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Research on a non-linear chaotic prediction model for urban traffic flow

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Abstract: In order to solve serious urban transport problems, according to the proved chaotic characteristic of traffic flow, a non-linear chaotic model to analyze the time series of traffic flow is proposed. This model reconstructs the time series of traffic flow in the phase space firstly, and the correlative information in the traffic flow is extracted richly, on the basis of it, a predicted equation for the reconstructed information is established by using chaotic theory, and for the purpose of obtaining the optimal predicted results, recognition and optimization to the model parameters are done by using genetic algorithm. Practical prediction research of urban traffic flow shows that this model has famous predicted precision, and it can provide exact reference for urban traffic programming and control.

Key words: traffic flow; chaotic theory; phase reconstruction; non-linear; genetic algorithm; prediction model

Along with the rapid development of the economy and society, traffic flow of each megalopolis is increasing at a very fast speed, and problems of traffic congestion have become more and more serious. In order to solve this problem, different measures have been adopted to improve urban traffic in succession. Among these, prediction of traffic flow[1-3] is extra-important, and its predicted results will directly influence the rationality of traffic programming and effect of urban control. But owing to the high-complexity, randomness and uncertainty of road traffic flow, traditional prediction methods don't yield superior results, and the effect of traffic control is also less than ideal.

Over the past years, along with the progress of research, scholars have found that notwithstanding the high degree of uncertainty and randomness, road traffic flow actually follows certain rules, this phenomenon has aroused the attention of traffic researchers[4]. Researches at home and abroad prove that chaotic phenomena certainly exist in traffic flow[5]. Based on this observation, a non-linear chaotic model to predict traffic flow using chaotic theory and a genetic algorithm is proposed in this paper, and a practical case is also demonstrated.

1 Mathematical Description of Non-Linear Chaotic Prediction Model

1.1 Phase reconstruction[6]

Phase reconstruction originally arises from the attempt to restore chaotic attractors in high-dimensional phase space. The track of a chaotic system is essentially decided, but owing to some random factors, the track will become too complex to stay on its course. However from long-time observation, we find that a chaotic system will eventually fall into a specific track, this specific track is the chaotic attractor, and it can be mixed through expansion and folding. Based on this theory, Takens, et al.[7] proposed the original idea of phase reconstruction. Takens, et al. believed that the evolution of any systemic variable was decided by other variables which effect them reciprocally, and the information from these mutual variables was kept in the developing process of any variable. Therefore, when a status space is reconstructed, we only require one variable to expand at some deferred points of fixed time. On the basis of this theory, we can restore the chaotic attractor by phase reconstruction in one-dimensional time space, and the gained chaotic attractor can reflect the character of the chaotic system.

According to the theory of phase reconstruction, we can construct phase space $\boldsymbol{X}_t = \{x(t), x(t+\tau), \cdots, x(t+(m-1)\tau)\}^{\mathrm{T}}$ to the time series $x(t_1)$, $x(t_2), \cdots, x(t_n)$, where $t_i = t_0 + i\Delta t$, τ is the delay, m is the embedded dimension. τ and m are two important parameters in phase space. In order to make phase space more reasonable, we must select suitable τ and m according to practical circumstances. To veiw the selection method in detail, the reader may consult Grassberger Procaccia algorithm[8].

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1.2 Non-linear chaotic model[9,10]

Suppose that time series $x_1, x_2, x_3, \cdots, x_N$ are the traffic flow of a road section before time T, then according to the chaotic theory, we can describe the chaotic time series by the following non-linear model:

$$x_t = F(\boldsymbol{Z}_t, \boldsymbol{\beta}_t) + \varepsilon_t$$

where $\boldsymbol{Z}_t = \{x_{t-1}, x_{t-2}, \cdots, x_{t-m}\} \in \mathbf{R}^m$; $\boldsymbol{\beta}_t = \{\beta_1, \beta_2, \cdots, \beta_n\}$ are the parameters to be decided; F is the non-linear function to be confirmed; ε_t is the white noise.

