

Purchasing policy model based on components/parts unification

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Abstract: This paper presents a mathematical model for components/parts unification (CPU) policy. This model considers two components/parts that are functionally interchangeable but purchased from suppliers with different prices and quality characteristics. Because of the buyer's quality preference and suppliers' discount rates for bulky purchases, the model assists the procurement manager to determine how best to purchase the components/parts to meet its demand while minimizing the total acquisition costs.

Key words: components/parts unification; purchasing; parametric example; total acquisition cost

Mass customization is an effective weapon for fighting the fierce competition in the global market where customer requirements are becoming increasingly diversified and individualized^[1]. The enormous challenge in mass customization is to resolve the conflicts among product diversification, production cost and response speed^[2]. One of the effective tactics is to maximize the product variety without dramatically increasing the number of components and parts. By so doing, enterprises are able to respond to market demands quickly with the right products and services of high quality and low costs. However, this is easy to say but difficult to implement. Even large reputable corporations have experienced difficulties in controlling the part varieties. For example, the Nissan company found in 1993, used 110 different kinds of radiators, 300 different kinds of ashtrays, 437 kinds of gauge boards, 1 200 kinds of inland blankets, and 6 000 kinds of fasteners^[2].

Increasing the kinds of components/parts leads to a reduction of economic benefits, to which is not paid attention. The phenomenon of increased part varieties causing reduced performance is at first evidenced at the product design stage. An explosion of part variety in product design leads to an explosion at the procurement stage. Purchasing engineers put undue emphasis on the purchase price of the components/parts (the direct cost), while ignoring the relevant costs of obtaining the components/parts which become a percentage of the

overhead costs. In fact, the proportion of overhead costs is relatively large, depending on the demand. Pareto's law applied to inflexible plants would say that 80% of the material overhead costs would be consumed on low-usage parts that may represent only 20% of total part volume^[2]. However, because overhead costs are often difficult to quantify accurately, the current practice of estimating overhead costs does not reflect the contribution of the cost involved in managing the part variety. Therefore, there is no management control over the ever-increasing part variety.

Now the challenge is how components/parts can be unified to reduce the part variety during procurement and under what condition such components/parts unification (CPU) should be best adopted. These problems have not been studied carefully in theory and practice. Only some preliminary qualitative analysis findings were brought forward^[1,2]. Only since the 1990's has when the tendency of the buyer reducing supplier quantities been studied mentioned; the above-mentioned phenomenon of standardization of components/parts has been in Refs. [3,4]. In other literature the components/parts standardization was emphasized only as a kind of means or precondition that enterprises should implement the strategy of customization manufacturing^[5]. None of these has solved the problem of CPU.

In this paper, a method will be proposed to minimize the total acquisition cost (TAC) by means of components/parts unification. The influencing factors to CPU are discussed via qualitative analysis. The condition that similar components/parts of two types of different prices are unified into one type of components/parts is confirmed via quantitative analysis.

Received 2002-09-11.

Foundation items: The National Natural Science Foundation of China (79900003) and the Research Fund of Southeast University (9214001196).

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According to the analytical findings, some concrete suggestions to the management of CPU are suggested.

1 Total Acquisition Cost Model of the Components/Parts

For structure total acquisition cost model for two similar different types of components/parts, we present the following suppositions first.

1) There are two different types of the same kindred components/parts that are u_1 and u_2 . They are functionally interchangeable but purchased from suppliers with different prices and quality characteristics. u_2 can be replaced by u_1 in terms of design and quality requirements. The unit purchasing cost/price of u_1 is higher than that of u_2 (otherwise u_2 has no reason to be purchased).

2) Annual demand quantity of u_1 and u_2 is stable.

3) X_1 and X_2 stand for the total acquisition cost when components/parts u_1 and u_2 are purchased, respectively. X shows the acquisition cost after u_1 and u_2 have been unified into u_1 (u_1 and u_2 unification).

4) D_1 and D_2 stand for annual demand quantity of u_1 and u_2 , respectively. D stands for the total annual demand quantity of u_1 and u_2 unification. So, $D = D_1 + D_2$. Let $D_2 = pD_1$, therefore, $D = (1 + p)D_1$.

