Journal of Southeast University (English Edition) Vol.19 No.1 Mar. 2003 ISSN 1003—7985

Direct adaptive fuzzy control based on integral-type Lyapunov function

Zhang Tianping Zhu Qing Zhang Huiyan Gu Haijun

(Department of Computer Science, College of Information Engineering, Yangzhou University, Yangzhou 225009, China)

Abstract: A new scheme of direct adaptive fuzzy controller for a class of nonlinear systems with unknown triangular control gain structure is proposed. The design is based on the principle of sliding mode control and the approximation capability of the first type fuzzy systems. By introducing integral-type Lyapunov function and adopting the adaptive compensation term of optimal approximation error, the closed-loop control system is proved to be globally stable, with tracking error converging to zero. Simulation results demonstrate the effectiveness of the approach.

Key words: fuzzy systems; fuzzy control; adaptive control; global stability

In recent years, the analytical study of nonlinear adaptive control systems using fuzzy universal function approximation has received much attention[1-4]. Utilizing the approximation capability of the fuzzy system, four design schemes of stable adaptive fuzzy controllers were proposed[1,2]. But the tracking error convergence depended upon the assumption that the approximation error should be square-integrable in Refs. [1,2]. The problem of indirect adaptive fuzzy control for a class of nonlinear systems with similar structure between subsystems was discussed in Ref.[3]. Based on a modified Lyapunov function method and multilayer neural networks, three design schemes of adaptive neural network control were proposed in Refs.[5 - 7]. The drawback was that tracking error converged to the residual set only. By introducing the compensation term of the optimal approximation error, an improved direct adaptive fuzzy control scheme for a class of SISO nonlinear systems was proposed[4].

In this paper, the problem of direct adaptive fuzzy control for a class of MIMO nonlinear systems with a triangular control structure is discussed. Based on a modified Lyapunov function and using the approximation capability of the first type fuzzy logic system, a novel scheme of adaptive fuzzy controller is presented. Adaptive law for the peak values of the consequence fuzzy sets in the fuzzy systems and the error compensation term is determined by using a Lyapunov function method. Two new features of the proposed scheme include that ① Tracking errors converge to zero; ② Control structure is simple without calculating the integrals of Eqs.(22) and (23) in Ref.[6]. Simulation results demonstrate the effectiveness of the approach and good tracking performance.

The paper is organized as follows. In section 1, we give the model of the plant and the basic assumptions. Novel update algorithms are given in section 2, and the robust properties of the adaptive fuzzy control algorithm with respect to the modeled uncertainties are analyzed in sections 3. In section 4, simulation results are demonstrated to show the effectiveness of the proposed method. The conclusion is included in section 5.

1 Problem Statement and Basic Assumptions

Consider the following nonlinear systems:

$$\left.\begin{array}{l}
\dot{x}_{1,j} = x_{1,j+1} \qquad j = 1,\cdots,n_1 - 1 \\
\dot{x}_{1,n_1} = f_1(\boldsymbol{X}) + b_{1,1}(\boldsymbol{X}_1)u_1(t) \\
\dot{x}_{2,j} = x_{2,j+1} \qquad j = 1,\cdots,n_2 - 1 \\
\dot{x}_{2,n_2} = f_2(\boldsymbol{X}) + b_{2,1}(\boldsymbol{X})u_1(t) + b_{2,2}(\boldsymbol{X}_2^+)u_2(t) \\
\vdots \\
\dot{x}_{i,j} = x_{i,j+1} \qquad j = 1,\cdots,n_i - 1 \\
\dot{x}_{i,n_i} = f_i(\boldsymbol{X}) + b_{i,1}(\boldsymbol{X})u_1(t) + \cdots + b_{i,i-1}(\boldsymbol{X})u_{i-1}(t) + b_{i,i}(\boldsymbol{X}_i^+)u_i(t) \qquad i = 3,\cdots,m \\
y_1 = x_{1,1},\cdots,y_m = x_{m,1}
\end{array}\right\} \tag{1}$$

Received 2002-10-11.

Foundation items: The National Natural Science Foundation of PRC (60074013) and the Natural Science Foundation of Education Bureau of Jiangsu Province (00KJB510006 & 00KJB470006).

