

Robust adaptive dynamic surface control for nonlinear uncertain systems

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Abstract: We propose a new method for robust adaptive backstepping control of nonlinear systems with parametric uncertainties and disturbances in the strict feedback form. The method is called dynamic surface control. Traditional backstepping algorithms require repeated differentiations of the modelled nonlinearities. The addition of n first order low pass filters allows the algorithm to be implemented without differentiating any model nonlinearities, thus ending the complexity arising due to the “explosion of terms” that makes other methods difficult to implement in practice. The combined robust adaptive backstepping/first order filter system is proved to be semiglobally asymptotically stable for sufficiently fast filters by a singular perturbation approach. The simulation results demonstrate the feasibility and effectiveness of the controller designed by the method.

Key words: nonlinear systems; robust control; adaptive control; dynamic surface control; uncertainties

A major contribution to the control of uncertain nonlinear systems, particularly those systems that do not satisfy matching conditions, is the use of “backstepping” methods^[1]. Many results have appeared in Refs. [1 – 4]. However, traditional backstepping algorithms suffer from an “explosion of terms” due to the necessity to perform repeated differentiations of nonlinear functions. A procedure similar to backstepping, called multiple surface sliding control (MSS)^[5,6], was developed to simplify the controller design of systems where model differentiation was difficult. Based on Refs. [5,6], Refs. [7,8] introduced dynamic surface control (DSC) that overcomes the problem of explosion of terms associated with the backstepping technique and the problem of finding derivatives of desired trajectories for the i -th state for the MSS scheme. So far, the research has not been carried out on robust adaptive control of a class of nonlinear systems with parametric uncertainties and disturbances by the DSC method. Hence, based on the above literatures, this paper studies the robust adaptive DSC problem of a class of nonlinear systems with parametric uncertainties and disturbances and designs a robust adaptive tracking controller. The controller designed guarantees the semi-global stability of the closed-loop systems and the output tracking of a given desired trajectory.

1 Problem Formulation

Consider the following uncertain nonlinear system:

$$\left. \begin{aligned} \dot{x}_1 &= x_2 + f_1(\bar{x}_1) + \theta_1 \phi_1(\bar{x}_1) + p_1(\bar{x}_1)w_1 \\ \dot{x}_2 &= x_3 + f_2(\bar{x}_2) + \theta_2 \phi_2(\bar{x}_2) + p_2(\bar{x}_2)w_2 \\ &\vdots \\ \dot{x}_n &= u + f_n(\bar{x}_n) + \theta_n \phi_n(\bar{x}_n) + p_n(\bar{x}_n)w_n \\ y &= x_1 \end{aligned} \right\} \quad (1)$$

where $\bar{x}_i = \{x_1, \dots, x_i\}^T \in \mathbf{R}^i$ is the state; $u \in \mathbf{R}$ is the control input; $\theta_i \in \mathbf{R}$ is an unknown constant parameter; $w_i \in \mathbf{R}$ is the external disturbance with $|w_i| \leq a_i$, a_i is a positive constant; $\phi_i(\bar{x}_i)$ and $p_i(\bar{x}_i)$ are known C^1 functions, $f_i(\bar{x}_i)$ is a smooth function with $f_i(0) = 0, 1 \leq i \leq n$.

We refer to x_{1d} as a feasible output trajectory in the desired ball of radius $r^{[8]}$, if

$$x_{1d}^2 + \left(\frac{dx_{1d}}{dt}\right)^2 + \dots + \left(\frac{d^{n-1}x_{1d}}{dt^{n-1}}\right)^2 \leq K_0(r)$$

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where $K_0(r) > 0$.

The objective of this paper is to design a robust adaptive dynamic surface controller for the nonlinear system (1) which guarantees output tracking of the desired feasible trajectory x_{1d} .

Throughout this paper, ϑ_i is the estimate of the unknown parameter θ_i , $i = 1, \dots, n$.