5) m_1 and m_2 stand for the unit price of u_1 and u_2 , respectively, and m stands for the unit price after u_1 and u_2 unification. Let $m_2 = am_1$, $m = bm_1$, ($a, b \leq 1$), where $a \leq 1$ stands for that the unit price of u_2 is lower than the unit price of u_1 , and $b \leq 1$ stands for that certain purchasing price discount (the preferential price) which can be obtained owing to the increasing of purchased quantity by means of parts unification.

6) Separately, k_1 and k_2 stand for the order cost (including production arranged cost) of u_1 and u_2 at a time. k stands for the order cost (including production arranged cost) after u_1 and u_2 unification at a time.

7) Separately, h_1 and h_2 stand for the average unit stock cost of u_1 and u_2 per year, and h stands for the average unit stock cost after CPU per year. In addition, $h_i = r_i m_i$ and $h = rm$, where r_i and r stand for the corresponding unit price percentages of the stock cost ($i = 1, 2$).

8) The overhead cost (including order cost and stock cost) of two types of components/parts is shared according to the different unit rate. The unit rate is related to the different demand quantity. The smaller the demand quantity is, the greater the unit rate is.

So, we can set:

$$\left. \begin{aligned} k &= g(1 + p)k_1, k_2 = g(p)k_1 \\ r &= f(1 + p)r_1, r_2 = f(p)r_1 \end{aligned} \right\}$$

where $g(p)$ and $f(p)$ are strictly monotonous declining functions of “ p ”.

9) The order point method and safe stock system are adopted to manage u_1 and u_2 ^[6,7]. Namely, one specific order point R_i is stipulated for u_i . Components/parts should be ordered when the stock level of u_i is up to R_i , and order batch is Q_i ($i = 1, 2$).

10) The safe stocks of u_1 and u_2 are S_1 and S_2 , respectively. The safe stock after CPU is S .

Based on the suppositions mentioned above, the TAC model of u_1 and u_2 can be structured while u_1 and u_2 are purchased, respectively.

$$X_i = m_i D_i + \frac{D_i}{Q_i} k_i + \left(\frac{Q_i}{2} + S_i \right) h_i \quad i = 1, 2 \quad (2)$$

where $m_i D_i$ represents annual purchasing cost of u_i ; $\frac{D_i}{Q_i} k_i$ represents annual order cost (including cost of order form and production arranged cost) of u_i . $\left(\frac{Q_i}{2} + S_i \right) h_i$ represents the annual stock cost (including the relevant management cost proportioned). Meanwhile, each item from all of the above is suitable for u_1 and u_2 , respectively.

The TAC after CPU is represented as follows.

$$X = mD + \frac{D}{Q} k + \left(\frac{Q}{2} + S \right) h \quad (3)$$

The meaning of each item on the right in formula (3) is similar to that of formula (2).

2 Analysis of CPU Condition

Whether or not the components/parts u_1 and u_2 should be unified can be established by testing the following formula:

$$\min X \leq \min X_1 + \min X_2 \quad (4)$$

If formula (4) is valid, u_1 and u_2 should be unified into one type of component/part u_1 .

Without considering the safe stock cost (viz. $S_1, S_2, S = 0$), the validity of formula (4) is analyzed below.

Proposition If the unit purchasing price of u_2 (m_2) is not lower by v times than the unit purchasing price of u_1 (m_1), these two types of components/parts should be unified into one type of component/part u_1 . And the TAC will be reduced by means of CPU (keeping the same only when $m_2 = vxm_1$). Among them:

$$v = \frac{(-\sqrt{\psi(p)} + \sqrt{\psi(p) + 4\sqrt{w}(\sqrt{w}(b-1+p) - \sqrt{1-p} + \sqrt{b\psi(1+p)})})^2}{4up} \quad (5)$$

where $\psi(\cdot) = f(\cdot)g(\cdot)$; $w = \frac{Dm_1}{2r_1k_1}$, which is called the “Form Parameter” of TAC in CPU.

