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Aperiodic stochastic resonance in Schmitt trigger

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Abstract: By the use of cross-correlation measures, the response of a symmetric Schmitt trigger (ST) driven by a random binary signal and white Gaussian noise is investigated. The results show that the information transmission can be enhanced when a certain amount of noise is presented, i.e., aperiodic stochastic resonance (ASR). Then, the influence of signal amplitude and the ST threshold on ASR is examined, the applicability of the ST in reducing the noise level of random signal transmission and improving the quality of output signal via ASR effect is illustrated. This research is of great interest in the field of digital communications.

Key words: aperiodic stochastic resonance; Schmitt trigger; cross-correlation

Stochastic resonance (SR) is a phenomenon wherein the transmission of a coherent signal is enhanced by adding random noise. The conception of SR was first introduced[1] in 1981 as an explanation for the switching of the earth's climate between ice ages and periods of relative warmth. Since then, it has been observed in a large variety of systems including optical systems, electronic and magnetic systems, excitable systems, chemical systems, spatially extended system, etc[2].

Usually, the three main ingredients[2] required to observe SR are a form of threshold, a source of noise, and a generally weak input source. In previous investigations, a large proportion of work has been limited to a periodic, or specially, sinusoid signal and the way to quantify SR is through the output signal-to-noise ratio (SNR). Because the information contained in a purely sinusoidal signal is zero, the applicability of SR in practice is limited. It is preferable to use a broadband aperiodic signal in order to improve the transmission of actual useful information via SR. The term ASR (aperiodic SR), first coined by Collins et al[3], describes an SR system that works with broadband informative signals. Clearly, the previous techniques using SNR are not appropriate for ASR. Cross-correlation measures and information-theoretical measures were proposed[3,4] in succession. Additionally, most of the literature regarding ASR to date has considered neuronal models. In this paper, we investigate using cross-correlation measures the behavior of ASR in the ST — the first and probably the simplest electronic circuit exhibiting periodic SR[5].

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1 The Schmitt Trigger

The ST's input signals $u(t)$ consist of two components $u(t) = s(t) + \eta(t)$, where $s(t)$ is an informational aperiodic signal of the form

$$s(t) = A\sum_{j=-\infty}^{\infty} S_j \Gamma(t - jT) \tag{1}$$

where $A > 0$ is a constant amplitude; $S_j = \pm 1$ is a sequence of binary symbols which are independent random variables with equal probabilities, j is an integer; $\Gamma(t)$ is a rectangular pulse of duration T and amplitude unity

$$\Gamma(t) = \begin{cases} 1 & t \in [0, T] \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$\eta(t)$ is zero-mean white Gaussian noise with the correlation

$$\langle \eta(t)\eta(t') \rangle = \sigma^2 \delta(t - t') \tag{3}$$

where $\langle \cdot \rangle$ and $\delta(\cdot)$ represent an ensemble average and the delta function, respectively; σ denotes noise intensity as root-mean-square (rms) amplitude.

Let us examine a symmetric ST[6] with two threshold levels $\pm K$ and two corresponding output states "±". The trigger output rests in "−" state as long as the input $u(t)$ is smaller than a threshold value K. As $u(t)$ crosses K, the trigger switches almost instantaneously (an ideal ST has infinitesimal switching time) into the '±' state and sits there until $u(t)$ gets smaller than $-K$, and vice versa. This can be easily realized by means of the electronic circuit with two operational amplifiers; a schematic diagram is presented in Fig.1. Summing amplifier C_1 adds signal $s(t)$ and noise $\eta(t)$, and its output is inverted to the operational amplifier C_2 which behaves as a comparator, the positive feedback of a fraction $\gamma = \frac{R_1}{R_1 + R_2} (\gamma < 1)$ of the output into V_+ serves to introduce hysteresis.