Biography: Zhang Tianping (1964—), male, doctor, professor, tpzhang@mail.yzu.edu.cn.

where $X = \{X_1^T, X_2^T, \cdots, X_m^T\}^T$ is the state vector; $X_i^+ = \{X_1^T, X_2^T, \cdots, X_i^T\}^T$, $X_i = \{x_{i,1}, x_{i,2}, \cdots, x_{i,n_i}\}^T, i = 1, \cdots, m$; $\{u_1(t), \cdots, u_m(t)\}^T$ is the control input; $f_i(X), i = 1, \cdots, m$, are the unknown continuous functions; $b_{1,1}(X_1), b_{2,1}(X), b_{2,2}(X_2^+), \cdots, b_{m,1}(X), \cdots, b_{m,m}(X)$ are the unknown control gains; $\{y_1, \cdots, y_m\}^T$ is the system output.

The control objective is to force the system output y_i to follow the specified desired trajectory $y_{di}, i = 1, \cdots, m$. Define X_{di}, e_i and s_i as

$$\left.\begin{aligned} X_{di} &= \{y_{di}, \dot{y}_{di}, \cdots, y_{di}^{(n_i-1)}\}^T \\ e_i &= X_i - X_{di} = \{e_{i1}, \cdots, e_{in_i}\}^T \\ s_i &= \left(\frac{d}{dt} + \lambda_i\right)^{n_i-1} e_{i1} = \sum_{j=1}^{n_i-1} c_{ij}e_{ij} + e_{in_i} \end{aligned}\right\} \tag{2}$$

where $c_{ij} = C_{n_i-1}^{j-1} \lambda_i^{n_i-j}, C_{n_i}^{j-1} (j = 1, \cdots, n_i - 1;\ i = 1, \cdots, m)$ is the combination coefficient; $\lambda_i > 0$ is the design constant.

Lemma[5] Let s_1 be defined by (2), then

1) If $s_1 = 0$, then $\lim_{t \to \infty} e_{11} = 0$;

2) If $|s_1| \leqslant c$, $e_1(0) \in \Omega_{1c}$, then $e_1(t) \in \Omega_{1c}, \forall t \geqslant 0$;

3) If $|s_1| \leqslant c$, $e_1(0) \notin \Omega_{1c}$, then $\exists T = (n_1 - 1)/\lambda_1, \forall t \geqslant T$, $e_1(t) \in \Omega_{1c}$.

where $c > 0, \Omega_{1c} = \{e_1(t) \mid |e_{1j}| \leqslant 2^{j-1} \lambda_1^{j-n_1} c,\ j = 1, \cdots, n_1\}$.

From Eqs.(1) and (2), we have

$$\dot{s}_i = f_i(X) + \sum_{j=1}^{i-1} b_{i,j}(X)u_j(t) + b_{i,i}(X_i^+)u_i(t) + \gamma_i \tag{3}$$

where $\gamma_i = \sum_{j=1}^{n_i-1} c_{ij}e_{i,j+1} - y_{di}^{(n_i)}$, $i = 1, \cdots, m$.

To design stable adaptive fuzzy control, we make the following assumptions:

1) $|f_i(X)| \leqslant F_i(X)$, $\forall X \in \mathbf{R}^n$;

2) $|b_{i,j}(X)| \leqslant B_{i,j}(X)$ and $0 < b_{0i} \leqslant b_{i,i}(X_i^+)$, $\forall X \in \mathbf{R}^n$, $j = 1, \cdots, i-1,$, $i = 1, \cdots, m$;

3) $\{X_{di}^T, y_{di}^{(n_i)}\}^T \in \Omega_{di} \subset \mathbf{R}^{n_i+1}$, $i = 1, \cdots, m$.

where $F_i(X)$ is a known positive continuous function; b_{0i} is a known positive constant; Ω_{di} is a known bounded compact set.

