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A new fuzzy edge detection algorithm

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Abstract: Based upon the maximum entropy theorem of information theory, a novel fuzzy approach for edge detection is presented. Firstly, a definition of fuzzy partition entropy is proposed after introducing the concepts of fuzzy probability and fuzzy partition. The relation of the probability partition and the fuzzy c-partition of the image gradient are used in the algorithm. Secondly, based on the conditional probabilities and the fuzzy partition, the optimal thresholding is searched adaptively through the maximum fuzzy entropy principle, and then the edge image is obtained. Lastly, an edge-enhancing procedure is executed on the edge image. The experimental results show that the proposed approach performs well.

Key words: edge detection; fuzzy entropy; image segmentation; fuzzy partition

Image segmentation is an important topic for image analysis, computer vision and pattern recognition. Until now, many classical edge detection algorithms have been put forward. In recent years, fuzzy set theory has been successfully applied to many areas, such as automation control, image processing, pattern recognition and computer vision, etc. It is generally believed that image processing bears some fuzziness in nature due to the following factors: ① Information loss while mapping 3-D objects into 2-D images; ② Ambiguity and vagueness in some definitions (such as edges, boundaries, regions, and textures, etc.); ③ Ambiguity and vagueness in interpreting low-level image processing results. Therefore, fuzzy techniques have frequently been used in image segmentation[1-4].

Jin Lizuo, et al.[5] proposed a new definition of fuzzy partition entropy using the conditional probability and conditional entropy, and designed a new thresholding selection algorithm based on the maximum fuzzy entropy. This paper extends the application of the work to the problem of the edge detection and presents a new fuzzy edge detection algorithm. In the algorithm, a gradient image is considered as being composed of an edge region and a smooth region. Based on the conditional probability and the fuzzy partition entropy, the optimal thresholding is searched adaptively through maximum fuzzy entropy principle. There are two major differences between the problems of the edge detection and the image thresholding segmentation. First, the problem is actually reduced to a two-level thresholding problem, where the purpose of thresholding is to partition the image into two regions: an edge region and a smooth region. Second, in order to find the best compact representation of the image edges and contours, the gradient image is processed. The experimental results show the effectiveness of the algorithm.

The rest of this paper is organized as follows. In section 1, we briefly outline the concept of fuzzy probability and fuzzy partition entropy. In section 2, we describe the fuzzy edge detection algorithm. In section 3, the experimental results and conclusions are presented.

1 Problem Formulation

1.1 Fuzzy probability description of image

Let an image have L gray-levels, i.e. $G = \{0,1,\cdots,L-1\}$. The histogram is denoted by h_k, where $k = 0,1,\cdots,L-1$. Let p be the probability of gray, then $p_k = p(\{k\}) = h_k$. A fuzzy set $\tilde{A} = \sum_{k=0}^{L-1}\frac{\mu_{\tilde{A}}(k)}{k}$ denotes some object in the image. A membership function $\mu_{\tilde{A}}(k)$ denotes the grade, where the gray k belongs to $\tilde{A}$. The probability of $\tilde{A}$ is computed simply by $p(\tilde{A}) = \sum_{k=0}^{L-1}\mu_{\tilde{A}}(k)h_k$, and the conditional probability, where the gray k is classified into $\tilde{A}$ due to $\tilde{A}$, is $p(\{k\}\mid\tilde{A}) = \mu_{\tilde{A}}(k)h_k/p(\tilde{A})$.

