Existence results for a class of parabolic evolution equations in Banach spaces

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Abstract: We discuss the existence results of the parabolic evolution equation $\mathrm{d}(x(t)+g(t,x(t)))/\mathrm{d}t+A(t)x(t)=f(t,x(t))$ in Banach spaces, where $A(t)$ generates an evolution system and functions f,g are continuous. We get the theorem of existence of a mild solution, the theorem of existence and uniqueness of a mild solution and the theorem of existence and uniqueness of an S-classical (semi-classical) solution. We extend the cases when $g(t)=0$ or $A(t)=A$.

Key words: evolution system; analytic semigroup; mild solution; semi-classical solution; classical solution

1 Introduction and Preliminaries

The class of equations considered in this paper has the form

$$\left.\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}(x(t)+g(t,x(t)))+A(t)x(t)=f(t,x(t)) \qquad t>0\\&x(0)=x_0\end{aligned}\right\} \tag{1}$$

We consider this system as a Cauchy problem on a Banach space X, where $A(t)$ generates an evolution system; $f,g:[0,T]\times X\rightarrow X$ are appropriate continuous functions. The case $g\equiv 0$ has an extensive literature. See Refs.[1,2] and the references contained therein. The present paper is related to some results when $A(t)\equiv A$ in Refs.[3-5]. In particular in Ref.[3], the author gave the existence of a mild solution, a semi-classical solution and a classical solution.

Throughout this paper X will be a Banach space equipped with the norm $\|\cdot\|$. Let $\{A(t)\mid t\in[0,T]\}$ be a family of linear operators and satisfy:

① The domain $D(A(t))=D$ of $A(t)$, $0\leqslant t\leqslant T$ is dense in X and independent of t.

② For each $t\in[0,T]$, the resolvent $R(\lambda;A(t))$ of $A(t)$ exists for all λ with $\mathrm{Re}\lambda\leqslant 0$ and there is a constant $M\geqslant 0$ such that

$$\|R(\lambda;A(t))\|\leqslant\frac{M}{|\lambda|+1}\qquad \text{for } \mathrm{Re}\lambda\leqslant 0,\ t\in[0,T]$$

③ There exist constants $H>0$ and $0<\alpha\leqslant 1$ such that

$$\|(A(t)-A(s))A(\tau)^{-1}\|\leqslant H|t-s|^{\alpha}\qquad \text{for } s,t,\tau\in[0,T]$$

④ For each $t\in[0,T]$, and some $\lambda\in\rho(A(t))$, the resolvent $R(\lambda,A(t))$ of $A(t)$ is a compact operator.

We remark that ② and ① imply that for every $t\in[0,T]$, $-A(t)$ is the infinitesimal generator of an analytic semigroup[1] and together with condition ④ insure that this semigroup is compact for $t>0$[6].

Definition 1 A two parameter family of bounded linear operators $U(t,s)$, $0\leqslant s\leqslant t\leqslant T$ on X is called an evolution system if the following two conditions are satisfied:

(i) $U(s,s)=I$, $U(t,r)U(r,s)=U(t,s)$ for $0\leqslant s\leqslant r\leqslant t\leqslant T$;

(ii) $(t,s)\rightarrow U(t,s)$ is strongly continuous for $0\leqslant s\leqslant t\leqslant T$.

Lemma 1[1] Under the assumptions ①-③, there is a unique evolution system $U(t,s)$ on $0\leqslant s\leqslant t\leqslant T$, satisfying:

(i) $\|U(t,s)\|\leqslant C$ for $0\leqslant s\leqslant t\leqslant T$;

(ii) For $0\leqslant s<t\leqslant T$, $U(t,s):X\rightarrow D$ and $t\rightarrow U(t,s)$ are strongly differentiable in X. The derivative $\frac{\partial}{\partial t}U(t,s)\in B(X)$ and it is strongly continuous on $0\leqslant s<t\leqslant T$.