2 Adaptive Fuzzy Controller Design

Let

$$\begin{aligned} h_i(Z_i) = {} & \frac{1}{b_{i,i}(X_i^+)}\left[f_i(X) + \sum_{j=1}^{i-1} b_{i,j}(X)u_j\right] + \\ & \frac{1}{s_i}\int_0^{s_i} \sigma\left[\sum_{j=1}^{i-1}\sum_{k=1}^{n_j} \frac{\partial b_{i,i}^{-1}(\bar{x}_i, \sigma + \beta_i)}{\partial x_{j,k}} x_{j,k+1} + \sum_{k=1}^{n_i-1} \frac{\partial b_{i,i}^{-1}(\bar{x}_i, \sigma + \beta_i)}{\partial x_{i,k}} x_{i,k+1}\right] d\sigma + \\ & \frac{1}{s_i}\int_0^{s_i} b_{i,i}^{-1}(\bar{x}_i, \sigma + \beta_i)\gamma_i d\sigma \end{aligned} \tag{4}$$

where $Z_1 = \{X^T, s_1, \beta_1, \gamma_1\}^T, Z_i = \{X^T, s_i, \beta_i, \gamma_i, u_1, \cdots, u_{i-1}\}^T, \bar{x}_i = \{X_1^T, \cdots, X_{i-1}^T, x_{i,1}, \cdots, x_{i,n_i-1}\}^T, i = 2, \cdots, m, \beta_i = y_{di}^{(n_i-1)} - \sum_{j=1}^{n_i-1} c_{ij}e_{ij}, \Omega_{Z_i} = \{Z_i \mid X_i \in \Omega_{\mu_i}, (X_{dj}^T, y_{dj}^{(n_j)})^T \in \Omega_{dj}, j = 1, \cdots, i\}, i = 1, \cdots, m$. Let $u_{hi}(Z_i, \theta_i)$ be the approximation of first-type fuzzy logic system on the compact set Ω_{Z_i} to $h_i(Z_i)$, $i = 1, \cdots, m$, i.e.

$$u_{hi}(Z_i, \theta_i) = \theta_i^T \varphi_i(Z_i) \qquad i = 1, \cdots, m \tag{5}$$

where $\theta_i = \{\theta_{i,1}, \cdots, \theta_{i,M_i}\}^T$ is an adjustable parameter vector; $\varphi_i(Z_i) = \{p_{i1}(Z_i), \cdots, p_{iM_i}(Z_i)\}^T$ is a fuzzy basis vector; M_i is the number of rules in the i-th fuzzy system; Ω_{μ_i} is given by the later theorem.

$$p_{il}(\boldsymbol{Z}_i) = \frac{\prod_{j=1}^{n+i+2} \exp\left(-\frac{(z_{i,j} - a_{i,j}^l)^2}{(b_{i,j}^l)^2}\right)}{\sum_{l=1}^{M_i}\prod_{j=1}^{n+i+2} \exp\left(-\frac{(z_{i,j} - a_{i,j}^l)^2}{(b_{i,j}^l)^2}\right)} \qquad n = \sum_{i=1}^{m} n_i \tag{6}$$

$$\left.\begin{aligned} &\Omega_{\boldsymbol{\theta}_i} = \{\boldsymbol{\theta}_i \mid \|\boldsymbol{\theta}_i\| \leqslant M_{\boldsymbol{\theta}_i}\} \\ &\boldsymbol{\theta}_i^* = \arg\min_{\boldsymbol{\theta}_i \in \Omega_{\boldsymbol{\theta}_i}}\left[\sup_{\boldsymbol{Z}_i \in \Omega_{\boldsymbol{Z}_i}} |u_{hi}(\boldsymbol{Z}_i, \boldsymbol{\theta}_i) - h_i(\boldsymbol{Z}_i)|\right] \qquad i = 1,\cdots,m \end{aligned}\right\} \tag{7}$$

where $M_{\boldsymbol{\theta}_i}$ is a positive constant, specified by the designer. Let $\boldsymbol{\varepsilon}_i = \max_{\boldsymbol{Z}_i \in \Omega_{\boldsymbol{Z}_i}} |u_{hi}(\boldsymbol{Z}_i, \boldsymbol{\theta}_i^*) - h_i(\boldsymbol{Z}_i)|, i = 1,\cdots, m$, then $\boldsymbol{\varepsilon}_i > 0$ is the unknown bounded constant.

