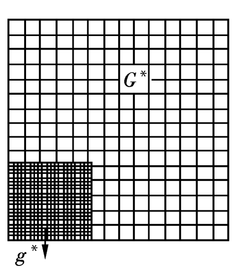

Fig.2 Global mesh G^* and local mesh g^*

1.1 Initial global analysis

The global analysis is to solve the equation: $(LU_c, v) = (f, v) - (Lg, v), \forall v \in H_c$, where H_c is the finite element space corresponding to the mesh G^* and the boundary being $\partial\Omega$. The global analysis must be an "adequate" analysis, which means the global behavior should be accurately determined and local details (i.e. local large variations) should be at least grossly incorporated. That is to say G^* must not be too coarse for the analysis.

After getting the solution U_c, firstly, a critical region Ω_{cr} requiring a more detailed analysis may be identified from U_c. Usually, U_c has large variations and is not accurate enough in this region. Ω_{cr} can be determined by way of using a posteriori error estimation or by trying a region and then modifying it. Then we enlarge Ω_{cr} to the local region Ω_f. The enlargement depends on the criterion: U_c is accurately determined on the boundary and outside of Ω_f, i.e. U_c is not accurate only in the interior of Ω_f. Finally, we separate Ω_f with a more refined local mesh g^* to predict more accurately the detailed state of the local region.

1.2 Local analysis

The local analysis is to solve the equation: $(LU_f, v) = (f, v) - (Lg_f, v)$, $\forall v \in H_f$, where H_f is the finite element space corresponding to g^*, and g_f represents the artificial boundary conditions:

$$g_f|_{\partial\Omega_f} := \left\{\begin{matrix} g & \text{on } \partial\Omega_f \cap \partial\Omega \\ U_c & \text{on } \Gamma \end{matrix}\right\}$$

It means if some part of the local region's boundary coincides with a part of the global area, we use the given Dirichlet boundary conditions. But for the interface Γ of Ω_f and Ω, we use the obtained solution U_c. Actually, for a point Q of $\partial\Omega_f$, its value is interpolated from U_c. We first find the element E_i of G^* where Q is, and then use the Newton iteration to get the local coordinates, finally we make use of the shape functions to obtain the interpolation value.

1.3 Modified global analysis

The modifications to the initial global analysis lie in:

1) We modify $\partial\Omega$ to be $\partial\Omega \cup \Omega_{cr}$.

2) We also impose the artificial Dirichlet boundary conditions on Ω_{cr}. As in the local analysis, the artificial boundary values of G_i's nodes are interpolated from U_f.

Thus our modified global analysis is to solve the equation $(LU^*, v) = (f, v) - (Lg_c, v), \forall v \in H^*$, where H^* is the finite element space corresponding to G^* and the boundary being $\partial\Omega \cup \Omega_{cr}$. Here,

$$g_c|_{\partial\Omega \cup \partial\Omega_{cr}} := \left\{\begin{matrix} g & \text{on } \partial\Omega \\ U_f & \text{on } \partial\Omega_{cr} \end{matrix}\right\}$$

After obtaining U^*, let $U_c = U^*$. Because the solutions of the critical regions are derived from those of the refined local analysis, U_c are much more improved. Then we can go to the local analysis, then the modified global analysis and iteratively achieve successively more accurate solutions as desired. Usually, five iterations are enough for the convergence.

2 Numerical Experiments

In this section, the effectiveness of the composite grid method (CGM) is demonstrated by its application to three examples: a two-dimensional Poisson equation with strong local behavior; a two-dimensional permanent magnetic electric machine; and the Team Problem-21. In the first example, the results of the CGM are compared with those from the regular FE analysis with a global coarse mesh, the regular FE analysis with a global fine mesh, and with the exact solution. Also, we present a table for showing the convergence state of CGM.

Example 1 The first example is a Poisson equation given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f \qquad 0 < x < 1;\ 0 < y < 1$$

$$u = g \qquad \text{on all boundaries}$$

where $f = 0$ and $g = \ln((x+0.01)^2 + (y+0.01)^2)$. And the exact solution is $u = \ln((x+0.01)^2 + (y+0.01)^2)$. For example, the global coarse mesh is a uniform 15×15 mesh; the global fine mesh is a uniform 100×100 mesh. The critical region is $[0, 0.2] \times [0, 0.2]$ and the local region is $[0, 0.25] \times [0, 0.25]$. And the local fine mesh is a uniform 25×25 mesh. We use bilinear quadrilateral elements in this example.

We have Tab.1 to list the comparative error norms from various FE analysis methods. In the table, the norm $\| \ \|$ is defined by $\|u\| = \left(\sum_1^n |u_i|^2\right)^{1/2}$ where n is the nodes number. And $\|e\|$ is the error norm of the FE solutions with the exact solutions. $\|u_{ex}\|$ is the norm of the exact solutions. It is evident that CGM's solutions agree very well with the exact solutions and can converge with the regular finite element solution with a global fine mesh.