In order to increase the veracity of the model, its non-linear form in this paper can be expressed as

$$x_{t+n} = (a_1 + \beta_1 e^{-\gamma_1 x_{t+n-1}^2}) x_{t+n-1} + (a_2 + \beta_2 e^{-\gamma_2 x_{t+n-2}^2}) x_{t+n-2} + \cdots + (a_n + \beta_n e^{-\gamma_n x_{t+n-n}^2}) x_{t+n-n} = \sum_{i=1}^{n} (a_i + \beta_i e^{-\gamma_i x_{t+n-i}^2}) x_{t+n-i}$$

where $a_1, a_2, \cdots, a_n; \beta_1, \beta_2, \cdots, \beta_n; \gamma_1, \gamma_2, \cdots, \gamma_n$ are the parameters to be decided.

1.3 Recognition of parameters

Recognition of the above parameters can be described as the following optimization problem:

$$\min_{\boldsymbol{D}} f(\boldsymbol{D}) = \frac{1}{2N} \sum_{i}^{n} \left| x_{t+n} - \sum_{j=1}^{n} (a_j + \beta_j e^{-r_j x_{\tau+n-j}^2}) x_{t+n-j} \right|^2$$

where $\boldsymbol{D} = \{a_1, a_2, \cdots, a_n, \beta_1, \beta_2, \cdots, \beta_n, \gamma_1, \gamma_2, \cdots, \gamma_n\}^{\mathrm{T}}$; N is the value of the sample.

Because the above optimum problem is probably not a single modal function, but a multimodal function, this condition presents huge difficulties to the recognition of the model parameters. However, most optimization methods recently in use such as grads method, Newton method and conjugate grads method can't solve a multimodal function effectively. Therefore, this paper adopts a genetic algorithm to recognize the parameters.

A genetic algorithm is a random optimization method based on natural selection and genetic theory, it was developed by Holland in the 1970's. Because the genetic algorithm adopts random operations, and it has some advantages, such as naught special requirements to search space, one need not seek a differential coefficient, simple calculation, fast convergent speed and global optimization, the genetic algorithm has become a research hotspot and an effective application method in recent years.