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Moreover,

$$\frac{\partial}{\partial t}U(t,s) + A(t)U(t,s) = 0 \qquad \text{for } 0 \leqslant s < t \leqslant T$$

$$\left\| \frac{\partial}{\partial t}U(t,s) \right\| = \| A(t)U(t,s) \| \leqslant \frac{C}{t-s}$$

and

$$\| A(t)U(t,s)A(s)^{-1} \| \leqslant C \qquad \text{for } 0 \leqslant s \leqslant t \leqslant T$$

(iii) For every $v \in D$ and $t \in [0,T]$, $U(t,s)v$ is differentiable with respect to s on $0 \leqslant s \leqslant t \leqslant T$ and

$$\frac{\partial}{\partial s}U(t,s)v = U(t,s)A(s)v$$

Lemma 2[7] Let $\{A(t) \mid 0 \leqslant t \leqslant T\}$ satisfy conditions ①–④. If $\{U(t,s) \mid 0 \leqslant s \leqslant t \leqslant T\}$ is the linear evolution system generated by $\{A(t)\}$, then $U(t,s)$ is a compact operator whenever $t-s>0$.

Definition 2 A function $x \in C([0,r]:X)$ is called a mild solution of the abstract Cauchy problem (1) if the following holds: $x(0)=x_0$; for each $0 \leqslant t < r$ and $s \in [0,t)$ the function $U(t,s)A(s)g(s,x(s))$ is integrable and

$$x(t) = U(t,0)(x_0 + g(0,x_0)) - g(t,x(t)) + \int_0^t U(t,s)A(s)g(s,x(s))\mathrm{d}s + \int_0^t U(t,s)f(s,x(s))\mathrm{d}s$$

Definition 3 A function $x \in C([0,r):X)$ is an S-classical (semi-classical) solution of the abstract Cauchy problem (1) if $x(0)=x_0$, $\frac{\mathrm{d}}{\mathrm{d}t}(x(t)+g(t,x(t)))$ is continuous on $(0,r)$, $x(t) \in D$ for all $t \in (0,r)$ and $x(\cdot)$ satisfies (1) on $(0,r)$.

Definition 4 A function $x \in C([0,r):X)$ is called a classical solution of the abstract Cauchy problem (1) if $x(0)=x_0$, $x(t) \in D$ for all $t \in (0,r)$, $\dot{x}$ is continuous on $(0,r)$, and $x(\cdot)$ satisfies (1) on $(0,r)$.

Lemma 3[1] Let $\{A(t)\}_{t\in[0,T]}$ satisfy the conditions ①–③ and let $U(t,s)$ be the evolution system provided by lemma 1. If f is Hölder continuous on $[s,t]$, then the initial value problem

$$\left.\begin{aligned} &\frac{\mathrm{d}u(t)}{\mathrm{d}t} + A(t)u(t) = f(t) \qquad \text{for } 0 \leqslant s < t \leqslant T \\ &u(s) = x \end{aligned}\right\} \tag{2}$$

has, for every $x \in X$, a unique solution u given by

$$u(t) = U(t,s)x + \int_s^t U(t,\sigma)f(\sigma)\mathrm{d}\sigma$$

For a function $\zeta \in C([0,a]:X)$ and $0 < t < a$, we will employ the notation

$$\| \zeta(\cdot) \|_t = \sup\{ \| \zeta(s) \| : s \in [0,t] \}$$

Finally for $x_0 \in X$, we will use the notation $x(\cdot,x_0)$ for the mild solution of (1).

2 Main Results

In this section, we will give three theorems about the existence results for the abstract Cauchy problem (1).

Theorem 1 Let $x_0 \in X$, ①–④ are satisfied and assume that the following conditions hold:

(a) The function $g \in D(A(0)) = D$ and there exists $L \geqslant 0$ such that $\| A(0)g(t,x) - A(0)g(s,y) \| \leqslant L(|t-s| + \|x-y\|)$ for every $0 \leqslant s,t \leqslant T$ and $x,y \in X$; moreover, $\| A(0)^{-1} \| L = \mu < 1$.

(b) The function f is continuous and takes bounded sets into bounded sets.

Then there exists a mild solution $x(\cdot,x_0)$ of the abstract Cauchy problem (1) defined on $[0,r]$ for some $0 < r < T$.

Proof Let $0 < r_1 < T$, $\delta > 0$ such that

$$V = \{(t,x(t)) \in [0,r_1] \times X : \| x(t) - x_0 \| \leqslant \delta\}$$

Assuming that the function f is bounded on V by $C_1 > 0$, with ③ we know that $A(s)A(0)^{-1}$ is bounded by $C_2 > 0$.

