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Coloring of some integer distance graphs

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Abstract: An integer distance graph is a graph $G(\mathbf{Z}, D)$ with the integer set $\mathbf{Z}$ as vertex set, in which an edge joining two vertices u and v if and only if $|u-v| \in D$, where D is a set of natural numbers. Using a related theorem in combinatorics and some conclusions known to us in the coloring of the distance graph, the chromatic number $\chi(G)$ is determined in this paper that is of the distance graph $G(\mathbf{Z}, D)$ for some finite distance sets D containing $\{2,3\}$ with $|D| = 4$ and containing $\{2,3,5\}$ with $|D| = 5$ by the method in which the combination of a few periodic colorings.

Key words: distance graph; chromatic number; combination

We consider the chromatic number of distance graphs on the integers. Let $G(\mathbf{Z}, D)$ denote the graph with the set $\mathbf{Z}$ of integers as vertex set and with an edge joining two vertices u and v if and only if $|u-v| \in D$. Such a graph $G(\mathbf{Z}, D)$ is called an integer distance graph or simply a distance graph (with a distance set D).

A proper coloring $f: V(G) \rightarrow \{c_1, c_2, \cdots, c_k\}$ of G is an assignment of colors to the vertices of G such that $f(u) \neq f(v)$ for all adjacent vertices u and v. The chromatic number $\chi(G)$ of G is the minimum number of colors necessary to color G, i.e. there exists a proper coloring f of G. The chromatic number $\chi(G(\mathbf{Z}, D))$ of the distance graph $G(\mathbf{Z}, D)$ is usually denoted by $\chi(D)$.

Distance graphs, first studied by Eggleton, et al.[1], were motivated by the well-known plane-coloring problem: What is the minimum number of colors needed to color all points of a Euclidean plane so that points at unit distance are colored with different colors? This problem is equivalent to determining the chromatic number of this distance graph $G(R^2, \{1\})$. It is well known that the chromatic number of this distance graph is between 4 and 7. However the exact number of colors needed remains unknown.

By now, the chromatic number $\chi(D)$ of the distance graph $G(\mathbf{Z}, D)$ has been completely determined when $|D| \leqslant 3$[2]. It is obvious that $\chi(D) = 2$ when D contains one element. For a 2-element set we have $\chi(D) = 2$ if D contains two positive odd integers and $\chi(D) = 3$ if D consists of two coprime integers of distinct parity. If $|D| = 3$ and the greatest common divisor of D, i.e. $\gcd(D) = 1$, then $\chi(D) = 4$ if and only if $D = \{1,2,3n\}$ or $D = \{x, y, x+y\}$ and $x \neq y \pmod 3$, and $\chi(D) \leqslant 3$ for all other 3-element distance sets. But there are not many results about it when $|D| \geqslant 4$. For example, $\chi(1,2,3,4n) = 5$, when $D = \{x, y, x+y, y-x\}, x<y, \gcd(x, y) = 1, (x, y) \neq (1,2)$, $\chi(D) = 4$ if x, y are of distinct parity and $\chi(D) = 5$ if x, y are odd numbers. Also $\chi(D)$ is determined for $D = \{2,3,s,s+u\}$ for many pairs (s, u)[3].

Moreover, some results are obtained for other distance sets. In Ref.[1], it is proved that $\chi(P) = 4$ where P denotes the sets of all primes. Some $\chi(D)$'s are determined when $D \subset P$ (see Refs.[4,5]). About the distance sets missing multiples, if $D = \{1,2,\cdots, n\} \setminus \{m, 2m, \cdots, sm\}$, then $\chi(D) = m$ if $n < (s+1)m$ and $\left\lceil \frac{n+sm+1}{s+1} \right\rceil \leqslant \chi(D) \leqslant \left\lceil \frac{n+sm+1}{s+1} \right\rceil + 1$ if $n \geqslant (s+1)m$. The cases when $\chi(D)$ coincides with the lower bound and when with the upper bound are determined in Ref.[6] (see Refs.[7,8]).

In this paper we determine the chromatic number $\chi(D)$ of the distance graph $G(\mathbf{Z}, D)$ for special 4-element distance sets D containing $\{2,3\}$ as a subset and the 5-element ones containing $\{2,3,5\}$.

1 Preliminaries

A coloring $f: \mathbf{Z} \rightarrow \{c_1, c_2, \cdots, c_k\}$ is called periodic with period P if $f(v) = f(v+p)$ for all $v \in \mathbf{Z}$. In the following, colors are denoted by $a, b, c, \cdots$. A p-periodic coloring is denoted by P_p, for example,

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$P_5 = abbcc$. A block of i colors of the same type is sometimes indicated by the exponent i, i.e. $P_5 = ab^2c^2$.

For any subset $M \subseteq \mathbf{N}$, we say that a coloring f: $\mathbf{Z} \to C$ is M-consistent if $f(i+m) \neq f(i)$ for every $i \in \mathbf{Z}$ and every $m \in M$. If $M = \{m\}$, we shall simply say that f is m-consistent. Thus f is a proper coloring of the distance graph $G(\mathbf{Z}, D)$ if and only if f is a D-consistent coloring of $G(\mathbf{Z}, D)$.

The following are several known results we shall use in the next two sections.

Lemma 1[9] If $M = \{m_1, m_2, \cdots, m_i\}$, P_p is a periodic M-consistent coloring of the graph G with period $p > m_j, j = 1, 2\cdots, i$, then P_p is also $(np \pm m_j)$-consistent for all $n \in \mathbf{N}$ where $j = 1, 2, \cdots, i$.