Adopt the following control law:

$$u_i(t) = -u_{hi}(\boldsymbol{Z}_i, \hat{\boldsymbol{\theta}}_i) - \hat{\varepsilon}_i \operatorname{sgn}(s_i) - k_i(t) s_i \qquad i = 1,\cdots,m \tag{8}$$

where

$$\left.\begin{aligned} &k_1 = \frac{1}{a_1}\sqrt{1 + \left(\frac{F_1(X)}{b_{01}}\right)^2 + \left(\frac{\gamma_1}{b_{01}}\right)^2 + u_{h1}^2(\boldsymbol{Z}_1, \hat{\boldsymbol{\theta}}_1) + \hat{\varepsilon}_1^{\,2}} \\ &k_i = \frac{1}{a_i}\sqrt{1 + \left(\frac{F_i(X)}{b_{0i}}\right)^2 + \left(\frac{\gamma_i}{b_{0i}}\right)^2 + \frac{1}{b_{0i}^2}\sum_{j=1}^{i-1} B_{i,j}^2(X) u_j^2 + u_{hi}^2(\boldsymbol{Z}_i, \hat{\boldsymbol{\theta}}_i) + \hat{\varepsilon}_i^2} \end{aligned}\right\} \tag{9}$$

where $\hat{\boldsymbol{\theta}}_i$ and $\hat{\varepsilon}_i (i = 2,\cdots,m)$ are the estimates of $\boldsymbol{\theta}_i^*$ and $\boldsymbol{\varepsilon}_i (i = 1,\cdots,m)$ at time t, respectively.

Choose the adaptive law as

$$\dot{\hat{\boldsymbol{\theta}}}_i = \begin{cases} \eta_{i1} s_i(t) \boldsymbol{\varphi}_i(\boldsymbol{Z}_i), \text{ if } \|\hat{\boldsymbol{\theta}}_i\| < M_i, \text{ or } \|\hat{\boldsymbol{\theta}}_i\| = M_i \text{ and } s_i(t) \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} \boldsymbol{\varphi}_i(\boldsymbol{Z}_i) \leqslant 0 \\ \eta_{i1} s_i(t) \boldsymbol{\varphi}_i(\boldsymbol{Z}_i) - \eta_{i1} s_i(t) \dfrac{\hat{\boldsymbol{\theta}}_i \hat{\boldsymbol{\theta}}_i^{\mathrm{T}}}{\|\hat{\boldsymbol{\theta}}_i\|^2} \boldsymbol{\varphi}_i(\boldsymbol{Z}_i), \text{ if } \|\hat{\boldsymbol{\theta}}_i\| = M_i \text{ and } s_i(t) \hat{\boldsymbol{\theta}}_i^{\mathrm{T}} \boldsymbol{\varphi}_i(\boldsymbol{Z}_i) > 0 \end{cases} \tag{10}$$

$$\dot{\hat{\varepsilon}}_i = \eta_{i2} |s_i(t)| \qquad i = 1,\cdots,m \tag{11}$$

where $\eta_{i1}, \eta_{i2} (i = 1,\cdots,m)$ are strictly positive constants which determine the adaptation rate.

3 Stability Analysis

Define a smooth scalar function:

$$V_{si}(t) = \int_0^{s_i} \frac{\sigma}{b_{i,i}(\bar{\boldsymbol{x}}_i, \sigma + \beta_i)} \mathrm{d}\sigma \tag{12}$$

where $\bar{\boldsymbol{x}}_i = \{\boldsymbol{X}_1^{\mathrm{T}},\cdots,\boldsymbol{X}_{i-1}^{\mathrm{T}}, x_{i,1},\cdots,x_{i,n_i-1}\}^{\mathrm{T}}$, $j = 1,\cdots,m$. By mean value theory, $V_{si}(t)$ can be rewritten as $V_{si}(t) = \dfrac{s_i^{\,2}}{2b_{i,i}(\bar{\boldsymbol{x}}_i, \lambda_{is}s_i + \beta_i)}$ with $\lambda_{is} \in (0,1)$. Since $b_{i,i}(\boldsymbol{X}_i^+) > 0$ and $\forall \boldsymbol{X}_i \in \mathbf{R}^{n_i}$, V_{si} is positive definitive with respect to s_i. For the function V_{si}, its time derivative is

