Journal of Southeast University (English Edition) Vol.19 No.4 Dec. 2003 ISSN 1003—7985

# Mathematical model and calculation for heat transfer during condensation on surfaces of corrugated plates

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**Abstract**: A mathematical model was established for condensation on surfaces of vertical corrugated plates based on the mechanism of heat transfer enhancement to thin down the liquid film due to surface tension effect between corrugated plate surfaces and liquid films. The relative heat transfer coefficients of condensation on corrugation plates were calculated in contrast with equivalent vertical plane ones. The heat transfer enhancement effects for the main geometric parameters such as pitch, height, corrugation angle, tilt angle, and fillet radii of corrugations were analyzed to guide the optimization of corrugation structure for application. A two-scale corrugation is suggested, which can compromise both the enhanced heat transfer effect and adequate cross section area for flows, and it makes the heat transfer coefficient 1 to 2 times more than that of an equivalent plane one.

**Key words**: condensation; enhancement of heat transfer; corrugated plates; plate-shell heat exchanger

The plate-shell heat exchanger (PSHE) is a kind of heat transfer equipment, which combines the features of high efficiency and compactness of plate heat exchanger (PHE) and the flexibility of a tube-shell heat exchanger (TSHE)[1,2]. It is suitable for use as a condenser, which can be arranged both vertically and horizontally. The condensation takes place usually on the shell-side surfaces of the corrugated plates, which form rows of wavy channels. When vertically arranged, the heat transfer plate bank is encased in a confined channel, and the steam flows downwards in a forced flow manner, so as to thin down the condensate film. This situation is just similar to what happens in a PHE condenser, which has been well studied before[3,4]. While in the horizontal arrangement, the same method might cause the condensate flood in some of the bottom part of plates, because the direction of horizontal flow is crosswise to that of the gravity, thus the heat transfer effect deteriorates. To solve the problem, the plate bank should be open at both top and bottom sides, which is a new situation and requires further study.

The method of heat transfer enhancement for condensation process by corrugated plate wall is mainly due to the surface tension effect which can thin down the liquid condensate film at corrugation peaks as well as their side slopes, thus decreasing the conduction resistance of the film. Simultaneously the corrugation channels also enhances heat transfer from cooling water to the wall at other side of the plates, thus the overall heat transfer coefficient of the condenser can be increased.

The heat transfer and fluid flow in the condensation process on these corrugated plate walls is very complicated and it is difficult to calculate because of the 3-D flows in the wavy channels as shown in Fig.1. Therefore it is usual to break down the process to two steps, i.e. first to treat it as film condensation

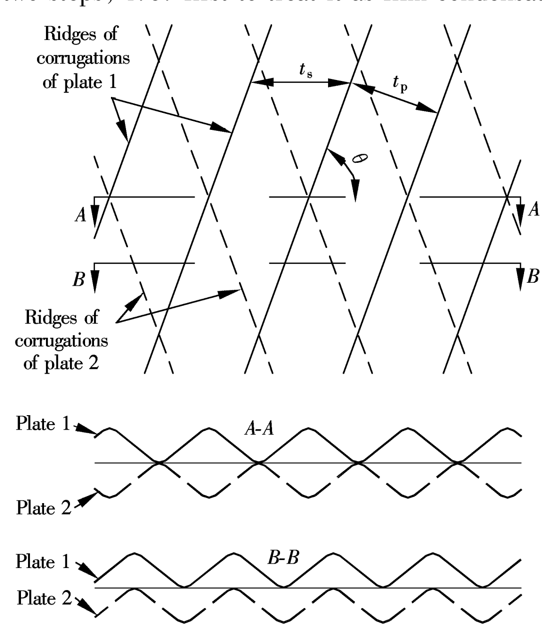


**Fig.1** Cross corrugated plate channels

**Received** 2003-06-18.

**Foundation items**: The National Key Fundamental Research Program of China (G2000026303) and the National Natural Science Foundation of China (50176008).

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on vertical plane plates, and then to consider the enhancement effect due to the corrugation of the plates. Since the first problem has already been solved with existing formulae[5-7], this paper mainly discusses the second one.