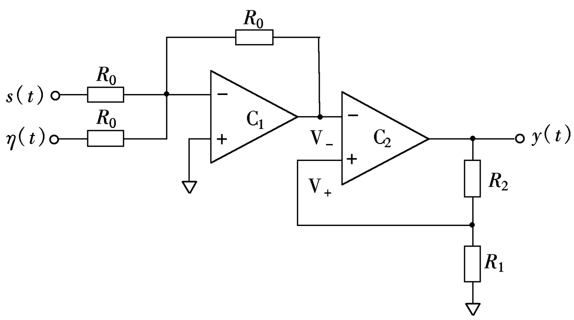

Fig.1 A schematic diagram of the ST circuit

An ideal ST is modeled by

$$y(t + \Delta t) = \mathrm{sgn}[Ky(t) + s(t) + \eta(t)] \quad (4)$$

where K characterizes the operating threshold levels of the trigger and may take values larger than one, Δt is a small step. Eq.(4) can well simulate the two important features of the ST — threshold and hysteretic nonlinearity.

2 Numerical Simulations

The sum $u(t)$ of random signal $s(t)$ and noise $\eta(t)$ is driving a symmetric ST, and the output is $y(t)$. Suppose that $s(t)$ consists of $N = 50$ pulse trains, each pulse persists for time $T = 5$ ms, and $s(t)$ is sampled at $\Delta t = 0.01$ ms, and then, $M = \frac{T}{\Delta t} = 500$ measures are performed in the duration of each pulse. In this paper, ASR is described in terms of a cross-correlation coefficient. A cross-correlation coefficient between $s(t)$ and $y(t)$ is denoted by C_{sy} and calculated by the following equation:

$$\left.\begin{aligned} C_{sy} &= \frac{\sum_{j=1}^{MN}[s(j) - \bar{s}][y(j) - \bar{y}]}{\sqrt{\sum_{j=1}^{MN}[s(j) - \bar{s}]^2 \sum_{j=1}^{MN}[y(j) - \bar{y}]^2}} \\ \bar{s} &= \frac{1}{MN}\sum_{j=1}^{MN} s(j) \\ \bar{y} &= \frac{1}{MN}\sum_{j=1}^{MN} y(j) \end{aligned}\right\} \quad (5)$$

Similarly, the cross-correlation coefficient between $s(t)$ and $u(t)$ is denoted by C_{su}. According to Eq.(4) and Eq.(5), the evolution of the correlation coefficient with noise intensity is investigated. The final results shown in Fig.2 are obtained by taking an average over 10 independent trials. Fig.2 (a) represents the variations of C_{sy} as a function of σ for different values of A. We observe in Fig.2 (a) a non-monotonic evolution of C_{sy} with σ and the resulting behavior is a form of ASR. In Fig.2 (b), we plot C_{sy} and C_{su} against noise by fixing A to 0.35. As visible in Fig.2 (b), C_{sy} is much larger than C_{su} for a certain range of noise level, and a positive gain can be obtained. This demonstrates that there is an actual benefit in passing the signal-plus-noise mixture through ST. Fig.2 (c) illustrates the influence of threshold K at a fixed $A = 0.35$ that the efficacy of the noise-assisted transmission decreases as K is increased from 1.24 to 2. Clearly, there is an optimal threshold distribution for different noise intensities, and the input-output information transmission can be optimized with a proper choice of K.