$$\begin{aligned} \dot{V}_{si} = {} & \frac{s_i(t)}{b_{i,i}(\boldsymbol{X}_i^+)}\dot{s}_i(t) + \int_0^{s_i}\sigma\left[\sum_{j=1}^{i-1}\sum_{k=1}^{n_j}\frac{\partial b_{i,i}^{-1}(\bar{\boldsymbol{x}}_i, \sigma + \beta_i)}{\partial x_{j,k}}x_{j,k+1} + \sum_{k=1}^{n_i-1}\frac{\partial b_{i,i}^{-1}(\bar{\boldsymbol{x}}_i, \sigma + \beta_i)}{\partial x_{i,k}}x_{i,k+1}\right]\mathrm{d}\sigma - \\ & \gamma_i\frac{s_i(t)}{b_{i,i}(\boldsymbol{X}_i^+)} + \gamma_i\int_0^{s_i} b_{i,i}^{-1}(\bar{\boldsymbol{x}}_i, \sigma + \beta_i)\mathrm{d}\sigma = \frac{s_i(t)}{b_{i,i}(\boldsymbol{X}_i^+)}\left[f_i(\boldsymbol{X}) + \sum_{k=1}^{i-1} b_{i,k}(\boldsymbol{x})u_k + b_{i,i}(\boldsymbol{X}_i^+)u_i\right] + \\ & \gamma_i\int_0^{s_i} b_{i,i}^{-1}(\bar{\boldsymbol{x}}_i, \sigma + \beta_i)\mathrm{d}\sigma + \int_0^{s_i}\sigma\left[\sum_{j=1}^{i-1}\sum_{k=1}^{n_j}\frac{\partial b_{i,i}^{-1}(\bar{\boldsymbol{x}}_i, \sigma + \beta_i)}{\partial x_{j,k}}x_{j,k+1} + \sum_{k=1}^{n_i-1}\frac{\partial b_{i,i}^{-1}(\bar{\boldsymbol{x}}_i, \sigma + \beta_i)}{\partial x_{i,k}}x_{i,k+1}\right]\mathrm{d}\sigma \end{aligned}$$

According to Eqs. (3) to (5), we obtain

$$\dot{V}_{si} = s_i[h_i(\boldsymbol{Z}_i) - u_{hi}(\boldsymbol{Z}_i, \hat{\boldsymbol{\theta}}_i) - \hat{\varepsilon}_i \operatorname{sgn}(s_i) - k_i s_i] \qquad i = 1,\cdots,m \tag{13}$$

We propose the following stability theorem.

Theorem Consider the nonlinear system (1) with control law defined by (2), (6), (8) and (9). Let the parameter vector $\hat{\boldsymbol{\theta}}_i, \hat{\varepsilon}_i$ be adjusted by the adaptation law determined by (10) and (11), and let the assumptions 1) to 3) be true. Then

1) If $\hat{\boldsymbol{\theta}}_i(0) \in \Omega_{\boldsymbol{\theta}_i}$, then $\|\hat{\boldsymbol{\theta}}_i(t)\| \leqslant M_{\boldsymbol{\theta}_i}, \forall t \geqslant 0$;

2) The overall closed-loop fuzzy control system is globally stable in the sense that all of the closed-loop signals are bounded, and the state vector $\boldsymbol{X}_i \in \Omega_{\mu_i} = \{\boldsymbol{X}_i(t) \mid |e_{ij}(t)| \leqslant 2^j \lambda_i^{j-n_i} \sqrt{i+4}\mu_i, j = 1,\cdots,n_i, X_{id} \in \Omega_{id}\}, i = 1,\cdots,m, \forall t \geqslant T_i$;

3) $\lim\limits_{t\to\infty} s_i = 0$, i.e. $\lim\limits_{t\to\infty} e_{i1}(t) = 0$, where μ_i is a positive constant, specified by the designer, $\|\hat{\boldsymbol{\theta}}_i(0)\| \leqslant M_{\boldsymbol{\theta}_i}$, T_i is a positive constant.

Proof ① Let $V_{\boldsymbol{\theta}_i}(t) = \hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\hat{\boldsymbol{\theta}}_i/2$. From Eq.(10), we have that if $\|\hat{\boldsymbol{\theta}}_i\| = M_{\boldsymbol{\theta}_i}$ and $s_i\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\varphi}_i(\boldsymbol{Z}_i) \leqslant 0$, then $\dot{V}_{\boldsymbol{\theta}_i}(t) \leqslant 0$; if $\|\hat{\boldsymbol{\theta}}_i\| = M_{\boldsymbol{\theta}_i}$ and $s_i\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\boldsymbol{\varphi}_i(\boldsymbol{Z}_i) > 0$, then $\dot{V}_{\boldsymbol{\theta}_i}(t) = 0$, hence $\|\hat{\boldsymbol{\theta}}_i\| \leqslant M_{\boldsymbol{\theta}_i}, \forall t \geqslant 0$. From the above analysis, we know that if $\hat{\boldsymbol{\theta}}_i(0) \in \Omega_{\boldsymbol{\theta}_i}$, then $\hat{\boldsymbol{\theta}}_i(t) \in \Omega_{\boldsymbol{\theta}_i}$.

