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Numerical simulation of bed degradation in alluvial channels

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Abstract: A numerical model to simulate the bed degradation process in a straight alluvial channel with respect to time and distance is introduced. The simulation takes into account the effect of non-uniformity of the bed material, and variations in the dimension of bed forms. The model predicts the changes in the grain size distribution with the time and space during degradation process. The numerical model proposes that the armoring process in degrading channels does not depend only on hydraulic characteristics of the flow but also on variation in the grain size distribution of sediments on the bed. The model was applied and compared with the results obtained from experiments conducted in 24 m recirculating flume for two sizes of sand; a good agreement was found between observed and calculated values.

Key words: degradation; armoring; numerical model

Whenever the rate of supply of sediment to a certain alluvial reach is less than the rate at which it transports the sediment, the deficiency sediment is usually picked from the bed and banks of the channel and will be transported downstream. As a result there are a general lowering in the streambed and an increase in the coarsening of the bed material while the sediment rate decreases with time. This phenomenon is known as degradation. Finally a new bed profile will form, the bed contains all the coarse particles that the following water is not able to remove and these particles lay on the bed surface forming a protective layer known as the armor layer, which prevents the under-laying finer materials from being transported, and may stop further degradation. This phenomena has been observed in many rivers in the world and investigated by several authors[1-3]; different analytical methods have been developed in order to evaluate the quantitative variation of the alluvial channels due to the degradation process, some of these models[4,5] are available but they are valid only for uniform bed material, neglecting the armoring process, which is a key factor in slowing the degradation rate. Koumura and Simons[6] used the same procedure as Tinney[4] but they took the armoring effect into account, Gessler[7], Ashida and Michine[8], Little and Mayer[9] are among the first who systematically studied the process of bed armoring. Recently Borah, et al.[10], Shen and Lu[11] tried to predict the grain size distribution of the final armor coat. Park and Jain[12] simulated the plane bed armoring under degradation conditions. Othman[13] made a series of laboratory experiments and analyses on degradation under armoring condition. Most of the aforementioned studies analyzed degradation problems by taking into account the effect of non-uniformity of particles of the bed material. These studies can be classified into two categories. The first category analyzes the armoring process considering only sediment properties. The second considers that the evolution of the process depends only on flow characteristics. Thus in this paper an effort is made to introduce a numerical model to predict the bed profile in degrading channels taking into account the armoring effect under consideration that armoring process depends on both the sediment and flow properties. The validity of the model is checked by experimental data[13].

1 Governing Equations

Riverbed degradation is a long-term process so the actual water movement can be assumed to be a steady uniform flow. Thus the governing equations to solve the problem can be summarized as follows:

The continuity equation for water can be reduced to the form:

$$q(x) = \text{constant} \tag{1}$$

The equation of motion for water can similarly be reduced to a parabolic form[14]:

$$\frac{\partial z}{\partial x} = -n^2 q^2 / y^{\frac{10}{3}} \tag{2}$$

The equation of continuity for sediment transport:

$$\frac{\partial q_s}{\partial x} = -\frac{\partial z}{\partial t} \tag{3}$$

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where y is the flow depth; n is the manning's roughness coefficient; q is the specific flow rate per unit width; z is the bed level; x is the axis which is taken with the flow direction; t is the time and q_s is the rate of sediment transport of material per unit time and width. As the bed load has a major role in this study, an equation for this load obtained by Bagnold was used[15]; after a simple passage we can express the variation in the bed level as

$$\frac{\partial z}{\partial t} = \frac{B_s \beta \gamma q}{c \gamma_s}\left[-\frac{1}{c}\frac{\partial c}{\mathrm{d}x}\left(\frac{\partial z}{\partial x} + \frac{y_f}{y}S_c\right) + \frac{\partial^2 z}{\partial^2 x}\right] \quad (4)$$

where y_f is the corresponding flow depth; c is the friction factors; S_c is the critical bed slope; γ is the specific weight of fluid and γ_s is the specific weight of grain size in fluid; β and B_s are coefficients depending respectively on the typical grain size and flow regime[16]. To determine the rate of degradation in the channel bed with respect to time it's necessary to calculate the value of friction factor c at every stage of the degradation process. This friction factor c can be expressed as[16]

$$\frac{1}{c^2} = \frac{1}{c_f^2} + \frac{1}{2}\sum_{i=1}^{2}\left(\frac{\Delta_i}{A_i}\right)^2 \frac{A_i}{y} \quad (5)$$

where c_f is the friction factor when the bed is flat, and it can be calculated from the following equation[16].