2 Controller Design

The design procedure for the robust adaptive dynamic surface controller is described below. Let the error in tracking a desired trajectory x_{1d} be S_1

$$S_1 = x_1 - x_{1d} \quad (2)$$

χ_2 is chosen to drive $S_1 \rightarrow 0$,

$$\chi_2 = -K_1 S_1 - f_1(\bar{x}_1) - \vartheta_1 \phi_1(\bar{x}_1) - \frac{a_1^2 S_1 p_1^2}{2\epsilon} + \dot{x}_{1d} \quad (3)$$

where ϵ is an arbitrarily small positive constant which will be chosen later.

Filtering χ_2 , x_{2d} is obtained.

$$\tau_2 \dot{x}_{2d} + x_{2d} = \chi_2, \quad x_{2d}(0) = \chi_2(0) \quad (4)$$

Define the second surface to be

$$S_2 = x_2 - x_{2d} \quad (5)$$

We choose χ_3 to drive $S_2 \rightarrow 0$,

$$\chi_3 = -K_2 S_2 - f_2(\bar{x}_2) - \vartheta_2 \phi_2(\bar{x}_2) - \frac{a_2^2 S_2 p_2^2}{2\epsilon} + \dot{x}_{2d} \quad (6)$$

and obtain x_{3d} by filtering χ_3 ,

$$\tau_3 \dot{x}_{3d} + x_{3d} = \chi_3, \quad x_{3d}(0) = \chi_3(0) \quad (7)$$

Proceeding similarly, define the i -th surface as

$$S_i = x_i - x_{id} \quad (8)$$

χ_{i+1} is chosen to drive $S_i \rightarrow 0$,

$$\chi_{i+1} = -K_i S_i - f_i(\bar{x}_i) - \vartheta_i \phi_i(\bar{x}_i) - \frac{a_i^2 S_i p_i^2}{2\epsilon} + \dot{x}_{id} \quad (9)$$

Filtering χ_{i+1} , x_{i+1d} are obtained.

$$\tau_{i+1} \dot{x}_{i+1d} + x_{i+1d} = \chi_{i+1}, \quad x_{i+1d}(0) = \chi_{i+1}(0) \quad (10)$$

Finally, define

$$S_n = x_n - x_{nd} \quad (11)$$

The controller u is chosen to be

$$u = -K_n S_n - f_n(\bar{x}_n) - \vartheta_n \phi_n(\bar{x}_n) - \frac{a_n^2 S_n p_n^2}{2\epsilon} + \dot{x}_{nd} \quad (12)$$

The update law for the parameter estimates is as follows:

$$\begin{cases} \dot{\vartheta}_1 = \rho_1 S_1 \phi_1 \\ \dot{\vartheta}_2 = \rho_2 S_2 \phi_2 \\ \vdots \\ \dot{\vartheta}_n = \rho_n S_n \phi_n \end{cases} \quad (13)$$

where $K_i, \tau_i, \rho_i > 0, i = 1, 2, \dots, n$, are design parameters that can be adjusted.

3 Stability Analysis

Define the boundary layer error as

$$y_i = x_{id} - \chi_i \quad i = 2, \dots, n \quad (14)$$

and let the i -th surface error be

$$S_i = x_i - x_{id} \quad i = 1, \dots, n \quad (15)$$

The estimate error is defined as

$$\tilde{\theta}_i = \theta_i - \vartheta_i \quad i = 1, \dots, n \quad (16)$$

Define the Lyapunov function as

$$V = \sum_{i=1}^n V_{is} + \sum_{i=2}^n V_{iy} + \sum_{i=1}^n V_{i\theta} = \sum_{i=1}^n \frac{1}{2} S_i^2 + \sum_{i=2}^n \frac{1}{2} y_i^2 + \sum_{i=1}^n \frac{1}{2\rho_i} \tilde{\theta}_i^2 \quad (17)$$

The closed-loop dynamics can be expressed as follows.