Proof Insert $S_i = 0$ into (2) and evaluate Q_i 's derivative is

$$\frac{dX}{dQ_i} = -k_i \frac{D_i}{Q_i^2} + \frac{h_i}{2} = 0 \quad i = 1, 2$$

$$Q_i = \sqrt{\frac{2D_i k_i}{h_i}} \quad i = 1, 2$$

Therefore,

$$\min X_i = D_i m_i + \sqrt{2D_i k_i h_i} \quad i = 1, 2 \quad (6)$$

Analogously,

$$\min X = Dm + \sqrt{2Dkh} \quad (7)$$

Insert $D_2 = pD_1$, $D = (1+p)D_1$, $m_2 = am_1$, $m = bm_1$, $k_2 = f(p)k_1$, $r_2 = g(p)r_1$, $k = f(1+p)k_1$, $r = g(1+p)r_1$ and formulae (6) and (7) into formula (4). We can deduce:

$$\begin{aligned} & bDm_1 + \sqrt{2Dm_1 k_1 r_1 b f(1+p) g(1+p)} \\ & \leq (1-p+ap)Dm_1 + \sqrt{2Dm_1 k_1 r_1 (\sqrt{1-p} + \sqrt{ap f(p) g(p)})} \end{aligned} \quad (8)$$

Define $w = \frac{Dm_1}{2r_1 k_1}$, then (8) can be changed into

$$\begin{aligned} & \sqrt{upa} + \sqrt{ap f(p) g(p)} - (\sqrt{b f(1+p) g(1+p)} + \\ & (b-1+p)\sqrt{w} - \sqrt{1-p}) \geq 0 \end{aligned} \quad (9)$$

Define $\psi(\cdot) = f(\cdot)g(\cdot)$, then we predigest (9) and get

$$a \geq \frac{(-\sqrt{\psi(p)} + \sqrt{\psi(p) + 4\sqrt{w}(\sqrt{w}(b-1+p) - \sqrt{1-p} + \sqrt{b\psi(1+p)})})^2}{4up} \quad (10)$$

Define the right function of (10) as $v(w, \psi, p, b)$, then $a \geq v$.

For $m_2/m_1 = a$, $m_2/m_1 \geq v$.

Therefore, if the unit purchasing price of $u_2(m_2)$ is not lower v times than the unit purchasing price of $u_1(m_1)$, u_2 should be replaced by u_1 (unification of u_1 and u_2).

In addition, because $(m_1 - m_2)/m_1 = 1 - a$, the condition of CPU is also expressed that the unit price difference rate of u_2 relative to u_1 cannot exceed $1 - v$. Shortly as follows, the permission scale of relative price difference rate is $1 - v$ if u_1 and u_2 should be unified.

3 Analysis of Influence Factor on CPU

From (5), the value of v is confirmed by the value of w , ψ , p and b . In order to analyze the change laws of the value of v following w , ψ , p and b

further, we should know the influence which various kinds of factors produced on the condition of CPU. The parametric example method is adopted to analyze the influence factors of CPU as follows.

In order to analyze the above problem further, we assume the value of $f(p)$ and $g(p)$ first. We know from (1) that $f(p)$ and $g(p)$ are strictly monotonous declining functions of p . The bigger value of $f(p)$ and $g(p)$ separately represent that the rate of purchasing cost and stock cost in of u_2 's TAC structure are heavier than u_1 's. Through real investigation and analysis, we know that the percentage of stock cost t is varies widely. The scale of t can be any from 9% to 50%, which is averaged at 25%^[6]. Possibly, the scale of t can be from 8.5% to 45%, which is averaged at 20%^[2]. These data are statistical data from some enterprises. Otherwise, different kinds of components/parts have different t (relating to the quantity of demand). Anderson and Pine II have done qualitative analysis about this^[2]. $g(\cdot)$ has similar characteristics as $f(\cdot)$. We can suppose as follows for consulting with t of enterprises.

$$\left. \begin{aligned} f(p) &= g(p) = 1 + \frac{t}{p} & 0 < p < 1 \\ f(p) &= g(p) = 1 & p \geq 1 \end{aligned} \right\} \quad (11)$$

Fig.1 illustrates changes of $f(p)$ when t was given different values (0, 0.03, 0.05, 0.08, 0.1). While $t = 0$, then $f(p) = 1$, which represents that overhead costs of u_1 and u_2 were proportioned equally according to the quantity of demand (traditional method of proportion). The bigger t is, the greater the relative change velocity of $f(p)$ is. That is to say, following reducing of the demand quantity, the unit overhead cost proportion is larger, and its change velocity will be larger following the change velocity of the demand quantity. The size of t is involved in production mode, management method and level of components/parts procurement, transport, and stock, etc. Generally, the more advanced the management method is, and the higher the management level is, then the smaller the value of t is. Contrarily, the greater the value of t is.