## 1 Mathematical Model for Film Condensation on Corrugated Plates

To analyze the enhancement effect, it is simplified by converting the real cross corrugated plate channels to vertically waved plate channels, neglecting the contact effect of two plates at ridges of corrugations. Fig.2 shows the film condensation model for a pitch of

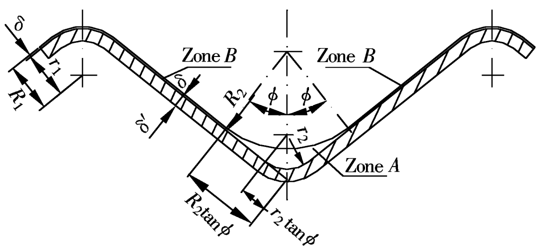


**Fig.2** Condensation model for a pitch of corrugation

triangular corrugation. Fig.3 shows the flow coordinates for a liquid film control element on a simplified vertical corrugation plate. Also the following assumptions are postulated. ① The thickness of the film is the same on the walls at ridges and at slopes of a corrugation, and the film there forms zone $B$, while the remaining part forms zone $A$ as shown in Fig.2. ② The condensate flows along the vertical corrugation by the component gravitational force $\rho g \sin\theta$. ③ The properties of steam and condensate are constant.

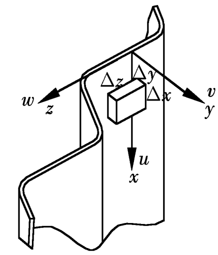


**Fig.3** Flow coordinates

Comparison is made for the results of condensation on a pitch of corrugation plate of unfolded width $l_0$ and height $H$, with those on an equivalent plane one. To keep the same heat flux for two types of plates, the temperature difference between steam and wall for the later case has to be greater than that of the former one.

From the continuity equation, with the same heat flux, the total condensate flow for both cases should be the same.

$$u_A A_A \rho + u_B A_B \rho = u_0 A_0 \rho \tag{1}$$

where $\rho$ is density of the condensate; $A$ is the cross section area of condensate film, the subscripts $A$, $B$, $0$ describe properties at zone $A$, zone $B$ for corrugated plate and at the equivalent flat plate, respectively. And they could be calculated with the formulae in Tab.1.

**Tab.1** Formulae of some parameters

| Parameters | A pitch of corrugation plate | | Equivalent flat plate |
|---|---|---|---|
| | Zone $A$ | Zone $B$ | |
| Unfolded width of boundary wall | $l_A = 2(R_2 - r_2)\tan\phi + \frac{\pi}{90} r_2 \phi$ | $l_B = l_0 - l_A$ | $l_0$ |
| Mean liquid film thickness | $\delta_A = \frac{A_A}{l_A}$ | $\delta_B$ | $\delta_0$ |
| Cross section area of condensate film | $A_A = 2\delta_B R_2 \tan\phi + (R_2^2 - r_2^2)\left(\tan\phi - \frac{\pi}{180}\phi\right)$ | $A_B = l_B \delta_B$ | $A_0 = l_0 \delta_0$ |
| Component velocity of condensate film | $u_A = \frac{g\sin\theta\, \delta_A^2}{3\nu}$ | $u_B = \frac{g\sin\theta\, \delta_B^2}{3\nu}$ | $u_0 = \frac{g\delta_0^2}{3\nu}$ |

From Eq.(1), it can be derived that:

$$\frac{\delta_0}{\delta_B} = \left\{\left[\frac{l_B}{l_0} + \left(\frac{\delta_A}{\delta_B}\right)^3 \frac{l_A}{l_0}\right]\sin\theta\right\}^{\frac{1}{3}} \tag{2}$$

$$\frac{\alpha}{\alpha_0} = \frac{\delta_0}{\delta_B}\left(\frac{l_B}{l_0} + \frac{l_A}{l_0}\frac{\delta_B}{\delta_A}\right) \tag{3}$$

For the control element shown as in Fig.3, the pressure difference due to the surface tension effect is the cause of drawing the liquid film to the valley, and it is given as

$$\frac{\mathrm{d}p}{\mathrm{d}z} = \frac{\Delta p}{\frac{l_B}{2}} = \frac{2\sigma}{l_B}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{4}$$

where $\sigma$ is strain coefficent of the condensate; $R_1$, $R_2$ are radii of the interface between condensate film and steam at ridge and at valley, respectively. With equilibrium of the active force and resistant drag, it is given as

$$\frac{2\sigma(\delta_B - y)}{l_B}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \mu\frac{\mathrm{d}w}{\mathrm{d}y} \tag{5}$$

In the condensate film, the component velocity $w$ varies with $y$, and its mean value can be integrated as

$$w = \frac{1}{\delta_B}\int_0^{w_\delta}(\delta_B - y)\mathrm{d}w = \frac{2\sigma}{\delta\,\mu\, l_B}\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

$$\int_0^{\delta_B} (\delta_B - y)^2 \mathrm{d}y = \frac{2\sigma\delta_B^2}{3\mu l_B}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{6}$$

where $\mu$ is dynamic viscosity of the condensate. At any $x$, the condensate flow at zone $A$ is

$$G_A(x) = \int_0^x \rho\delta_B w \mathrm{d}x = \frac{2\sigma\delta_B^3 x}{3\nu l_B}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{7}$$

where $\nu$ is kinetic viscosity of the condensate. Assuming the condensate flow at zone $A$ is linear, the mean values of flow parameters are those at the midpoint of the plate.