$$c_f = 2.5\ln\left(a\frac{y}{2d_{50}}\right) \quad (6)$$

where d_{50} is the median sediment diameter; a is a coefficient depending on the Reynolds roughness number, Δ_i and A_i are respectively the height and length of the bed form i that can be calculated as a function of the flow depth and bed roughness[16].

2 Armor Layer Development

Due to the degradation process, the friction factor will vary with time and space due to coarsening of median sediment diameter and erosion process along the channel. Observing the considered alluvial reach one can find out that the armoring process is not uniform and median sediment diameter decreases along the reach[3,6,13]. Thus, to calculate the value of the friction factor correctly it is imperative to evaluate the variation of the median sediment diameter with respect to time and distance. Therefore it is necessary to consider all the factors affecting the resistance to movement of an individual grain, which not only depend on its size and shape, but also on its sheltering and exposure to flow, which can change during the degradation process.

3 Probability of the Particle to Stay on the Bed

Shen and Lu[11] supposed that the probability of grain remaining stationary on the bed is a function of the relative shear stress $\hat{\tau}$, which is the ratio of $\hat{\tau} = \tau_c/\tau$. Thus the particle that stays on the bed must have an average bottom shear stress τ not exceeding the critical shear stress τ_c. Gessler[17] found that the relation between relative shear stress and the probability P of the particle that stays on the bed can be approximated by Guassian distribution with mean and standard deviation which is equal to 1 and 0.75, respectively. In this study, it can be proposed that the relative shear stress at each time step for each grain size i can be calculated as the inverse of flow intensity η_i, which is the ratio between critical mobility number $Y_{c,i}$ of the considered grain and the mobility number Y_i for the grain size i, which is calculated from the following equations[16].

$$Y_i = \frac{\tau}{\gamma_s d_i} = \frac{\gamma y S}{\gamma_s d_i} \quad (7)$$

$$Y_{c,i} = \frac{0.1349}{\zeta_i^{0.392}}\mathrm{e}^{-0.02\zeta_i^2} + 0.05\left(1 - \mathrm{e}^{-0.068\zeta_i}\right) \quad (8)$$

where S is the bed slope; ζ_i is calculated by the following equation

$$\zeta_i = \frac{\gamma s d_i^3}{\rho \upsilon^2} \quad (9)$$

where ρ and υ are the density and kinematic viscosity of fluid. Thus according to Gessler[17] and Shen and Lu[11] the probability of the grain of diameter d_i that stays on the bed can be determined by using the following equation

$$P_i = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{X_i}\exp\left(\frac{X^2}{2}\right)\mathrm{d}X \quad (10)$$

with

$$X_i = \frac{\left(\xi\frac{\tau_c}{\tau}\right)_i - 1}{\sigma} \quad (11)$$

where σ is the geometric standard deviation of grain size distribution of bed material, and X is the variable of integration. The parameter ξ is called hiding factor which reflects the influence of the grains of the whole mixture on the transport of the grain of diameter d_i, which can be calculated by the equation proposed by Karim and Kennedy[18] as

$$\xi = \left(\frac{d_i}{d_{50}}\right)^{0.85} \quad (12)$$

4 Computation of the Median Sediment Diameter

After determining the probability of the particle that stays on the bed of each grain of diameter d_i and knowing the initial fraction by weight or by volume in each diameter d_i of sediment on the bed, the new corresponding fractions of the sediment remaining on the bed at each time step can be calculated as