The basic computing process of simple genetic algorithm[11] is described as follows:

```
Begin
  Confirm fitness function and coding rule;
      Confirm generation size, population size, crossover
      possibility Pc and mutation possibility Pm;
      Generate an initial population randomly, and calculate the
      fitness of each individual;
      Repeat
        Select two (or more) individuals from the parent
        population, and let them crossover at possibility Pc;
        Select individual from parent population, and let it
        mutate at possibility Pm;
        Decode each individual, and calculate its fitness;
        Select offspring population (propagate suitable
        individual, wash out unsuitable individual);
  Until (achieve set generation or satisfied result)
End
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In order to avoid the problems of a simple genetic algorithm such as poor local searching ability, premature convergence, large amount of calculation and bad adaptability to large search space, a hybrid genetic algorithm, which imports simplex search, niche elimination, elitist preservation and accelerated cycle, is used in this paper, and its specific computing process is described in Ref. [12].

2 Examples

2.1 Predicted error index of traffic flow

In order to compare the predicted results with the real value, some error indices are imported in this paper:

1) Average absolute error

$$e_{\mathrm{aa}} = \frac{1}{N} \sum_{t} \left| Y_{\mathrm{pred}}(t) - Y_{\mathrm{real}}(t) \right|$$

2) Maximal absolute error

$$e_{\mathrm{ma}} = \max_{t} \left| Y_{\mathrm{pred}}(t) - Y_{\mathrm{real}}(t) \right|$$

3) Average relative error

$$e_{\mathrm{ar}} = \frac{1}{N} \sum_{t} \left| \frac{Y_{\mathrm{pred}}(t) - Y_{\mathrm{real}}(t)}{Y_{\mathrm{real}}(t)} \right|$$

4) Maximal relative error

$$e_{\mathrm{mr}} = \max_{t} \left| \frac{Y_{\mathrm{pred}}(t) - Y_{\mathrm{real}}(t)}{Y_{\mathrm{real}}(t)} \right|$$

5) Similar degree

$$d_{\mathrm{s}} = 1 - \frac{\sqrt{\sum_{t} [Y_{\mathrm{pred}}(t) - Y_{\mathrm{real}}(t)]^2}}{\sqrt{\sum_{t} Y_{\mathrm{pred}}^2(t)} + \sqrt{\sum_{t} Y_{\mathrm{real}}^2(t)}}$$

where $Y_{\mathrm{real}}(t)$ is the real value; $Y_{\mathrm{pred}}(t)$ is the predicted value.

2.2 Prediction of traffic flow

In order to test the validity of the prediction model in this paper, we chose an intersection in the city of Changshu for case research. This intersection lies in the center of the city of Changshu, and its traffic flow is large. For the convenience of prediction, this paper adopts a loop sensor to measure traffic flow.

In the course of prediction, direction of vehicle flow in the case research is from south to north, the sampling interval is 5 min, and the sample data is the dynamic updating traffic flow of 4 d before testing time. According to generic advice and computing experiences[8,12], in this paper, the embedded dimension of phase space chosen is 10, fitness function selects the error of prediction, binary coding depends on the range and required precision of each parameter, generation size selected is 500, population size selected is 100, required precision $E = 10^{-4}$, crossover possibility $P_c = 65\%$, mutation possibility $P_m = 4\%$, elitist individuals of accelerated cycle $N_A = 10$, individuals of simplex search $N_S = 10$, elitist individuals of niche $N_P = 15$, penalty function of fitness in niche selected is 10^{-30}. Through continuous observation of a whole day, predicted results in detail are shown in Fig.1 and Tab.1.

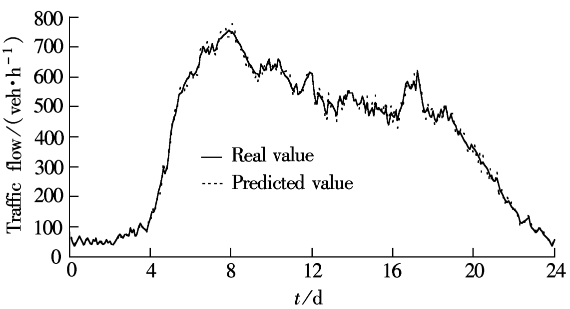

Fig.1 Contrast figure of real value and predicted value

Tab.1 Predicted error of traffic flow

Error index	Predicted value
e_{aa}/veh	12.800
e_{ma}/veh	29.600
e_{ar}/%	5.630
e_{mr}/%	12.080
d_s	0.982

Through comparison we can find that the model's prediction precision is quite good, and its prediction error can completely satisfy the requirements of urban traffic programming and control. Therefore, this model is an effective approach for predicting traffic flow.

3 Conclusion

Prediction of urban traffic flow has extremely important impact in traffic programming, control and management. In order to predict it well and accurately, this paper proposes a non-linear chaotic model which integrates chaotic theory with a genetic algorithm. Practical case research shows that this model has the advantages of veracity, robustness and understandability, so it has very wide application potential in the field of traffic management.

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城市交通流量的非线性混沌预测模型研究

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摘　要　为了解决日益严重的城市交通问题,本文根据交通流已被证明的混沌特性,尝试采用非线性混沌模型来分析交通流时间序列.该模型首先将交通流时间序列在相空间中重构,以充分提取交通流中的相关信息,在此基础上,应用混沌理论对重构信息构建了预测方程,并运用遗传算法对模型参数进行了优化辨识,以获得最佳的预测效果.实际的城市交通流量预测研究表明,该模型具有较高的预测精度,可以为城市交通规划和控制提供准确的参考.

关键词　交通流量；混沌理论；相空间重构；非线性；遗传算法；预测模型

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