1.2 New definition of fuzzy partition entropy[5]

Definition 1 Let $X = \{X_1, X_2, \cdots, X_n\} \subset \mathbf{R}^p$ (p-dimensional real space) be a randomly finite set and $P = \{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_c\}$ be a fuzzy partition of X. $\tilde{B}$ is a fuzzy event of the probability space, then, on

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condition that $\tilde{B}$ is given the conditional entropy of P is as follows:

$$H(P|\tilde{B}) = -\sum_{i=1}^{c} p(\tilde{A}_i|\tilde{B})\log p(\tilde{A}_i|\tilde{B}) = -\sum_{i=1}^{c} \frac{p(\tilde{A}_i\tilde{B})}{p(\tilde{B})}\log\frac{p(\tilde{A}_i\tilde{B})}{p(\tilde{B})} \quad (1)$$

The sequence of set $Q_i = \{X_i\}(i = 0,1,\cdots,n)$ is constructed. $Q = \{Q_1, Q_2, \cdots, Q_n\}$ is apparently a fuzzy partition of X; Q is called a natural fuzzy partition of X, By definition 1, on condition that $\tilde{B}$ the conditional entropy of Q is as follows:

$$H(Q|\tilde{B}) = -\sum_{i=1}^{n} p(Q_i|\tilde{B})\log p(Q_i|\tilde{B}) = -\sum_{i=1}^{n} \frac{p(Q_i\tilde{B})}{p(\tilde{B})}\log\frac{p(Q_i\tilde{B})}{p(\tilde{B})} \quad (2)$$

Definition 2 Let $X = \{X_1, X_2, \cdots, X_n\} \subset \mathbf{R}^p$ be a randomly finite set. Let $P = \{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_c\}$ be a fuzzy partition of X. Let $Q = \{Q_1, Q_2, \cdots, Q_n\}$ be a naturally fuzzy partition of X. Then, the entropy of P is

$$H(P) = \sum_{i=1}^{c} H(Q|\tilde{A}_i) = -\sum_{i=1}^{c}\sum_{j=1}^{n} \frac{p(Q_j\tilde{A}_i)}{p(\tilde{A}_i)}\log\frac{p(Q_j\tilde{A}_i)}{p(\tilde{A}_i)} \quad (3)$$

2 The Proposed Algorithm

2.1 Gradient image

Let $g(x,y)$ denote the gradient of an image, $m(x,y)$ at the pixel location (x,y). The gradient of the image can be defined as

$$g(x,y) = \mathrm{grad}\{m(x,y)\} = \sqrt{[m(x,y) - m(x-1,y)]^2 + [m(x,y) - m(x,y-1)]^2} \quad (4)$$

2.2 Fuzzy partition of gradient image

Let a gradient image $g(x,y)$ have L gray-levels, i.e. $G = \{0,1,\cdots,L-1\}$ and let the histogram be denoted by $h_k, k = 0,1,\cdots,L-1$. The sequence of set $Q_k = \{k\}$, $k = 0,1,\cdots,L-1$, is constructed. $Q = \{Q_0, Q_1, \cdots, Q_{L-1}\}$ is apparently a fuzzy partition of G.

Here, a gradient image is considered as being composed of edge region D_e and smooth region D_s. Based on the definition of fuzzy partition entropy, the problem of the edge detection is to find the unknown probabilistic fuzzy 2-partition of P, where $P = \{D_e, D_s\}$. The probability distributions of D_e and D_s are denoted by p_e and p_s, i.e. $p_e = p(D_e), p_s = p(D_s)$. The two fuzzy partitions of D_e and D_s are characterized by two membership functions, $\mu_e(g;a,b)$ and $\mu_s(g;a,b)$, respectively. Parameter couple (a,b) are used to control the shape of membership functions. $\mu_e(g;a,b) + \mu_s(g;a,b) = 1$, $g = 0,1,\cdots,L-1$. $a < b$. The functions are shown in Fig.1.

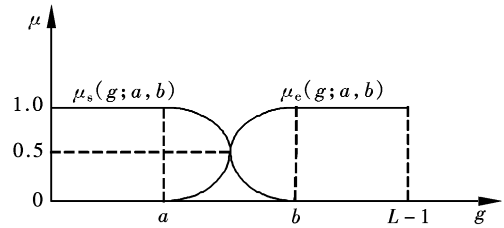


Fig.1 Membership functions of edge and smooth region

In the proposed edge detection scheme, $\mu_e(g;a,b)$ is an S-function, $\mu_s(g;a,b)$ is a Z-function.