$$G_A\left(\frac{H}{2}\right) = \rho A_A u_A = \rho l_A \delta_A^3 \frac{g\sin\theta}{3\nu} = \frac{\sigma\delta_B^3 H}{3\nu l_B}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{8}$$

therefore

$$\delta_B = \left[\frac{\rho g \sin\theta l_A l_B R_1 R_2 \delta_A{}^3}{\sigma H(R_1 + R_2)}\right]^{\frac{1}{3}} \tag{9}$$

Eq. (9) reveals the relationship between $\delta_B$ and $R_2$, which can be solved by trial method. The heat flux of condensation on unit area is

$$q = \frac{G(H)r}{Hl_0} \approx \frac{2G\left(\frac{H}{2} + G_B\right)}{Hl_0} r \tag{10}$$

## 2 Calculation and Results Analysis

### 2.1 Characteristics of a certain corrugation plate

Fig.4 shows the results of condensation on a certain corrugation plate (corrugation angle $\phi = 40°$, tilt angle $\theta = 60°$, unfolded width of a pitch of corrugation $l_0 = 20$ mm, inner fillet radius $r_2 = 1.5$ mm, thickness of the plate $\delta' = 0.5$ mm, height of the corrugated plate $H = 300$ mm). It shows that the thickness of liquid film at zone $B$ is much smaller than that at the equivalent flat plate, which enhances heat transfer, while the area of zone $A$ is limited, thus a net gain in efficiency is ensured.

### 2.2 Effects of geometric parameters of corrugation

Fig.5 shows that the film thickness at zone $B$ of plates is very sensitive to the size of corrugation, the smaller the size, the thinner the thickness, other things being equal. Thus the results of condensation enhancement effect on corrugation plates is also sensitive to the corrugation size as shown in Fig.6, the smaller the size, the greater the enhancement effects. Fig.6 also reveals that a very small size corrugation plate is easily flooded at greater heat flux, especially when the height of the plate is greater. By comparison