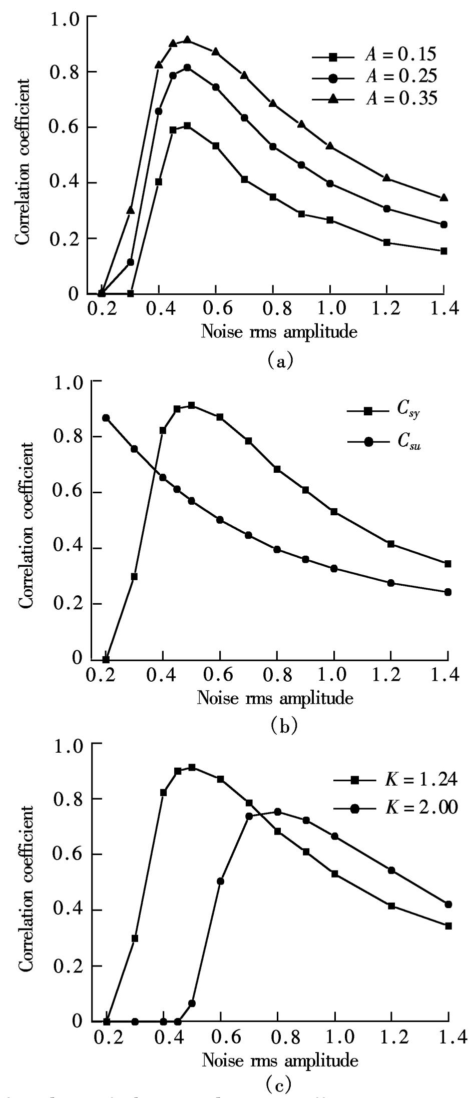

Fig.2 Plots of the correlation coefficient against noise intensity. (a) C_{sy} versus σ at $K = 1.24$; (b) C_{sy} and C_{su} versus σ at $K = 1.24$ and $A = 0.35$; (c) C_{sy} versus σ at $A = 0.35$

Fig.3 shows a typical time evolution of the different signals including the random binary signal $s(t)$, input noisy signal $u(t)$ and output signal $y(t)$ under certain conditions. We see that $y(t)$ is almost the same as $s(t)$. Then, we can draw a conclusion that ST can be used to reduce the noise level of random signal transmission and improve the quality of output signal via ASR, which is of great interest in the field of digital communications.

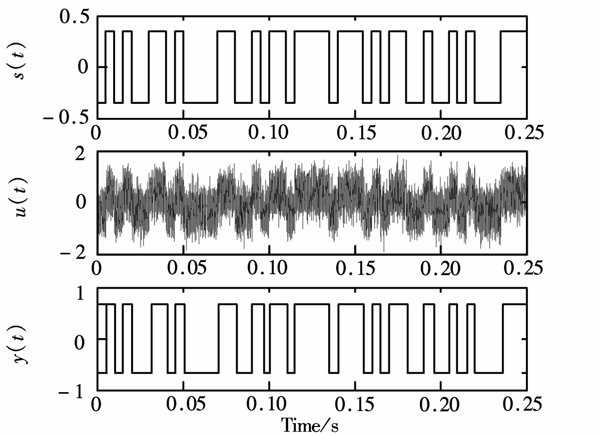

Fig.3 Time evolution of the signals at $A = 0.35$, $K = 1.24$, and $\sigma = 0.4$

3 Conclusions

1) This paper investigates the behavior of ASR in the ST by the use of cross-correlation measures. The particular kind of signal considered here, a random binary (telegraph) signal is not only useful for confirming that the SR phenomenon is general enough to be observable for broadband signals as well as periodic signals, but also is of direct engineering application in the field of digital communications.

2) An important feature of the ST is that it has hysteresis — there is a range of the input for which it is bistable. In fact, the conclusion that ST can be used to reduce the noise level of random signal transmission and improve the quality of the output signal via ASR is true for other bistable systems such as the quartic double-well system, etc. This has been verified theoretically and experimentally[7,8]. The emphasis of future research is on how to realize adaptive processing of noisy random signal by fine-tuning the corresponding system parameters.

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施密特触发器中非周期随机共振的研究

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摘　要　采用互相关方法，研究了随机二进制信号和高斯白噪声同时激励施密特触发器时的输出响应.结果表明，一定量的噪声有助于信息传输，即非周期随机共振.然后，考察了信号幅值和触发器阈值对非周期随机共振的影响，证实了利用施密特触发器中的非周期随机共振效应，可以减小随机信号传输中的噪声水平和改善输出信号质量.该研究在数字通信领域具有十分重要的意义.

关键词　非周期随机共振；施密特触发器；互相关

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