② Let $V_i = s_i^2/2, i = 1,\cdots,m$, then

$$\dot{V}_i = s_i b_{i,i}(\boldsymbol{X}_i^+)\left[u_i(t) + \sum_{j=1}^{i-1}\left(\frac{b_{i,j}(\boldsymbol{X})}{b_{i,i}(\boldsymbol{X}_i^+)}u_j(t)\right) + \frac{f_i(\boldsymbol{X}) + \gamma_i}{b_{i,i}(\boldsymbol{X}_i^+)}\right] = s_i b_{i,i}(\boldsymbol{X}_i^+)[-k_i(t)s_i - u_{hi} \times (\boldsymbol{Z}_i, \hat{\boldsymbol{\theta}}_i) - \hat{\varepsilon}_i \operatorname{sgn}(s_i)] + s_i b_{i,i}(\boldsymbol{X}_i^+)\left[\sum_{j=1}^{i-1}\left(\frac{b_{i,j}(\boldsymbol{X})}{b_{i,i}(\boldsymbol{X}_i^+)}u_j(t)\right) + \frac{f_i(\boldsymbol{X}) + \gamma_i}{b_{i,i}(\boldsymbol{X}_i^+)}\right] \leqslant -b_{i,i}(\boldsymbol{X}_i^+)k_i(t)\left\{s_i^2 - \frac{|s_i|}{k_i(t)}\left[\frac{|f_i(\boldsymbol{x})| + |\gamma_i|}{b_{i,i}(\boldsymbol{X}_i^+)} + \sum_{j=1}^{i-1}\left|\frac{b_{ij}(\boldsymbol{X})}{b_{i,i}(\boldsymbol{X}_i^+)}u_j(t)\right| + |u_{hi}(\boldsymbol{Z}_i, \hat{\boldsymbol{\theta}}_i)| + |\hat{\varepsilon}_i|\right]\right\} \leqslant -b_{i,i}(\boldsymbol{X}_i^+)k_i(t)[s_i^2 - \sqrt{i+3}\mu_i|s_i|] \tag{14}$$

From $2\sqrt{i+3}\mu_i|s_i| \leqslant s_i^2/2 + 2(i+3)\mu_i^2, s_i^2 = 2V_i$, it follows that $2\dot{V}_i = \mathrm{d}(2V_i - 4(i+3)\mu_i{}^2)/\mathrm{d}t \leqslant -b_{i,i}(x_i^+)k_i(t)[2V_i - 4(i+3)\mu_i^2]$, hence $2V_i - 4(i+3)\mu_i^2 \leqslant [2V_i(0) - 4(i+3)\mu_i^2] \times \exp\left(-\int_0^t b_{i,i}(\boldsymbol{X}_i^+)k_i(\tau)\mathrm{d}\tau\right)$. Because $b_{i,i}(\boldsymbol{X}_i^+)k_i(t) \geqslant b_{0i}/\mu_i > 0$, we obtain

$$s_i^2 \leqslant s_i^2(0)\exp(-b_{0i}t/\mu_i) + 4(i+3)\mu_i^2 \tag{15}$$

From (15), we have that $\forall t \geqslant T_{i1} = \max\left\{0, \frac{2\mu_i}{b_{0i}}\ln\frac{|s_i(0)|}{2\mu_i}\right\}$, $|s_i(t)| \leqslant 2\sqrt{i+4}\mu_i$. Using lemma, we have that if $t \geqslant T_i = T_{i1} + (n_i - 1)/\lambda_i$, then $|e_{ij}(t)| \leqslant 2^j\lambda_i^{j-n_i}\sqrt{i+4}\mu_i, j = 1,\cdots,n_i, i = 1,\cdots,m$. Therefore, system state vector $\boldsymbol{X}_i \in \Omega_{\mu_i}, \forall t \geqslant T_i$. Let $T = \max\{T_1,\cdots,T_m\}$, then $\boldsymbol{X}_i \in \Omega_{\mu_i}, \forall t \geqslant T$, $i = 1,\cdots,m$.