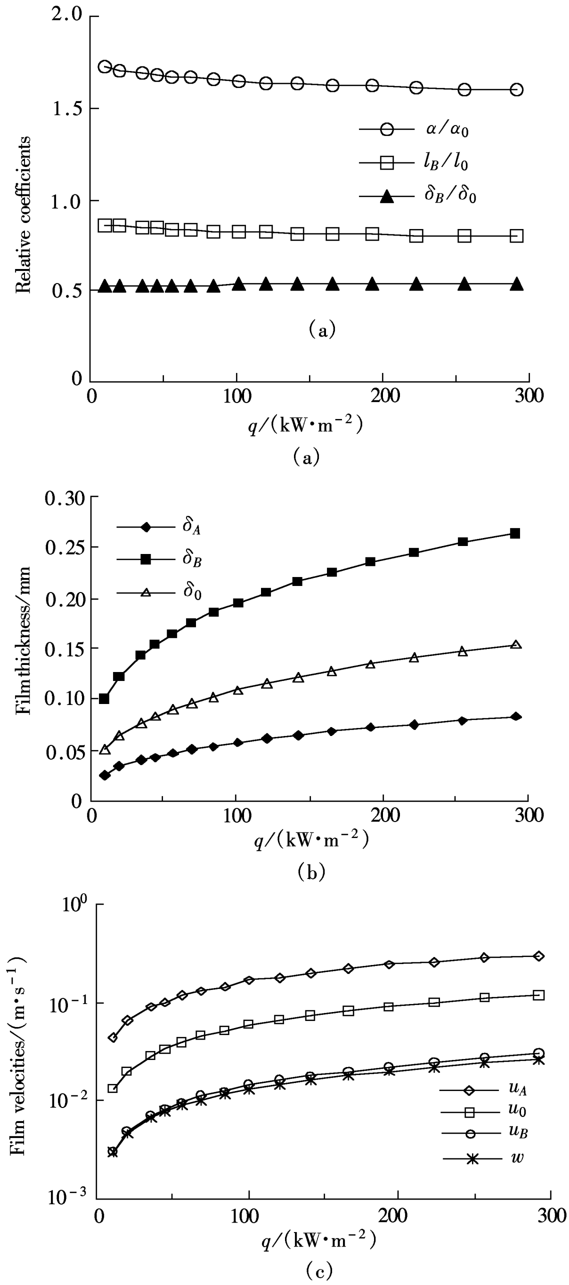


**Fig.4** Results of certain corrugation plates

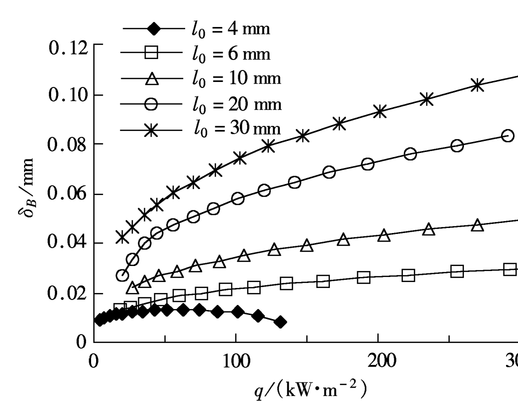


**Fig.5** $\delta_B$ vs. $q$ with different $l_0$ at $H = 300$ mm, $\phi = 40°$, $\theta = 60°$, $r_2 = 1.5$ mm

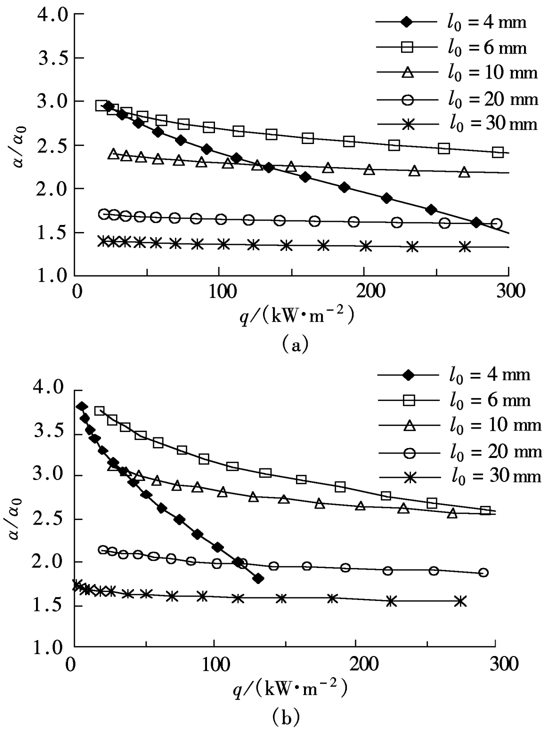


**Fig.6** $\alpha/\alpha_0$ vs. $q$ with different $l_0$ at $\phi = 40°, \theta = 60°$, $r_2 = 1.5$ mm, $\delta' = 0.5$ mm. (a) $H = 300$ mm; (b) $H = 600$ mm

of Fig.6(a) and Fig.6(b), it seems that a taller plate has a greater enhancement effect. However, because the heat transfer coefficient during condensation on a vertical plane wall is proportional to $H^{-\frac{1}{4}}$, actually the height of the corrugation plate has little effect on the heat transfer coefficient.

Eq.(5) shows that the pressure difference due to surface tension effect which draws liquid film to valleys is mainly determined by the radii of the liquid film at both peaks and valleys of corrugation, and therefore the heat transfer enhancement effect is related to the structural parameters $r_1$ and $r_2$ as shown in Fig.7.

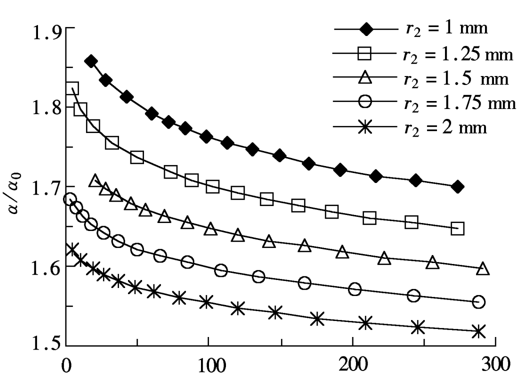


**Fig.7** $\alpha/\alpha_0$ vs. $q$ with different $r_2$ at $\phi = 40°, \theta = 60°$, $l_0 = 20$ mm, $H = 300$ mm

Although smaller radii benefit from the enhancement effect, they are often limited by the stretch property and structural strength of the plate material.

Corrugation angle $\phi$ (see Fig.2) represents the sharpness of corrugation. The greater the angle, the sharper the corrugation, thus the stronger the surface tension effect and higher the heat transfer coefficient as shown in Fig.8. However, it is also limited by the stretch property of the plate material.