Tab.1 Comparative error norms of various analysis schemes in example 1

Item		$\|e\| / \|u_{ex}\|$
FE analysis with a global fine mesh		4.702×10^{-4}
FE analysis with a global coarse mesh		4.143×10^{-3}
CGM	One iterations	1.281×10^{-3}
	Three iterations	5.130×10^{-4}
	Five iterations	5.021×10^{-4}

Example 2 The second example is a permanent magnetic resistance electric machine, which has strong local behavior in the air gap and neighboring areas. For this, we apply CGM to it. Fig.3 shows a quarter of the global coarse mesh. After the initial global analysis, the critical region is chosen to be the air gap, and the local region to be properly larger than the air gap. Using CGM, we find the solutions of the FE analysis agree well with the results of the physical experiments on electric potentials, density and eddy current distribution.

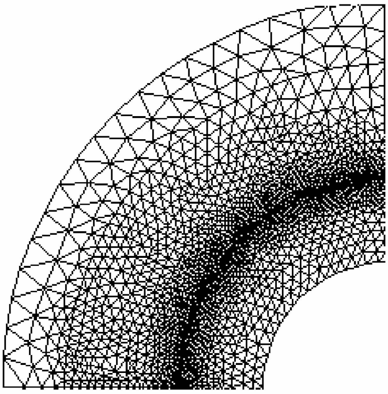

Fig.3 A quarter of the global mesh for CGM analysis of an electric machine

Example 3 The third example is Team Problem-21 which is about the electromagnetic analysis of a 3-D stray field loss model. Model B is chosen for FE analysis. And considering the symmetry of the problem, a 1/2 region is taken for computing. The most striking point of Model B is: The eddy currents in the steel plate area vary rapidly, i.e. with very strong local interest. Hence, we perform an FE analysis of it with CGM. In our CGM analysis the whole model (i.e. 1/2 region) is chosen to be the global analysis area; the steel plate is chosen to be the critical region; a cubic area covering the steel plate and including the plate's neighboring gap area is chosen to be the local region.

Fig.4 and Fig.5 show the global coarse mesh and a part of the local fine mesh, respectively. Even though our CGM analysis of Problem-21 is just a beginning, we have obtained satisfying results about the eddy current distribution of the steel plate and the magnetic flux densities distribution on the surface of the steel plate. And we find CGM is an effective method for further dealing with Problem-21.

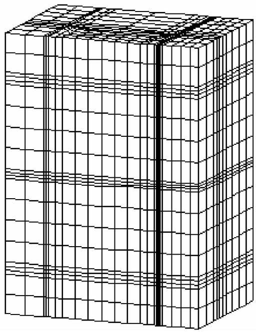

Fig.4 Global coarse mesh for CGM analysis of a transformer of Team Problem-21

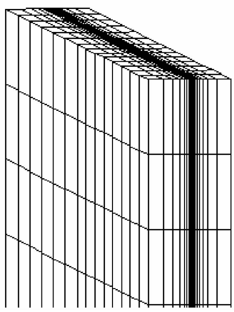

Fig.5 A part of the local fine mesh for CGM analysis of a transformer of Team Problem-21

3 Conclusion and Future Work

In this paper, a composite grid method is developed for FE analysis of an electromagnetic field with strong local interest. Great computational efficiency of CGM is achieved by using a coarse mesh for the global region, and refined mesh for the local region. And the addition of the modified global analysis enhances the effectiveness. Moreover, the procedure allows one to follow an iterative procedure to gain further accuracy in the solution. Three numerical examples were used to demonstrate the efficiency of CGM. The results indicated that the present method not only produced accurate solutions, but also realized considerable time-savings in computation.

1) The improvement of the defining method of the critical region and the local region to increase the efficiency of the method;

2) The mathematical analysis of errors estimation and convergence rate of CGM;

3) The parallel implementation of CGM;

4) Further FE analysis (using CGM) of Problem-21.

References

[1] McCormick S F, Thomas J. The fast adaptive composite grid method (FAC) for elliptic boundary value problems[J]. *Math Comp*, 1986,**46**:439 - 456.

[2] Ferket P J J, Reusken A. Further analysis of the local defect correction method [J]. *Computing*, 1996, **56**(2): 117 - 139.

[3] Mao K M, Sun C T. A refined global-local finite element analysis method [J]. *Int J Numer Methods Eng*, 1991,**32**: 29 - 43.

[4] Whitcomb J D. Iterative global/local finite element analysis [J]. *Computers and Structures*, 1991, **40**(4): 1027 - 1031.

利用复合网格进行电磁场分析

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摘 要 本文描述了对有需要着重分析的局部区域的电磁场进行有限元分析的复合网格法.这种方法基于通常的有限元方法,有总体分析,局部分析和修正后的总体分析 3 个基本步骤.在前 2 步中,利用较粗的网格进行总体分析,得到节点的电势,将其作为后续进一步分析的局部区域的人工边界条件.将这些有了边界条件的局部区域用更精细的网格进行分析,得到更为精确的电势与密度分布.在第 3 步中,进行修正后的总体分析,通过迭代不断改进结果,直到满足给定的求解精度,得到更好的电势与密度分布的结果.数值实验表明,与通常的有限元方法相比,复合网格法在得到同样求解精度的结果时所耗的计算时间要少得多.

关键词 复合网格; 总体分析; 局部分析

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