$$F(i) = F(i)_o P_i \qquad i = 1, \cdots, N_d \qquad (13)$$

where $F(i)$ and $F(i)_o$ are respectively the new and old fractions in each diameter d_i; N_d is the number of considered grain diameters. Then the cumulative fraction of the grains that remained on the bed finer than diameter d_i is determined according to

$$F_{c,I} = \sum_{i=1}^{I} F_i P_i \qquad I = 1, \cdots, N_d \qquad (14)$$

Then the new median sediment diameter is determined by the following equation[19]

$$\ln d_{50} = \ln d_j + \frac{\ln d_{j+1} - \ln d_j}{F_{c,j+1} - F_{c,j}}(0.5 - F_{c,j}) \qquad (15)$$

where j and $j+1$ are chosen so that both $F_{c,j} \leqslant 0.5$ and $F_{c,j+1} > 0.5$.

5 Numerical Scheme

The differential Eq. (4) is solved numerically by using the finite difference method and approximating the second order derivative with the Crank-Nicolson scheme, so Eq. (4) is described as

$$Z_i^{t+1} = Z_i^t + \frac{\alpha_k \Delta t}{2\Delta x^2}[(Z_{i-1}^{t+1} - 2Z_i^{t+1} + Z_{i+1}^{t+1}) + (Z_{i-1}^t - 2Z_i^t + Z_{i+1}^t)] - W_k^t \qquad (16)$$

where

$$\alpha_k = -\frac{\beta B_s \gamma q}{c_k \gamma} \qquad (17)$$

$$W_k^t = -\frac{1}{c_i^t}\frac{c_i^t - c_{i-1}^t}{\Delta x}\left[S_k^t + \frac{(y_f)_k^t}{y_k^t}(S_c)_k^t\right] \qquad (18)$$

where $i = 1, \cdots, N$; $K = 1, \cdots, M$; M is the number of stretches Δx in which the considered channel reach is described; N is the number of the grid points; t and $t+1$ are known and unknown time steps, respectively.

6 Procedure for the Calculation of Degradation

The procedure for the calculation of degradation that will occur is based on the variability of certain pertinent parameters. The procedure starts with the calculation of the grain size distribution at each time step by using Eq. (14) and then the median sediment diameter for each stretch is determined by using Eq. (15). Then Eq. (16) will be solved to obtain the new bed slope and flow depth of the new bed slope, and the new flow depth is calculated. After that the flow intensity is calculated as the ratio of the mobility number and critical mobility number which can be calculated by using Eqs. (7) and (8) as a function of median sediment diameter. The above step should be repeated until the flow intensity becomes insignificant or equals zero for each stretch (end of degradation). Fig. 1 shows the flow chart of the proposed procedure.

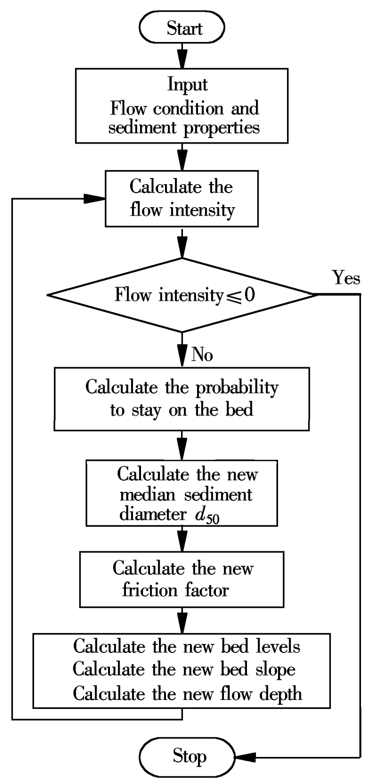

Fig. 1 Flow chart proposed model

6.1 Initial and boundary condition

The problem under investigation is as follows: a wide rectangular alluvial channel of semi-infinite length carries a constant discharge per unit width q and mean flow depth. The equilibrium sediment discharge per unit width under uniform flow conditions is q_{s_i} and the sediment discharge at the upstream end of the channel $x = 0$ is suddenly interrupted, consequentially the equilibrium between the water and sediment discharge is disturbed and the incoming clear water flow shall erode the channel bed. The initial and boundary conditions are

$$Y(x,0) = y_o, \; q(x,0) = q_o, \; S(x,0) = S_o$$
$$c(x,0) = c_o, d_{50}(x,0) = d_{50}$$