$$\mu_s(g;a,b) = \begin{cases} 1 & 0 \leqslant g \leqslant a \\ 1 - 2\left(\dfrac{g-a}{b-a}\right)^2 & a \leqslant g \leqslant \dfrac{a+b}{2} \\ 2\left(\dfrac{g-b}{b-a}\right)^2 & \dfrac{a+b}{2} \leqslant g \leqslant b \\ 0 & b \leqslant g \leqslant L-1 \end{cases} \quad (5)$$

$$\mu_e(g;a,b) = 1 - \mu_s(g;a,b) \quad (6)$$

In fact, under the condition that they have a gradient of g, the membership functions, $\mu_e(g;a,b)$ and $\mu_s(g;a,b)$, represent respectively the conditional probability that a pixel is classified into the edge region D_e and the smooth region D_s, i.e:

$$\left.\begin{aligned} \mu_s(g) &= p_s|_g \\ \mu_e(g) &= p_e|_g \end{aligned}\right\} \quad (7)$$

According to definition 1 and Eq. (2), the conditional entropy of Q, on condition that the edge region D_e is given, is determined by

$$H(Q|D_e) = -\sum_{k=0}^{L-1} \frac{p(Q_kD_e)}{p(D_e)}\log\frac{p(Q_kD_e)}{p(D_e)} = -\sum_{k=0}^{L-1} \frac{\mu_e(k)h_k}{p(D_e)}\log\frac{\mu_e(k)h_k}{p(D_e)} \quad (8)$$

where

$$p(D_e) = \sum_{k=0}^{L-1} \mu_e(k)h_k \quad (9)$$

The conditional entropy of Q, on condition that the smooth region D_s is given, is determined by

$$H(Q|D_s) = -\sum_{k=0}^{L-1} \frac{p(Q_kD_s)}{p(D_s)}\log\frac{p(Q_kD_s)}{p(D_s)} = -\sum_{k=0}^{L-1} \frac{\mu_s(k)h_k}{p(D_s)}\log\frac{\mu_s(k)h_k}{p(D_s)} \quad (10)$$

where

$$p(D_s) = \sum_{k=0}^{L-1} \mu_s(k) h_k \tag{11}$$

According to definition 2, Eqs.(8) and (10), the fuzzy partition entropy $H(P)$ is given by

$$H(P) = H(Q \mid D_e) + H(Q \mid D_s) = - \sum_{k=0}^{L-1} \left[\frac{\mu_e(k)h_k}{p(D_e)} \log \frac{\mu_e(k)h_k}{p(D_e)} + \frac{\mu_s(k)h_k}{p(D_s)} \log \frac{\mu_s(k)h_k}{p(D_s)} \right] \tag{12}$$

2.3 Maximum fuzzy entropy criterion

In Ref.[5], the aim of the algorithm is to find a fuzzy partition of the maximum information, i.e. to search optimal parameters in image space to maximize Eq.(12). However, in the problem of edge detection, we aim at finding the parameters' values that result in the best compact edge-representation of images. Hence the parameters' values that correspond to the maximum entropy are selected and then the best selected parameters, $\tilde{a}$ and $\tilde{b}$, are the ones satisfying:

$$H(\tilde{a}, \tilde{b}) = \max_{g=0,1,2,\cdots,L-1} (H(a(g), b(g))) \tag{13}$$

Eq.(12) can be regarded as the total information measurement on the basis of fuzzy partition P, i.e. $P = \{D_e, D_s\}$. In the course of searching fuzzy partition which has maximum information entropy, one encounters the problem of parameter optimization, the algorithm is as follows:

1) Let the gradient image have L grayscales and compute the gray histogram, which is denoted by h_k, where $k = 0, 1, \cdots, L-1$;

2) Determine the minimum grayscale $g_{\min}$ and the maximum grayscale $g_{\max}$;

3) Search for a fuzzy partition P, which has a maximum entropy;

For $a = g_{\min}$ to $g_{\max} - 1$

For $b = a + 1$ to $g_{\max}$

According to Eqs.(5) and (6), compute the membership functions of the edge region D_e and the smooth region D_s;

According to Eq.(9) and (11), compute the probability of the edge region D_e and the smooth region D_s;

According to Eq. (12), compute the entropy of fuzzy partition P, $P = \{D_e, D_s\}$;

If $H \geqslant H_{\max}$, then $H_{\max} = H$, $a_{opt} = a$, $b_{opt} = b$;

4) Compute the thresholding T, which is equal to $\left[\frac{a_{opt} + b_{opt}}{2}\right]$ ([·]denotes an integer).