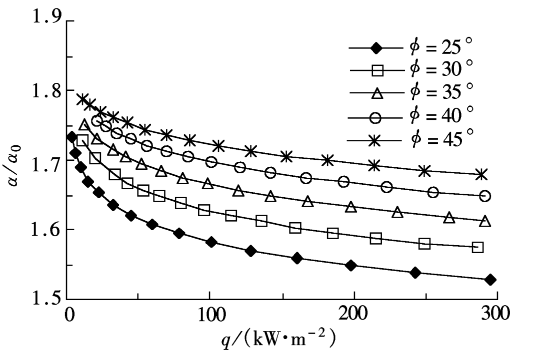


**Fig.8** $\alpha/\alpha_0$ vs. $q$ with different $\phi$ at $r_2 = 1.5$ mm, $\theta = 60°$, $l_0 = 20$ mm, $H = 300$ mm

Though actual tilt corrugation ridges are simplified as vertical ones in establishment of the mathematical model, still it is assumed that condensate flows along the corrugation by the component gravitational force $\rho g \sin\theta$. Therefore the effect of the tilt angle of corrugation can be calculated with the model; the results are shown in Fig.9. It can be seen that the closer the angle to 90°, the better the effect. Nevertheless, this will raise the resistance in the other side of the plate channels, so a moderate value of 60° is usually applicable.

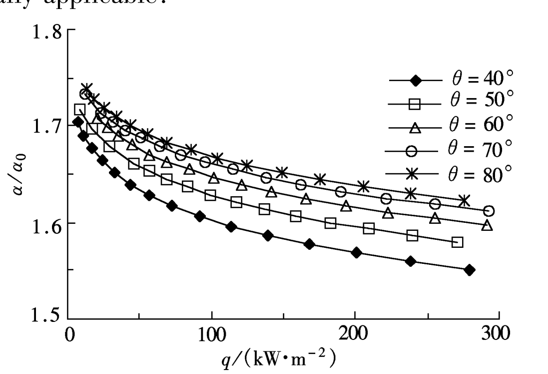


**Fig.9** $\alpha/\alpha_0$ vs. $q$ with different $\theta$ at $r_2 = 1.5$ mm, $\phi = 40°$, $l_0 = 20$ mm, $H = 300$ mm

## 3 Two-Scale Corrugation Plates with the Same Direction

Smaller corrugation size is desirable but not applicable on account of resistance, because the corrugation size often determines the cross section area of the flow in both sides of the plates. To solve the problem, the two-scale corrugation plates shown in

Fig.10 are designed for PSHE condensers, which can compromise the heat transfer enhancement effect and cross section area for both side flows. Thus a much higher heat transfer coefficient than that of an equivalent flat plate or on an ordinary corrugation plate can be expected. Experimental research on this kind of corrugated plates also shows promising results[8].

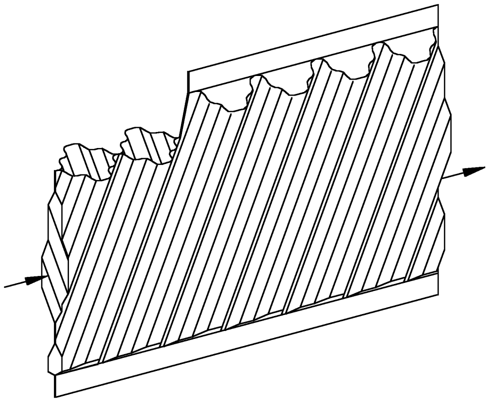

**Fig.10** Two-scale corrugation plates with the same direction

## 4 Conclusion

A plate-shell heat exchanger with combined features of high efficiency, compactness and flexibility can be used as a condenser. By theoretical calculation, the influences of the main geometric parameters of the corrugation are investigated, of which the corrugation size is the key factor. A two-scale corrugation with the same direction plate pattern is suggested for condensation plate-shell heat exchanger, which can compromise the enhanced heat transfer and adequate flow resistance requirements.

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# 波纹板表面凝结传热的数学模型与计算

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**摘　要**　基于波纹板表面与液膜之间的表面张力作用能减薄液膜厚度而强化传热的机理,建立了竖直波纹板表面的凝结过程的数学模型.计算了波纹板对应于当量平板的凝结传热增强系数.分析了波纹的主要几何参数如波纹节距、波纹板高度、波纹角、波纹倾斜角、波纹圆角半径等对强化传热的影响效应以指导波纹结构的优化.提出了一种能兼顾强化传热效果和适当的流动截面积的双尺度波纹,其凝结传热系数可比当量平板增强 1~2 倍.

**关键词**　凝结; 强化传热; 波纹板; 板壳式换热器

**中图分类号**　TK124