For the surface errors,

$$\begin{aligned} \dot{S}_1 &= x_2 + f_1(\bar{\mathbf{x}}_1) + \theta_1 \phi_1(\bar{\mathbf{x}}_1) + p_1 w_1 - \dot{x}_{1d} = S_2 + x_{2d} + f_1(\bar{\mathbf{x}}_1) + \theta_1 \phi_1(\bar{\mathbf{x}}_1) + p_1 w_1 - \dot{x}_{1d} = \\ &S_2 + y_2 - K_1 S_1 + \tilde{\theta}_1 \phi_1(\bar{\mathbf{x}}_1) - \frac{a_1^2 S_1 p_1^2}{2\epsilon} + p_1 w_1 \end{aligned} \quad (18)$$

\vdots

$$\dot{S}_i = S_{i+1} + y_{i+1} - K_i S_i + \tilde{\theta}_i \phi_i(\bar{\mathbf{x}}_i) - \frac{a_i^2 S_i p_i^2}{2\epsilon} + p_i w_i \quad (19)$$

\vdots

$$\dot{S}_n = -K_n S_n + \tilde{\theta}_n \phi_n(\bar{\mathbf{x}}_n) - \frac{a_n^2 S_n p_n^2}{2\epsilon} + p_n w_n \quad (20)$$

For the boundary layers,

$$\left. \begin{aligned} \dot{y}_2 &= -y_2/\tau_2 - \dot{\chi}_2 \\ \dot{y}_3 &= -y_3/\tau_3 - \dot{\chi}_3 \\ &\vdots \\ \dot{y}_n &= -y_n/\tau_n - \dot{\chi}_n \end{aligned} \right\} \quad (21)$$

For the estimate errors,

$$\left. \begin{aligned} \dot{\tilde{\theta}}_1 &= -\rho_1 S_1 \phi_1 \\ \dot{\tilde{\theta}}_2 &= -\rho_2 S_2 \phi_2 \\ &\vdots \\ \dot{\tilde{\theta}}_n &= -\rho_n S_n \phi_n \end{aligned} \right\} \quad (22)$$

The proof uses the technique of singular perturbations; interested readers are referred to [9,10]. In this constructive proof, $\Theta_i(\cdot)$, $\varphi_i(\cdot)$, $\psi_i(\cdot)$, $\eta_i(\cdot)$, $\mu_i(\cdot)$ are used to denote functions at the i -th step of the induction.

From Eqs. (9), (10), (14) and (15), we can get

$$\left. \begin{aligned} x_1 &= S_1 + x_{1d} \\ x_2 &= S_2 + y_2 - K_1 S_1 - f_1(x_1) + \vartheta_1 \phi_1(x_1) - \frac{a_1^2 S_1 p_1^2}{2\epsilon} + \dot{x}_{1d} \\ &\vdots \\ x_{i+1} &= S_{i+1} + y_{i+1} - K_i S_i - f_i(\bar{\mathbf{x}}_i) + \vartheta_i \phi_i(\bar{\mathbf{x}}_i) - \frac{a_i^2 S_i p_i^2}{2\epsilon} - \frac{y_i}{\tau_i} \quad \forall i \geq 2 \end{aligned} \right\} \quad (23)$$

By induction, for all $i \geq 2$,

$$x_{i+1} = \psi_i(S_1, \dots, S_{i+1}, y_2, \dots, y_{i+1}, \vartheta_1, \dots, \vartheta_i, K_1, \dots, K_i, \tau_2, \dots, \tau_i, x_{1d}, \dot{x}_{1d}) \quad (24)$$

By Eqs. (19), (23) and (24), we have

$$\begin{aligned} \dot{S}_i &= S_{i+1} + y_{i+1} - K_i S_i + \tilde{\theta}_i \phi_i(S_1 + x_{1d}, \psi_1, \psi_2, \dots, \psi_{i-1}) - \\ &a_i^2 S_i p_i^2 (S_1 + x_{1d}, \psi_1, \psi_2, \dots, \psi_{i-1}) / (2\epsilon) + p_i w_i \end{aligned} \quad (25)$$

Assuming that there exists an upper bound function $\Theta_i(\cdot)$ on \dot{S}_i , then we have

$$|\dot{S}_i| \leq \Theta_i(S_1, \dots, S_{i+1}, y_2, \dots, y_{i+1}, \tilde{\theta}_1, \dots, \tilde{\theta}_i, K_1, \dots, K_i, \tau_2, \dots, \tau_i, x_{1d}, \dot{x}_{1d}) \quad (26)$$

Thus, the bound on \dot{S}_i depends only $S_1, \dots, S_{i+1}, y_2, \dots, y_{i+1}, \tilde{\theta}_1, \dots, \tilde{\theta}_i, K_1, \dots, K_i, \tau_2, \dots, \tau_i, x_{1d}$ and \dot{x}_{1d} .