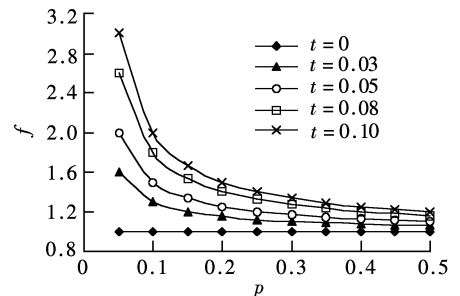


Fig.1 Various kinds of overhead cost proportion method changed with demand

The influence of other factors on relative price difference rate $1 - v$ permitted is analyzed, under the price discount (viz. $b = 1$) is not existent after CPU of u_1 and u_2 . See example 1 and example 2.

Example 1 Influence analysis of $f(p)$ and $g(p)$ changing on $1 - v$.

Suppose $w = 1\,000$, separately insert the value of variable t (0, 0.03, 0.05, 0.08, 0.1) into $f(p)$ and $g(p)$, and discuss the influence of t changes on $1 - v$. Fig.2 has shown the change laws of $1 - v$ with the functional relation of p for different values of t . When w is steady, we can get the following findings:

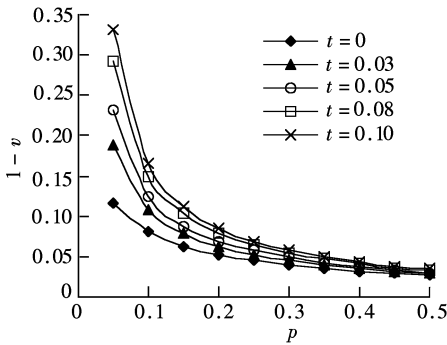


Fig.2 The influence of value of p on $1 - v$ ($w = 1\,000$)

Finding 1 The smaller p is, the greater the value of $1 - v$ is. Viz., the smaller the demand quantity of u_2 relative to u_1 is, the larger the unification permitted scale of t is. That is to say, the components/parts of smaller demand quantity should be replaced by the components/parts of larger demand quantity so as to reduce the overhead cost, thus achieving the goal of economizing TAC.

Finding 2 The bigger t is, the smaller p is, and the faster the value of $1 - v$ increasing speed is. That is to say, the permission scale of relative price difference rate is influenced by the cost proportion method of components/parts. The permission scale of relative price difference rate which gets through the traditional overhead costs proportion method, is smaller than the rate which gets through the overhead costs proportion method according to demand quantity, which is closer to practical situation. However, even if the traditional overhead cost proportion method has already been adopted, u_2 also should be replaced by u_1 (so long as relative price difference rate is not over 8%) when the demand quantity of u_2 is very small to u_1 (for example under 10%), whether or when the percentage of overhead cost in TAC is less (for example, $w \leq 1\,000$). See Fig.2.

Example 2 Influence analysis of w on $1 - v$.

Fig.3 shows: ① No matter how we choose the

value of t by changing $f(p)$ or $g(p)$, the value of will have obvious influence on $1 - v$; ② The smaller w is, the bigger $1 - v$ is, that means the relative price difference rate scale which unification permits is larger. Combining the definition of w , we can draw the following finding:

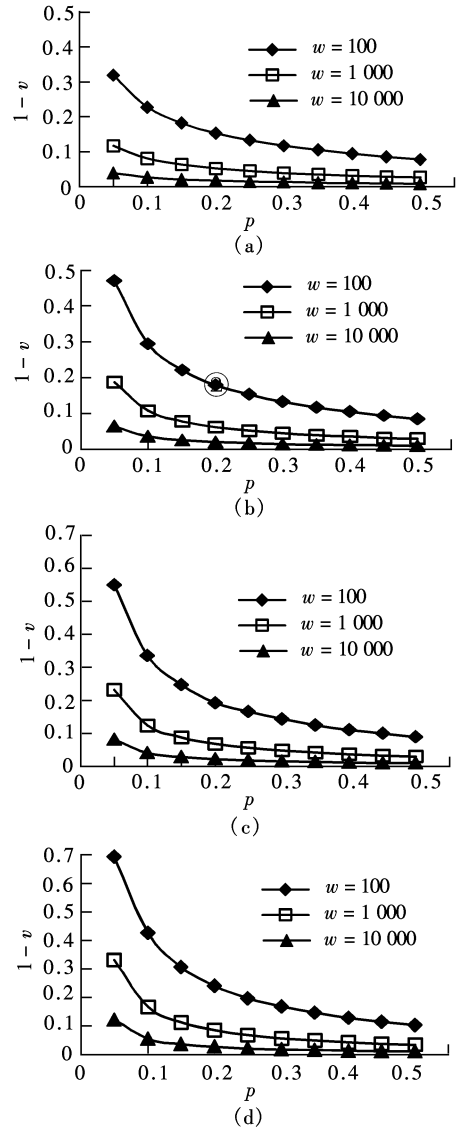


Fig.3 Influence of the TAC parameter w on $1 - v$. (a) $t = 0$; (b) $t = 0.03$; (c) $t = 0.05$; (d) $t = 0.1$

Finding 3 The smaller w is (viz. the total purchasing cost (Dm_1) is smaller, also viz. the percentage of overhead cost in TAC ($k_1 r_1$) is bigger.), the larger is the relative price differential rate scale which unification permits. For example, when $w = 100$, suppose the annual demand quantity of u_2 is 30% of u_1 , then u_2 also should be replaced by u_1 so long as the relative price difference rate is not over 13.3% (when $f = g = 1.1$, viz. $t = 0.03$) (See Fig.3(b)), or the relative price difference rate is not over 16.8% (when $f = g = 1.33$, viz. $t = 0.1$) (See Fig.3(d)).

Example 3 The influence analysis of p on $1 - v$.

After components/parts u_1 and u_2 are unified into one type of component/part u_1 , the annual demand quantity of u_1 will increase. If u_1 is regarded as a benchmark, the annual demand quantity of u_1 before CPU is D_1 and the unit-purchasing price is m_1 . The annual demand quantity of u_1 after unification is $D = (1 + p)D_1$. With the demand quantity increasing, the amount of purchase each time will also increase. So certain price discounts can be enjoyed. According to the above assumptions, the unit purchasing price of u_1 after CPU is $m = bm_1$. Then according to the above analysis, we can suppose, $b = 1 - zp$.

Fig.4 shows the influence of the purchasing price discount proportion $1 - b$ after unification on the value of $1 - v$ ($w = 1\,000, t = 0.05$). $z = 0$ means the purchasing price discount by means of CPU cannot be enjoyed. $z > 0$ means the purchasing price discount by means of CPU can be enjoyed. With the demand quantity proportion p with u_2 relative to u_1 increasing, the relative price difference degree at which u_1 and u_2 should be unified is smaller and smaller. However, the reduction scale of the relative price difference while $z > 0$ is much smaller than on $z = 0$. The larger the discount rate of purchasing price enjoyed by means of CPU is, the smaller the reduction scale of the relative price difference is.

Therefore, above findings can be drawn as follows.

Finding 4 When the purchasing price discount is enjoyed by means of CPU, two different types of components/parts can be unified. And by means of unification, the larger the discount rate of purchasing price enjoyed is, the bigger the rate scale of the relative price difference $1 - v$ is (See Fig.4).