Choose $0 < r < r_1$, such that

$$\| (U(t,0) - I)x_0 \| \leqslant \frac{1-\mu}{6}\delta \tag{3}$$

$$\|A(0)^{-1}\| Lr + CC_2 Lr^2 + CC_1 r \leqslant \frac{1-\mu}{6}\delta \tag{4}$$

$$CC_2 Lr \leqslant \frac{1-\mu}{6} \tag{5}$$

Define the set

$$S = \{x \in C([0,r]:X) \mid x(0) = x_0, \|x(t) - x_0\| \leqslant \delta,\ t \in [0,r]\}$$

It is easy to see that S is a nonempty bounded, closed and convex subset of $C([0,r]:X)$.

We define the operator:

$$Tx(t) = U(t,0)(x_0 + g(0,x_0)) - g(t,x(t)) + \int_0^t U(t,s)A(s)g(s,x(s))\mathrm{d}s + \int_0^t U(t,s)f(s,x(s))\mathrm{d}s$$

For the mapping T we consider the decomposition $T = T_1 + T_2$, where

$$T_1 x(t) = U(t,0)(x_0 + g(0,x_0)) - g(t,x(t)) + \int_0^t U(t,s)A(s)g(s,x(s))\mathrm{d}s$$

$$T_2 x(t) = \int_0^t U(t,s)f(s,x(s))\mathrm{d}s$$

Next we will prove that T_1 and T_2 are well defined in S, and T_1 is a contraction mapping and T_2 is compact.

It is clear that $U(t,s)A(s)g(s,x(s))$ is continuous on $s \in [0,t)$ and

$$\|A(0)g(t,x(t))\| \leqslant \|A(0)g(t,x(t)) - A(0)g(0,x_0)\| + \|A(0)g(0,x_0)\| \leqslant L(r_1 + \delta) + \|A(0)g(0,x_0)\|$$

So,

$$\|U(t,s)A(s)g(s,x(s))\| = \|U(t,s)A(s)A(0)^{-1}A(0)g(s,x(s))\| \leqslant CC_2(L(r+\delta) + \|A(0)g(0,x_0)\|)$$

which implies that $\|U(t,s)A(s)g(s,x(s))\|$ is integrable on $[0,t)$. We thus conclude that T_1 is well defined and with values in $C([0,r):X)$. Samely, $U(t,s)f(s,x(s))$ is continuous on $[0,t)$, $s \in [0,t)$ and $\|U(t,s)f(s,x(s))\| \leqslant CC_1$ if $x \in S$, which implies that T_2 is well defined and with values in $C([0,r]:X)$.

Let $x, y \in S$, then for $t \in [0,r]$, with (3)–(5) we have

$$\begin{aligned}\|T_1 x(t) + T_2 y(t) - x_0\| &= \|(U(t,0) - I)x_0\| + \|g(0,x_0) - g(t,x(t))\| + \\ &\int_0^t \|U(t,s)A(s)(g(s,x(s)) - g(0,x_0))\|\mathrm{d}s + \int_0^t \|U(t,s)f(s,y(s))\|\mathrm{d}s \leqslant \\ &\|(U(t,0) - I)x_0\| + \|A(0)^{-1}\| L(r+\delta) + CC_2 L(r+\delta)r + CC_1 r \leqslant \frac{1-\mu}{6}\delta + \\ &\mu\delta + \frac{1-\mu}{6}\delta + \frac{1-\mu}{6}\delta \leqslant \delta\end{aligned}$$

Moreover,

$$\begin{aligned}\|T_1 x(t) - T_1 y(t)\| &\leqslant \|g(t,x(t)) - g(t,y(t))\| + \int_0^t \|U(t,s)A(s)(g(s,x(s)) - g(s,y(s)))\|\mathrm{d}s \leqslant \\ &\|A(0)^{-1}\| L\|x(t) - y(t)\| + \int_0^t CC_2 L\|x(s) - y(s)\|\mathrm{d}s \leqslant (\|A(0)^{-1}\| L + CC_2 Lr)\|x - y\|_r\end{aligned}$$

Thus,

$$\|T_1 x - T_1 y\|_r \leqslant (\|A(0)^{-1}\| L + CC_2 Lr)\|x - y\|_r$$

The estimate (5) and $\mu < 1$ imply that T_1 is a contraction mapping on S.

Now, we shall show that T_2 is compact.

By lemma 2 we know that $U(t,s)$ is a compact operator whenever $t > s$.