For the convenience, let $\mathbf{Z} = \{S_1, \dots, S_{i+1}, y_2, \dots, y_{i+1}, \tilde{\theta}_1, \dots, \tilde{\theta}_i, K_1, \dots, K_i, \tau_2, \dots, \tau_i, x_{1d}, \dot{x}_{1d}\}^T$. Similarly, one can show that

$$\begin{aligned} \frac{dp_i}{dt} &= \sum_{j=1}^i \frac{\partial p_i}{\partial x_j} \dot{x}_j \\ \left| \frac{dp_i}{dt} \right| &\leq \varphi_i(\mathbf{Z}) \end{aligned} \quad (27)$$

$$\left| \frac{d\phi_i}{dt} \right| \leq \phi_i(\mathbf{Z}) \quad (28)$$

$$\left| \frac{df_i}{dt} \right| \leq \eta_i(\mathbf{Z}) \quad (29)$$

By (3), we have

$$\dot{\chi}_2 = -K_1 \dot{S}_1 - \frac{\partial f_1}{\partial x_1} \dot{x}_1 - \dot{\vartheta}_1 \phi_1(x_1) - \frac{\partial_1 \partial \phi_1}{\partial x_1} \dot{x}_1 - \frac{a_1^2 \dot{S}_1 p_1^2}{2\epsilon} - a_1^2 S_1 p_1 \frac{\partial p_1}{\partial x_1} \frac{\dot{x}_1}{\epsilon} + \dot{x}_{1d} \quad (30)$$

Then by Eqs. (26)–(30), there exists an upper bound function μ_2 such that

$$|\dot{\chi}_2| \leq \mu_2(S_1, S_2, y_2, \vartheta_1, K_1, x_{1d}, \dot{x}_{1d}, \ddot{x}_{1d}) \quad (31)$$

In addition, by (9), we have

$$\dot{\chi}_{i+1} = -K_i \dot{S}_i - \sum_{j=1}^i \frac{\partial f_i}{\partial x_j} \dot{x}_j - \dot{\vartheta}_i \phi_i(\bar{x}_i) - \vartheta_i \sum_{j=1}^i \frac{\partial \phi_i}{\partial x_j} \dot{x}_j - \frac{a_i^2 \dot{S}_i p_i^2}{2\epsilon} - a_i^2 S_i p_i \sum_{j=1}^i \frac{\partial p_i}{\partial x_j} \frac{\dot{x}_j}{\epsilon} - \frac{\dot{y}_i}{\tau_i} \quad (32)$$

By induction, there exists an upper bound function μ_{i+1} such that

$$|\dot{\chi}_{i+1}| \leq \mu_{i+1}(\mathbf{Z}, \ddot{x}_{1d}) \quad (33)$$

From Eqs. (17)–(22), (33) and $|S_i| |p_i w_i| \leq \frac{S_i^2 p_i^2 a_i^2}{2\epsilon} + \frac{\epsilon}{2}$, we get

$$\begin{aligned} \dot{V}_{is} &= S_i \dot{S}_i = S_i \left(S_{i+1} + y_{i+1} - K_i S_i + \tilde{\theta}_i \phi_i(\bar{x}_i) - \frac{a_i^2 S_i p_i^2}{2\epsilon} + p_i w_i \right) \leq \\ &- K_i S_i^2 + |S_i| (|S_{i+1}| + |y_{i+1}|) + S_i \tilde{\theta}_i \phi_i + \frac{\epsilon}{2} \quad i = 1, 2, \dots, n-1 \end{aligned} \quad (34)$$

$$\dot{V}_{ns} \leq -K_n S_n^2 + S_n \tilde{\theta}_n \phi_n + \frac{\epsilon}{2} \quad (35)$$

$$\dot{V}_{\theta} = -\frac{1}{\rho_i} \tilde{\theta}(\rho_i S_i \phi_i) = -S_i \tilde{\theta}_i \phi_i \quad i = 1, 2, \dots, n \quad (36)$$

$$\dot{V}_{iy} = y_{i+1} \dot{y}_{i+1} \leq \frac{-y_{i+1}^2}{\tau_{i+1}} + |y_{i+1}| \mu_{i+1}(\mathbf{Z}, \ddot{x}_{1d}) \quad i = 1, 2, \dots, n-1 \quad (37)$$