2.4 Edge detection

Let the edge image be $e(x, y)$, then calculate it as

$$e(x,y) = \begin{cases} 200 & \text{if } g(x,y) \geqslant T \\ 0 & \text{if } g(x,y) \leqslant T \end{cases} \tag{14}$$

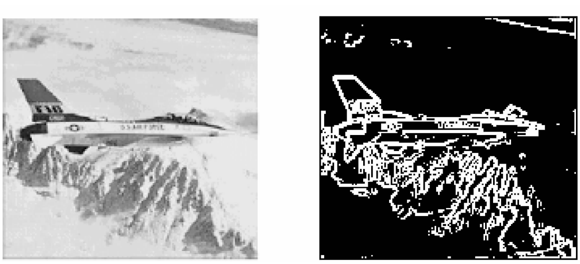
Spurious or weak edges (intensity discontinuities) may result in the image edge representation due to many factors; among them are noise and breaks in the boundary between two regions due to non-uniform illumination. In this section, we introduce a simple yet effective procedure for removing spurious or weak edges. The procedure is as follows:

1) Run a 3×3 pixel window on the edge image, where the center of the window imposed on each location (x, y);

2) Sum the number of points which have been classified as edge in the window, if the number is greater than four, leave these edge points, else they represent weak or spurious edges.

3 Experimental Results and Conclusions

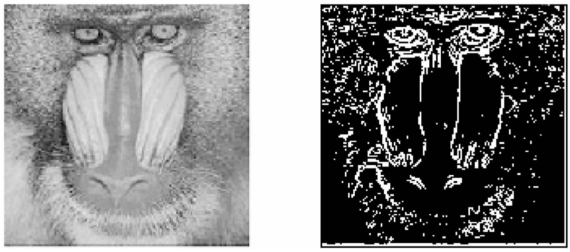
In this section, the experiments on various kinds of images have been carried out with the proposed method. The three original images are selected and shown in Figs.2 – 4. Fig.2 is an airplane image, the size of which is 212×200 pixel. The membership function parameters set $(a, b) = (5, 157)$ and the image thresholding is 81. Fig.3 is a baboon image, the size of which is 202×200 pixel. The membership functions parameter set $(a, b) = (6, 164)$ and the image thresholding is 85. Fig.4 is a Lena image, the size of which is 212×208 pixel. The membership function parameters set $(a, b) = (3, 147)$ and the image thresholding is 75.



(a) (b)

Fig.2 Airplane images. (a) Original image; (b) Result image

In this paper, we combine conditional probability with fuzzy maximum entropy to introduce a new fuzzy edge detection algorithm. The experimental results show that this algorithm performs well. It is verified that segmentation methods, which combine fuzzy theory with probability statistics, are suitable for further research.



(a) (b)

Fig.3 Baboon images. (a) Original image; (b) Result image

(a) (b)

Fig.4 Lena images. (a) Original image; (b) Result image

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一种新的模糊边缘检测算法

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摘 要 基于信息论中最大熵原理,提出了一种新的模糊边缘检测算法.首先介绍了模糊概率、用条件概率与条件熵定义模糊划分熵的概念以及模糊划分的原理.算法利用了自然划分以及梯度图像模糊划分的关系,在条件概率与模糊划分熵的基础上,通过最大模糊熵原则实现图像分割中最优阈值的自动提取,从而实现图像的边缘检测.对不同测试图像的边缘检测结果进行比较,表明了该算法的有效性.

关键词 边缘检测; 模糊熵; 图像分割; 模糊划分

中图分类号 TP391.4