Let $0 < \varepsilon < t$, for $x \in S$, we define

$$T_{2,\varepsilon}x(t) = \int_0^{t-\varepsilon} U(t,s)f(s,x(s))\mathrm{d}s = U(t,t-\varepsilon)\int_0^{t-\varepsilon} U(t-\varepsilon,s)f(s,x(s))\,\mathrm{d}s$$

The set $\{T_{2,\varepsilon}x(t)\}_{x\in S}$ is precompact in X because $U(t,t-\varepsilon)$ is compact for $0 < \varepsilon < t$ and $\int_0^{t-\varepsilon} U(t-\varepsilon,s)f(s,x(s))\mathrm{d}s$ is bounded in S.

We observe that

$$\| T_{2,\varepsilon}x(t) - Tx(t)\| \leqslant \int_{t-\varepsilon}^{t} \| U(t,s)f(s,x(s))\| \mathrm{d}s \leqslant CC_1\varepsilon$$

which implies that $\{T_2x(t)\}_{x\in S}$ is precompact.

For any $t_1 < t_2 \in (0,r)$, we have

$$\| T_2x(t_1) - T_2x(t_2)\| = \left\| \int_0^{t_1} U(t_1,s)f(s,x(s))\mathrm{d}s - \int_0^{t_2} U(t_2,s)f(s,x(s))\mathrm{d}s \right\| \leqslant$$

$$\int_0^{t_1} \| (U(t_1,s) - U(t_2,s))f(s,x(s))\| \mathrm{d}s + \int_{t_1}^{t_2} \| U(t_2,s)f(s,x(s))\| \mathrm{d}s \leqslant$$

$$C_1\int_0^{t_1} \| U(t_1,s) - U(t_2,s)\| \mathrm{d}s + CC_1|t_1 - t_2|$$

which implies that $T_2x(t)$ is equicontinuous. From Arzela-Ascoli's theorem we know that T_2 is compact.

Using an extended fixed point theorem, we know that T has a fixed point $x(t)$ in S. This fixed point is the desired mild solution of the abstract Cauchy problem (1).

Theorem 2 Assume that $\{A(t)\}$ satisfy ①–③. Let $x_0 \in X$ and the following conditions hold:

(a) The function $g \in D(A(0)) = D$ and there exists $L \geqslant 0$ such that $\| A(0)g(t,x) - A(0)g(s,y)\| \leqslant L(|t-s| + \|x-y\|)$ for every $0 \leqslant s,t \leqslant T$ and $x,y \in X$; moreover, $\| A(0)^{-1}\| L = \mu < 1$.

(b) The function f is continuous and there exists $N > 0$ such that $\| f(t,x) - f(s,y)\| \leqslant N(|t-s| + \|x-y\|)$ for $0 \leqslant s,t \leqslant T$ and $x,y \in X$.

Then there exists a unique mild solution $x(\cdot,x_0)$ of the abstract problem (1) defined on $[0,r]$ for some $0 < r < T$.

Proof The proof is given in two steps.

Step 1 First we consider the Cauchy problem

$$\left.\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}(x(t) + g(t,x(t))) + A(t)x(t) = h(t) \\ &x(0) = x_0 \end{aligned}\right\} \tag{6}$$

where $h(t)$ is continuous. Let $0 < r_1 < T$, $\delta > 0$ such that

$$V = \{(t,x(t)) \in [0,r_1] \times X : \| x(t) - x_0\| \leqslant \delta\}$$

So, $h(t)$ is bounded on $[0,r_1]$, assume that $\| h(t)\| \leqslant C_1'$.

For $(t,x(t)) \in V$, we have

$$\| A(0)g(t,x(t))\| \leqslant \| A(0)g(t,x(t)) - A(0)g(0,x_0)\| + \| A(0)g(0,x_0)\| \leqslant$$

$$L(r_1 + \delta) + \| A(0)g(0,x_0)\|$$

$$\| f(t,x(t))\| \leqslant \| f(t,x(t)) - f(0,x_0)\| + \| f(0,x_0)\| \leqslant N(r_1 + \delta) + \| f(0,x_0)\|$$

Let $C_1 = \max\{C_1'; L(r_1+\delta) + \| A(0)g(0,x_0)\|; N(r_1+\delta) + \| f(0,x_0)\|\}$, so h, f and $A(0)g$ are bounded on V by $C_1 > 0$. With the condition ③ we know that $A(s)A(0)^{-1}$ is bounded by $C_2 > 0$.