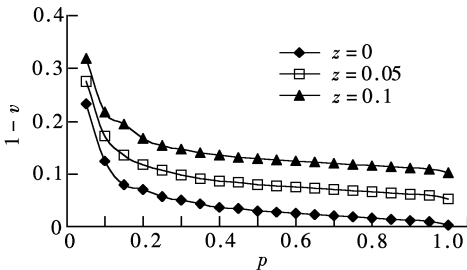


Fig.4 The influence of the value of b on $1 - v$ ($w = 1\,000, t = 0.05$)

Finding 5 The rate scale of relative price difference $1 - v$ that is permitted by CPU is far larger than the purchasing price discount $1 - b$ that can be enjoyed by means of CPU. Namely, there is a magnified effect of $1 - b$ to $1 - v$.

For example, when $w = 1\,000, t = 0.05$ and $p = 0.7$, if $z = 0.05$, the relative price difference which can be enjoyed by means of CPU is less. Namely, if $1 - v$ is not over 6.94%, u_2 should be replaced by u_1 and the purchasing price will drop 3.5%.

4 Managerial Implications for CPU

Following components/parts types sharply increasing, enterprises urgently need to standardize them. Based on the above analysis findings, reducing the types of components/parts by means of CPU is an effective method. From TAC view, some same kinds of components/parts should be unified. Furthermore, the scale of CPU (viz. the rate scale of relative price difference of components/part unit price) is quite large. Not only can similar components/parts of low value be unified (viz. the unit purchasing price is lower, or viz. the annual total purchasing cost Dm is lower), but also the similar components/parts of high value (annual total purchasing cost Dm higher) can be unified. But the scale of relative price difference rate is smaller than low value components/parts.

From the above analysis findings, we can know that the following factors may impact on CPU decision-making:

- 1) Relative demand quantity of u_2 to u_1 ;
- 2) Relative unit purchasing price difference rate;
- 3) Total demand quantity of u_1 and u_2 ;
- 4) Percent contribution to overhead cost (including purchasing cost and stock cost);
- 5) TAC structure of u_1 (percentages of direct cost and overhead cost in TAC), viz. the structure parameter of TAC w ;
- 6) Purchasing price discount enjoyed by means of CPU.

So, we propose the following suggestions to the management of enterprise CPU as follows:

- 1) When making purchasing decisions, enterprises should not only consider the relative unit purchasing price differences. They should consider more important thing that is other influence factors of TAC synthetically the problem of CPU.
- 2) Based on the premise that two components/parts are functionally interchangeable, the two types of components/parts should be unified into one type of component/part if one of the following conditions is valid. ① The relative demand quantity rate of u_2 to u_1 is small (for example, under 15%), and the relative price difference rate of u_2 to u_1 is not too much heavier (for example, not over 10%). ② The total demand quantity of u_1 and u_2 is much

smaller. Meanwhile, the relative demand quantity rate is not too much heavier (for example, not over 50%), and the relative price difference rate is also not too heavy (for example, not over 10%). ③ Overhead cost (including purchasing cost and stock cost) is much heavier, or the percentage of overhead cost of u_1 in TAC structure is very big (viz. structure parameter w of TAC is very small, such as $w \leq 100$), and the relative price difference rate of u_2 to u_1 is not too large (for example, not over 10%). ④ Certain purchasing price discounts can be enjoyed by means of CPU, and the relative price difference rate of u_2 to u_1 is not too large (for example, not over 10%).

Certainly, the components/parts cannot be unified. With the reducing of the components/parts types and the increasing of demand quantity, the relative price difference rate, which CPU permitted, will be smaller and smaller. Finally one best component/part type count will be achieved (when the relative price difference rate reaches zero).

5 Conclusion

This paper has discussed a mathematical model of components/parts unification to assist procurement engineers to minimize total acquisition costs. The model, however, is limited to considering only two components that are functionally interchangeable but with different unit prices and quality characteristics. The model can be extended for considering a more general situation where multiple functionally interchangeable components are involved. In addition,

safe stock level and suppliers' capacities have not been considered in the model. Extensions may also be made to include these factors in the model.

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基于零部件归并问题的采购策略模型

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摘 要 针对不同价格和不同质量要求的 2 种功能可互换的零部件, 提出了一个研究零部件归并问题的数学模型. 在对零部件质量要求不同而供应商根据购买量提供的折扣不同的情形下, 该模型可以帮助采购经理决定如何将零部件归并后统一购买以使得总购置成本最低.

关键词 零部件归并; 采购; 参数示例; 总购置成本

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