6.2 Application and comparison with experimental results

The proposed numerical method was applied and compared with experimental results obtained by Othman[13] who preformed and analyzed a series of experiments conducted in recirculating flume of rectangular cross section of 24 m length, 0.81 m wide and 0.70 m depth by using two sizes of sand of median diameter 0.47 mm (sample A) and 0.79 mm (sample B)and geometric standard deviation 4.65 and 3.54, respectively. Fig.2 shows the grain size distribution of used bed materials, his study focused on the

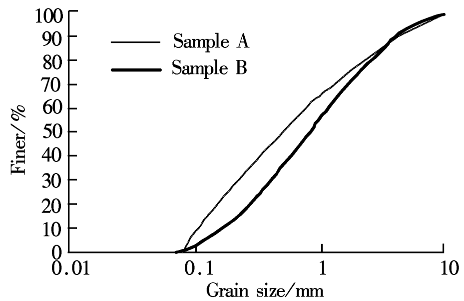

Fig.2 Grain size distribution of used bed of material

degradation problem in alluvial channels under armoring. The bed was considered armored when the sediment transport rate was less than 1% of the initial sediment rate. Six runs from his experiments were chosen arbitrary to apply the suggested numerical model. Runs No. A_1, A_5, A_{10}(d_{50} = 0.47 mm, σ = 4.65), and B_1, B_6, B_{10}(d_{50} = 0.79 mm, σ = 3.54). The reach was divided into three stretches. Tab.1 shows the initial and final conditions at the end of degradation for the selected runs, while Tab.2 shows the rate of degradation and variation in median sediment diameter for run A_5 at different times and distances. Fig.3 and Fig.4 show respectively the calculated and observed bed profiles and size distribution curves for three locations at different time steps for run B_6. From these figures and tables it can be seen that there is a good agreement between the observed values and values obtained by using the proposed numerical model. Also it is found from these results that the rate of degradation in non-uniform bed material channels is affected greatly by the armoring process that will stop further erosion of the bed. In addition the median sediment diameter becomes bigger and bigger with time and decreases along x; the most changes in the bed characteristics take place at the beginning of the degradation, and then the rate of the change slows down as the time proceeds. Further, it is also found that the rate of change in type B is more than in type A. This is due to the uniformity of particle size distribution of type B (σ = 3.54) and type A (σ = 4.65) which will affect in hiding and sorting of the particles during the degradation process.

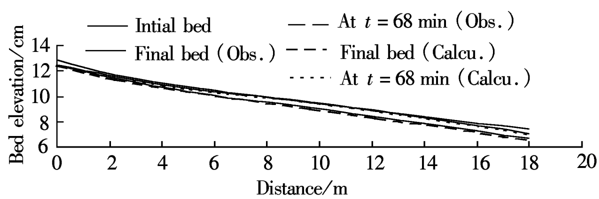

Fig.3 Calculated and observed bed profile for run B_6 at different times

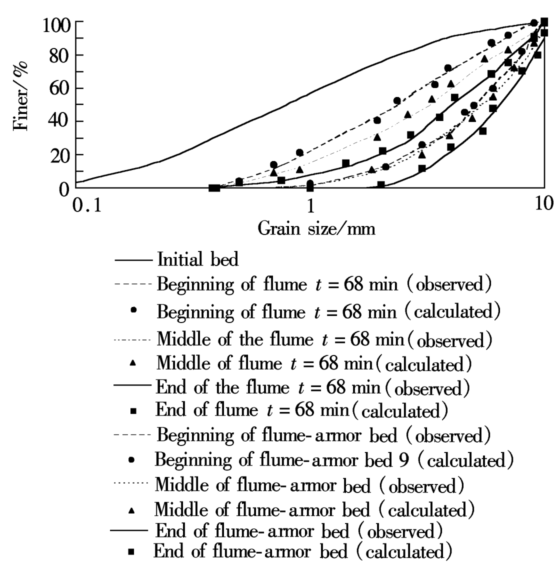

Fig.4 Calculated and observed size distribution curves of bed material for run B_6 at different times