Choose $0 < r < r_1$ such that

$$\| (U(t,0) - I)x_0\| \leqslant \frac{1-\mu}{6}\delta \tag{7}$$

$$\| A(0)^{-1}\| Lr + CC_2Lr^2 + CC_1r \leqslant \frac{1-\mu}{6}\delta \tag{8}$$

$$CC_2Lr \leqslant \frac{1-\mu}{6} \tag{9}$$

$$CNr < 1 \tag{10}$$

Define the set

$$S = \{x \in C([0,r]:X) \mid x(0) = x_0,\ \| x(t) - x_0\| \leqslant \delta, t \in [0,r]\}$$

It is easy to see that S is a nonempty bounded, closed and convex subset of $C([0,r]:X)$.

We define the operator:

$$Tx(t) = U(t,0)(x_0 + g(0,x_0)) - g(t,x(t)) + \int_0^t U(t,s)A(s)g(s,x(s))\mathrm{d}s + \int_0^t U(t,s)h(s)\mathrm{d}s$$

The same as in theorem 1 we can see that T is well defined and with values in $C([0,r]:X)$. Let $x \in S$, with (7)–(8) we have

$$\|Tx(t)-x_0\| \leqslant \|(U(t,0)-I)x_0\| + \|g(0,x_0)-g(t,x(t))\| + \int_0^t \|U(t,s)A(s)(g(s,x(s))-g(0,x_0))\|\mathrm{d}s + \int_0^t \|U(t,s)h(s)\|\mathrm{d}s \leqslant \|(U(t,0)-I)x_0\| + \|A(0)^{-1}\|L(r+\delta) + CC_2L(r+\delta)r + CC_1 r \leqslant \frac{1-\mu}{6}\delta + \mu\delta + \frac{1-\mu}{6}\delta + \frac{1-\mu}{6}\delta \leqslant \delta$$

Therefore $T: S \to S$.

Furthermore, for any $x, y \in S$, we have

$$\|Tx(t)-Ty(t)\| \leqslant \|g(t,x(t))-g(t,y(t))\| + \int_0^t \|U(t,s)A(s)(g(s,x(s))-g(s,y(s)))\|\mathrm{d}s \leqslant \|A(0)^{-1}\|L\|x(t)-y(t)\| + \int_0^t CC_2L\|x(s)-y(s)\|\mathrm{d}s \leqslant (\|A(0)^{-1}\|L + CC_2Lr)\|x-y\|_r$$

Thus,

$$\|Tx-Ty\|_r \leqslant (\|A(0)^{-1}\|L + CC_2Lr)\|x-y\|_r$$

The estimate (9) and $\mu < 1$ imply that T is a contraction mapping on S.

By the Banach contraction principle T has a unique fixed point $x(t)$ in S. This fixed point is the mild solution of (6).

Step 2 Define the solution operator K by $Kh(t) = x(t)$, here $x(t)$ is the mild solution of (6). Let $Gx(t) = Kf(t,x(t))$. Next we prove that G has a fixed point in S.

For any continuous functions $h_1(t)$, $h_2(t)$, $t \in [0,r]$, we have

$$\|Kh_1(t)-Kh_2(t)\| \leqslant \int_0^t \|U(t,s)(h_1(s)-h_2(s))\|\mathrm{d}s \leqslant Cr\|h_1-h_2\|_r$$

Then for any $x, y \in S$,

$$\|Gx(t)-Gy(t)\| = \|Kf(t,x(t))-Kf(t,y(t))\| \leqslant Cr\|f(t,x(t))-f(t,y(t))\|_r \leqslant CNr\|x-y\|_r$$

So,

$$\|Gx-Gy\|_r \leqslant CNr\|x-y\|_r$$

The estimate (10) implies that G is a contraction mapping.

And if $\|x(t)-x_0\| \leqslant \delta$, then $\|f(t,x(t))\| \leqslant C_1$, from step 1 we can see that $G: S \to S$.

By the Banach contraction principle we conclude that G has a unique fixed point in S, and this fixed point is the desired mild solution of (1).

In fact, we can prove that the unique mild solution is a semi-classical solution. We state it out as a theorem.

Theorem 3 Under the assumptions in theorem 2, if $x_0 \in D$, then there exists a unique S-classical solution $x(\cdot, x_0) \in C([0,r]:X)$ for some $0 < r < T$.