③ Define the Lyapunov function candidate

$$\bar{V}_i(t) = \int_0^{s_i}\frac{\sigma}{b_{i,i}(\bar{\boldsymbol{x}}_i, \sigma + \beta_i)}\mathrm{d}\sigma + \frac{1}{2}[\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_i/\eta_{i1} + (\hat{\varepsilon}_i - \boldsymbol{\varepsilon}_i)^2/\eta_{i2}] \tag{16}$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i^* - \hat{\boldsymbol{\theta}}_i$. Differentiating $\bar{V}_i(t)$ with respect to time t, we have

$$\dot{\bar{V}}_i(t) = \dot{V}_{si} - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_i/\eta_{i1} + (\hat{\varepsilon}_i - \boldsymbol{\varepsilon}_i)\dot{\hat{\varepsilon}}_i/\eta_{i2} \tag{17}$$

Substituting (10),(11) and (13) into (17), we obtain

$$\dot{\bar{V}}_i(t) = s_i[-k_i(t)s_i - \hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\varphi(\boldsymbol{Z}_i) - \hat{\varepsilon}_i\operatorname{sgn}(s_i) + h_i(\boldsymbol{Z}_i)] - \tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\dot{\hat{\boldsymbol{\theta}}}_i/\eta_{i1} + (\hat{\varepsilon}_i - \boldsymbol{\varepsilon}_i)\dot{\hat{\varepsilon}}_i/\eta_{i2} \leqslant -k_i(t)s_i^2 + I_i s_i\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\hat{\boldsymbol{\theta}}_i\hat{\boldsymbol{\theta}}_i^{\mathrm{T}}\varphi(\boldsymbol{Z}_i)/\|\hat{\boldsymbol{\theta}}_i\|^2 \qquad \forall t \geqslant T \tag{18}$$

where $I_i = 0\ (1)$, if the first (second) condition of (10) is true. If the second condition of (10), then $\|\hat{\boldsymbol{\theta}}_i\| = M_{\boldsymbol{\theta}_i}$ and $\tilde{\boldsymbol{\theta}}_i^{\mathrm{T}}\hat{\boldsymbol{\theta}}_i = [\|\boldsymbol{\theta}_i^*\|^2 - \|\hat{\boldsymbol{\theta}}_i\|^2 - \|\boldsymbol{\theta}_i^* - \hat{\boldsymbol{\theta}}_i\|^2]/2 \leqslant 0$. Therefore

$$\dot{\bar{V}}_i(t) \leqslant -k_i(t)s_i^2 \leqslant -s_i^2/\mu_i \leqslant 0 \qquad \forall t \geqslant T \tag{19}$$

Similar to the proof of theorem in Ref.[3], it is easy to show that the conclusion is true.

4 Simulation Results

Consider the following nonlinear system:

$$\left.\begin{aligned}\dot{x}_{1,1} &= x_{1,2}\\ \dot{x}_{1,2} &= x_{2,1} - 0.3\sin(x_{1,2}) + (2 - \sin^2(x_{1,1}))u_1\\ \dot{x}_{2,1} &= x_{2,2}\\ \dot{x}_{2,2} &= 0.2x_{1,1}^2 + (1 + 0.8|x_{1,2}|)u_1 + (1 + 0.5\sin(x_{2,1}))u_2\end{aligned}\right\} \tag{20}$$

Control objective is to force $\boldsymbol{X}_i = \{x_{i,1}, x_{i,2}\}^{\mathrm{T}}$ to follow the desired trajectory $\boldsymbol{X}_{di}, i = 1,2$. $\boldsymbol{X}_{d1} = \left(\cos\frac{\pi t}{2}, -\frac{\pi}{2}\sin\frac{\pi t}{2}\right)^{\mathrm{T}}, \boldsymbol{X}_{d2} = \left(\sin\frac{\pi t}{20}, \frac{\pi}{20}\cos\frac{\pi t}{20}\right)^{\mathrm{T}}$. $\lambda_1 = 8, \lambda_2 = 3, \eta_{11} = \eta_{12} = 1.5, \eta_{21} = \eta_{22} = 2, b_{01} = 1, b_{02} = 1, B_{2,1}(\boldsymbol{X}) = 1 + |x_{1,2}|, F_1(\boldsymbol{X}) = |x_{2,1}| + 0.5, F_2(\boldsymbol{X}) = 0.2x_{1,1}^2 + 0.1, M_1 = M_2 = 5, M_{\boldsymbol{\theta}_1} = M_{\boldsymbol{\theta}_2} = 10$, $x_{1,1}(0) = 0.6, x_{1,2}(0) = 0.1, x_{2,1}(0) = 0.3, x_{2,2}(0) = -0.1, \hat{\varepsilon}_1(0) = \hat{\varepsilon}_2(0) = 0.5$, $\hat{\boldsymbol{\theta}}_1(0)$, $\hat{\boldsymbol{\theta}}_2(0)$ are randomly taken in the interval $[-1,1]$. The simulation results are shown in Fig. 1.