Thus by (34)–(37), we have

$$\dot{V} \leq -\sum_{i=1}^n K_i S_i^2 + \sum_{i=1}^{n-1} \left[|S_i| (|S_{i+1}| + |y_{i+1}|) - \frac{y_{i+1}^2}{\tau_{i+1}} + |y_{i+1}| \mu_{i+1} \right] + \frac{n\epsilon}{2} \quad (38)$$

By choosing $K_i = 2 + \gamma$ and $\frac{1}{\tau_{i+1}} = 1 + \frac{\lambda_{i+1}^2}{2\epsilon} + \gamma$, we have

$$\begin{aligned} \dot{V} &\leq -(2 + \gamma) \sum_{i=1}^n S_i^2 + \sum_{i=1}^{n-1} \left[\frac{S_{i+1}^2 + y_{i+1}^2}{2} + S_i^2 - \left(1 + \gamma + \frac{\lambda_{i+1}^2}{2\epsilon} \right) y_{i+1}^2 + \frac{y_{i+1}^2 \mu_{i+1}^2}{2\epsilon} \right] + n\epsilon \leq \\ &- \gamma \sum_{i=1}^n (S_i^2 + y_{i+1}^2) + n\epsilon - \sum_{i=1}^{n-1} \left(1 - \frac{\mu_{i+1}^2}{\lambda_{i+1}^2} \right) \frac{\lambda_{i+1}^2 y_{i+1}^2}{2\epsilon} \end{aligned} \quad (39)$$

By choosing τ_{i+1} , ϵ is small enough and K_i is large enough, we have $\dot{V} \leq 0$. Using LaSalle's invariance theorem, we can prove asymptotic stability with $\{S_i, y_i, \tilde{\theta}_i\}^T \rightarrow 0$. Hence, in view of the above analysis, we get the following theorem.

Theorem Consider the nonlinear uncertain system described by (1). For all admissible uncertainties, there exists a set of surface gains K_1, \dots, K_n and filter time constants τ_2, \dots, τ_n such that robust adaptive dynamic surface controller guarantees the semi-global stability of the closed-loop system and the output tracking of a given desired trajectory.

4 Simulation Results

Consider the following 3-order nonlinear system:

$$\left. \begin{aligned} \dot{x}_1 &= x_2 + \theta_1 x_1^3 \\ \dot{x}_2 &= x_3 + \theta_2 (x_1^2 + x_2^2) + (\sin x_2) w_2 \\ \dot{x}_3 &= u + 2x_3^2 \sin(x_1 + x_2) + w_3 \end{aligned} \right\} \quad (40)$$

In simulation, $\theta_1 = \theta_2 = 1$, $\theta_3 = 0$, $p_1 = 0$, $p_2 = \sin x_2$, $p_3 = 1$, $w_1 = 0$, $w_2 = \cos 2t$, $w_3 = 2\sin 3t$,

$f_1(x_1) = f_2(\bar{x}_2) = 0$, $f_3(\bar{x}_3) = 2x_3^2 \sin(x_1 + x_2)$, $\vartheta_1(0) = 0$, $\vartheta_2(0) = 0$, $\rho_1 = 1$, $\rho_2 = 0.01$, $K_1 = K_2 = K_3 = 10$, $x_{1d} = 1$, $x_1(0) = 2$, $x_2(0) = 1$, $x_3(0) = -10$, $\varepsilon = 1.0$, $\tau_2 = \tau_3 = 0.01$, $x_{2d}(0) = \chi_2(0) = 0$, $x_{3d}(0) = \chi_3(0) = 0$, $x_{1d} = 1$.