Proof From theorem 2 we get the mild solution of (1)

$$x(t) = U(t,0)(x_0+g(0,x_0)) - g(t,x(t)) + \int_0^t U(t,s)A(s)g(s,x(s))\mathrm{d}s + \int_0^t U(t,s)f(s,x(s))\mathrm{d}s$$

We affirm that $x(\cdot)$ is locally Hölder continuous. We shall make use of the following inequalities[8]:

$$\|U(t+h,\tau)-U(t,\tau)\| \leqslant \frac{Mh^\gamma}{|t-\tau|^\gamma} \qquad \text{if } 0 < h < 1,\ |t-\tau| \geqslant h \text{ and } 0 < \gamma < 1$$

For any $0 < h < t-s$ sufficiently small, $t \in [\epsilon, r)$, $\forall \epsilon > 0$, we have

$$\|x(t+h)-x(t)\| \leqslant \|(U(t+h,0)-U(t,0))(x_0+g(0,x_0))\| + \|g(t+h,x(t+h))-g(t,x(t))\| + \int_0^t \|(U(t+h,s)-U(t,s))A(s)(g(s,x(s))\|\mathrm{d}s + \int_t^{t+h} \|U(t+h,s)A(s)(g(s,x(s))\|\mathrm{d}s + \int_0^t \|(U(t+h,s)-U(t,s))f(s,x(s))\|\mathrm{d}s + \int_t^{t+h} \|U(t+h,s)f(s,x(s))\|\mathrm{d}s \leqslant \frac{C}{\epsilon}\|(x_0+g(0,x_0))\|h + \|A(0)^{-1}\|L(h+\|x(t+h)-x(t)\|) + C_2C_1M\frac{r^{1-\gamma}}{1-\gamma}h^\gamma + CC_2C_1h + C_1M\frac{r^{1-\gamma}}{1-\gamma}h^\gamma + CC_1h$$

The last inequality can be rewritten in the form

$$\| x(t+h) - x(t) \| \leqslant \frac{P(t,x_0)}{1-\mu} h^{\gamma}$$

Since $\mu = L\| A(0)^{-1} \| N < 1$. Therefore the function $x(\cdot)$ is locally γ-Hölder continuous on $(0,r)$. Now it is easy to show that $t \rightarrow A(t)g(t,x(t))$ and $t \rightarrow f(t,x(t))$ are ρ-Hölder continuous on $(0,r)$, where $\rho = \min\{\alpha, \gamma\}$. So, $f(t,x(t)) + A(t)g(t,x(t))$ is ρ-Hölder continuous on $(0,r)$. From lemma 3, we conclude that $x(t) + g(t,x(t))$ is a C^1 function on $(0,r)$. The proof is completed.

Remark Assume that $g(\cdot) \in D(A)$, f and g are continuously differential functions on $[0,T] \times X$, then $x(\cdot, x_0)$ is a classical solution.

References

[1] Pazy A. *Semigroups of linear operators and applications to partial differential equations*[M]. New York: Springer-Verlag, 1983.

[2] Goldstein Jerome A. *Semigroups of linear operators and applications*[M]. New York: The Clarendon Press, Oxford University Press, 1985.

[3] Eduardo Hernández M. Existence results for a class of semi-linear evolution equations[J]. *Electronic Journal of Differential Equations*, 2001(24):1－14.

[4] Eduardo Hernández, Henríquez Hernán R. Existence of periodic solutions of partial neutral functional-differential equations with unbounded delay[J]. *J Math Anal Appl*, 1998, **221**(2):499－522.

[5] Eduardo Hernández, Henríquez Hernán R. Existence results for partial neutral functional-differential equations with unbounded delay[J]. *J Math Anal Appl*, 1998, **221**(2): 452－475.

[6] Hale J. *Functional differential equations*[M]. New York: Springer-Verlag, 1971.

[7] Fitzgibbon W E. Semilinear functional differential equations in Banach space[J]. *Journal of Differential Equations*, 1978, **29**:1－14.

[8] Friedman A. *Partial differential equations*[M]. New York: Holt, Rinehart and Winston Press, 1969.

Banach 空间中一类半线性发展方程的存在结果

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摘　要　讨论了 Banach 空间中抛物发展方程 $d(x(t)+g(t,(x)))/dt + A(t)x(t) = f(t,x(t))$ 的存在结果，这里 $A(t)$ 生成一个发展系统，函数 f,g 是连续的．笔者分别给出适度解定理，适度解存在惟一性定理和半古典解存在惟一性定理，推广了前人 $g(t) \equiv 0$ 或 $A(t) \equiv A$ 的结果．

关键词　发展系统；解析半群；适度解；半古典解；古典解

中图分类号　O177.2