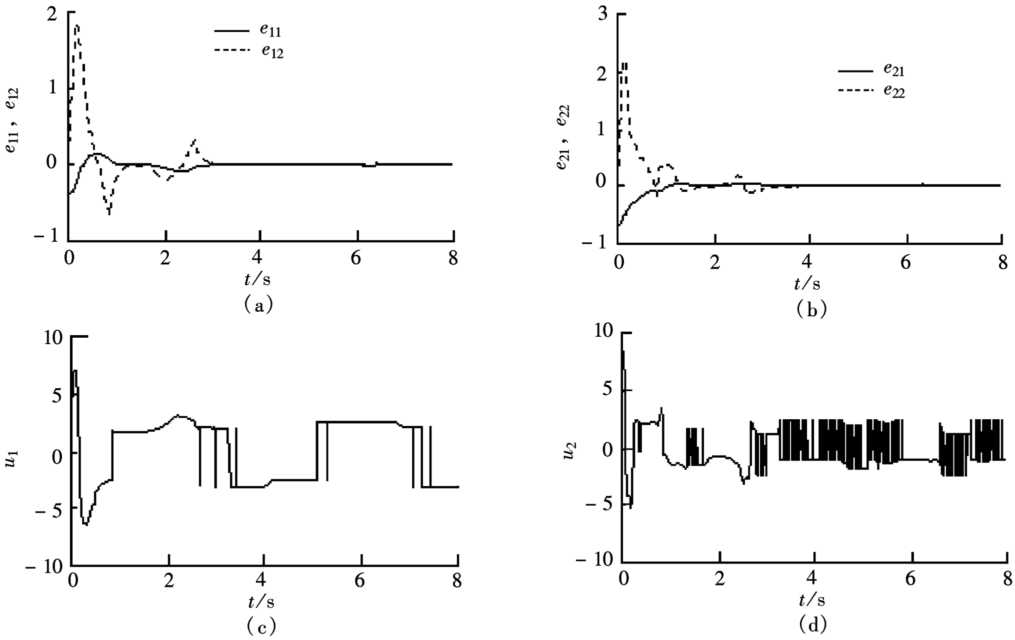

Fig. 1 Tracking errors and control signals. (a) Tracking errors e_{11}, e_{12}; (b) Tracking errors e_{21}, e_{22}; (c) Control signal u_1; (d) Control signal u_2

5 Conclusion

A direct recursive adaptive fuzzy control scheme has been presented for a class of MIMO nonlinear systems. The design is based on an integral-type Lyapunov function, utilizing the system triangular property. The adaptive law of the adjustable parameter vector in the fuzzy system and the optimal approximation error are determined by using a Lyapunov method. The developed controller can guarantee the global stability of the resulting closed-loop fuzzy system in the sense that all signals involved are uniformly bounded and the asymptotic convergence of the tracking error approaches zero.

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基于积分型李亚普诺夫函数的直接自适应模糊控制

张天平　朱　清　张惠艳　顾海军

(扬州大学信息工程学院计算机系,扬州 225009)

摘　要　针对一类具有未知下三角形函数控制增益矩阵的非线性系统,根据滑模控制原理,并利用I型模糊系统的逼近能力,提出了一种直接自适应模糊控制器设计的新方案.通过引入积分型李亚普诺夫函数及逼近误差自适应补偿项,证明了闭环系统是全局稳定的,跟踪误差收敛到零.仿真结果表明了该方法的有效性.

关键词　模糊系统; 模糊控制; 自适应控制; 全局稳定性

中图分类号　TP273.4^{+}