Simulation results are shown in Figs.1 - 4.

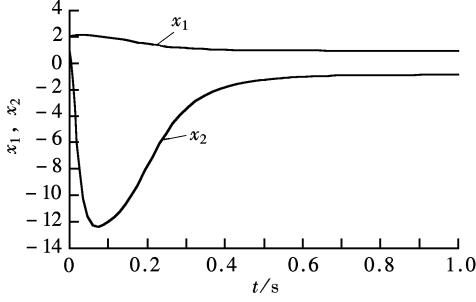


Fig.1 States x_1, x_2 response curves

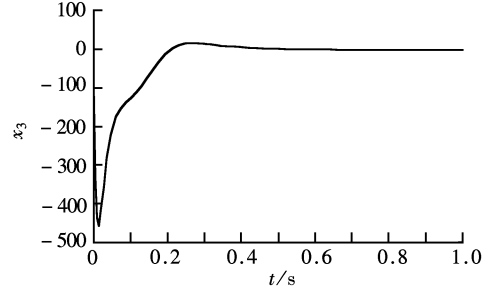


Fig.2 State x_3 response curve

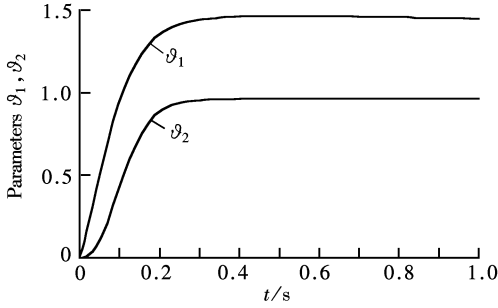


Fig.3 Parameters estimate ϑ_1, ϑ_2

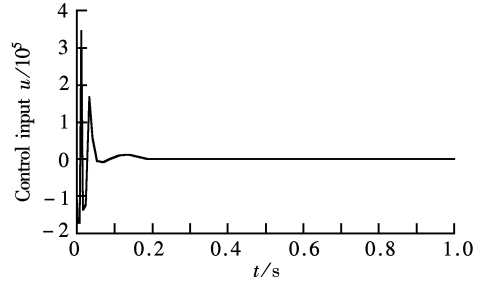


Fig.4 Control input u

Fig.1 shows that the output x_1 semi-globally tracks the desired trajectory x_{1d} . Figs.1 - 2 show the states x_2 and x_3 response curve. Fig.3 shows that the parameters estimate ϑ_1 and ϑ_2 converge to some constants. Fig.4 shows the control input u . Evidently, the controller designed guarantees the semi-global stability of the closed-loop system and the output tracking of a given desired trajectory. This also demonstrates that the scheme proposed is effective and feasible.

5 Conclusion

In this paper, we propose a new design method for robust adaptive control of a class of nonlinear uncertain systems. This method is called dynamic surface control. Current backstepping algorithms require repeated differentiations of the modeled non-linearities. The addition of n first order low pass filters allows the new algorithms to be implemented without differentiating any model non-linearities, thus ending the complexity arising due to the “explosion of terms”. The proposed approach guarantees the semi-global stability of the closed-loop systems and the output tracking of a given desired trajectory. The simulation results also show the effectiveness and feasibility of the scheme proposed.

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非线性不确定系统鲁棒自适应动态面控制

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摘 要 本文针对具有未知参数不确定和干扰的严格反馈型的非线性系统鲁棒自适应控制提出了一种新的设计方法,即动态面控制.传统的递推算法要求对建模的非线性反复多次微分.本文方法由于加入了 n 个低通滤波器使得算法不用对模型非线性进行多次微分,因而避免了因“微分项的爆炸”引起的算法复杂性从而简化了算法.理论分析证明了所设计的鲁棒自适应动态面控制器保证了闭环系统的半全局渐近稳定并且使得输出跟踪期望轨迹.仿真结果表明了所设计的控制器的有效性和可行性.

关键词 非线性系统; 鲁棒控制; 自适应控制; 动态面控制; 不确定

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