Tab.1 The initial and final conditions for selected runs

Run No.	Flow rate/ $(L \cdot (s \cdot m)^{-1})$	Initial bed slope/10^3	Initial water depth (X = 0)/cm	Final bed slope/10^3	Final water depth (X = 0)/cm	$\partial z/\partial t$ at the beginning of degradation (X = 0)/ $(\mu m \cdot min^{-1})$	d_{50} of armored bed at beginning of flume/mm	d_{50} of armored bed at middle of flume/mm	d_{50} of armored bed at end of flume/mm
A_1	8.810	3.39	3.64	3.29	3.95	15.6	3.3	2.8	2.0
A_5	17.76	3.96	4.30	3.65	5.00	20.0	3.9	3.7	3.4
A_{10}	25.77	3.37	4.45	3.00	6.00	70.0	5.2	4.8	3.9
B_1	13.17	2.61	4.50	2.30	5.10	16.0	3.4	3.3	2.9
B_6	22.27	2.87	3.70	2.25	4.40	117.0	6.25	5.5	5.0
B_{10}	29.60	3.50	4.60	1.50	5.80	333.0	6.8	6.3	6.0

Tab.2 The rate of degradation and variation of median sediment diameter for run A_5 at different times and distances

X/m			Time/min							
			18	38	78	138	258	498	858	1 278
2.0	$\frac{\partial z}{\partial t}$/(μm · min^{-1})	Observed	11.1	90.0	52.5	20.0	3.3	1.25	0.00	0.00
		Calculated	12.0	83.5	48.1	24.0	2.5	0.90	0.00	0.00
	d_{50}/mm	Observed	1.64	2.40	3.61	3.15	3.96	3.97	3.97	3.97
		Calculated	1.57	2.42	2.85	2.95	3.90	3.96	3.98	3.99
7.4	$\frac{\partial z}{\partial t}$/(μm · min^{-1})	Observed	55.5	40.0		15.0	10.8	3.75	0.55	0.00
		Calculated	50.2	45.0		14.1	12.3	4.1	0.00	0.00
	d_{50}/mm	Observed	1.05	0.78	1.73	3.02	3.63	3.64	3.80	3.80
		Calculated	1.00	0.82	1.75	3.01	3.50	3.70	3.82	3.84
19.5	$\frac{\partial z}{\partial t}$/(μm · min^{-1})	Observed	127.0		132.0	10.0	24.1	0.00	0.00	0.00
		Calculated	90.0		105.0	8.0	25.0	0.90	0.48	0.00
	d_{50}/mm	Observed	0.64	0.66	1.07	2.08	3.61	3.10	3.40	3.62
		Calculated	0.40	0.50	1.20	2.15	2.90	3.00	3.28	3.60

7 Conclusions

1) The proposed numerical model computes the degradation rate in nonuniform bed material channels. The model takes into account the variation of friction factor with time and distance as well as predicting the variation in grain size distribution at each time step.

2) The degradation rate in nonuniform bed material is high at the beginning of the process, but after a short time the rate reduces owing to the armoring effect and this rate increase as the discharge is increased.

3) A good agreement is found between the observed and calculated values after applying the proposed numerical model.

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冲积河道河床冲刷过程的数值模拟

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摘 要 本文建立了一维数值模型来模拟顺直冲积渠道的冲刷过程.模型考虑了床沙的非均匀性、床面形态变化,应用该模型预测计算了冲刷过程中的床沙级配变化.模型计算结果表明床沙粗化过程不仅与水流的水力特性有关,还与床沙的颗粒大小及分布有关.模型计算结果与2组非均匀床沙水槽试验结果进行了比较,结果表明模型计算结果与试验结果符合得较好.

关键词 冲刷; 粗化; 数值模型

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