Lemma 2[10] If D is finite, $D = \{d_1, d_2, \cdots, d_r\}$, then $\chi(D) \leqslant \min_{n \in \mathbf{N}} n(|D^n| + 1)$ where $D^n = \{d_i \in D : n \mid d_i, i = 1, 2, \cdots, r\}$.

Lemma 3[10] If $n \in \mathbf{N}$, then $\chi(d_1, d_2, \cdots, d_r) = \chi(nd_1, nd_2, \cdots, nd_r)$.

Lemma 4[11] If $D = \{x, y, x+y\}$, $\gcd(x, y) = 1$, $x \not\equiv y \pmod 3$, then $\chi(D) = 4$.

Lemma 5[12]

$$\chi(2,3,s,s+2) = \begin{cases} 3 & \text{if } s \equiv 2 \pmod 6 \\ 4 & \text{otherwise} \end{cases}$$

Lemma 6[12]

$$\chi(2,3,s,s+3) = \begin{cases} 3 & \text{iff } s \equiv 3 \pmod 9 \\ 4 & \text{iff } s \not\equiv 3 \pmod 9 \text{ and } s \neq 5 \\ 5 & \text{iff } s = 5 \end{cases}$$

Lemma 7 (Theorem of Frobenius[13]) Let a and b be two positive integers such that $\gcd(a, b) = 1$, if t is an integer such that $t > ab - a - b$, then the equation $t = ma + nb$ has at least one solution with n and m non-negative integers.

2 The Case When the Distance Set D Is 4-Element Set Containing $\{2,3\}$

In Ref.[13], the following theorem was given with the distance set $D = \{2, 3, s, s+u\}$.

Theorem 1 If $D = \{2, 3, x, x+s\}$, $x > 3$, then $\chi(D) = 3$ if one of the following cases occurs: ① $s = 1$ and $x > 21$; ② $s = 4$ or 5 and $x > 17$; ③ $s = 6$ and $x > 16$; ④ $s = 7$ and $x > 40$; ⑤ $s = 8$ and $x > 92$; ⑥ $s = 9$ and $x > 19$.

In the proof, a proper 3-coloring was given in each case by using the three periodic 3-colorings $P_5 = aabcc$, $P_6 = aabbcc$, $P_9 = aabccabbc$ and their combination.

Determined by computer, the authors listed the table to obtain $\chi(2, 3, x, x+s) = 4$ and $s = 1, 4, 5, \cdots, 9$ for all these graphs. See Tab.1.

Tab.1 All (x, s)'s such that $\chi(2, 3, x, x+s) = 4$

s	x
1	4,5,10
4	5,6
5	5
6	6
7	4,5,6,10,11,12,16,17,22
8	4,5,6, 9,10,11,13,15,18,19,23,24,29,33,37,42,47
9	4,5,10

Studying this table, we can see that $\chi(2, 3, 5, 5+s) = 4$, $s = 1, 4, 5, \cdots, 9$. Moreover generally, we have the following theorem.

Theorem 2 $\chi(2,3,5,n) = \begin{cases} 5 & \text{if } n = 8 \\ 4 & \text{otherwise} \end{cases}$

Proof According to Tab.1, we have $\chi(2, 3, 5, n) = 4$, $n = 6, 9, 10, \cdots, 14$.

Lemma 5 implies that $\chi(2, 3, 5, 7) = 4$ and $\chi(2, 3, 5, 8) = 5$ by lemma 6. It is obvious that $\chi(2, 3, 5) = 4$ due to lemma 4. Since the distance graph $G(\mathbf{Z}, \{2, 3, 5\})$ is a subgraph of $G(\mathbf{Z}, \{2, 3, 5, n\})$, we get $\chi(2, 3, 5, n) \geqslant \chi(2, 3, 5) = 4$, $n \geqslant 15$.

Let $n = 5 + s$, $P_7 = aabbccd$, $P_8 = aabbbccdd$ are all $\{2, 3, 5\}$-consistent, which is easily checked. Moreover P_7P_8 and P_8P_7 are also $\{2, 3, 5\}$-consistent. Since $\gcd(7, 8) = 1$ the equation $t = 7l + 8m$ has at least one solution in non-negative integers l and m whenever $t = 3 + s > 7 \times 8 - 7 - 8$, i.e. $s > 38$ by lemma 7.

For such a pair (l, m) we define a periodic 4-coloring of this form: $P_t = P_{s+3} = P_7{}^l P_8{}^m$, $s > 38$, P_t is also $\{2, 3, 5\}$-consistent because of the consistence of P_7, P_8, P_7P_8 and P_8P_7. So $P_t = P_{s+3} = P_7{}^l P_8{}^m$ is a $\{2, 3, 5, n\}$-consistent coloring by application of lemma 1 where $n > 43$, i.e. $\chi(2, 3, 5, n) \leqslant 4$, $n > 43$.

For the case when $D = \{2, 3, 5, n\}$, $15 \leqslant n \leqslant 43$, it is obtained that $\chi(2, 3, 5, n) \leqslant 4$ when $s \neq 4k+3$, $k \in \mathbf{N}$ due to lemma 2 choosing $n = 4$. It suffices to prove that $\chi(2, 3, 5, 5+s) \leqslant 4$, $15 \leqslant s \leqslant 38$, $s = 4k + 3$, $k \in \mathbf{N}$. We will construct periodic 4-colorings to prove this case by the combination of P_7 and P_8. In the following, choosing $P_t = P_j = P_7{}^l P_8{}^m$ as a periodic coloring of the distance graph $G(\mathbf{Z}, D)$ when $n = i$ is denoted by $n = i$, $P_t = P_j = P_7{}^l P_8{}^m$.

$s + 5 = n = 20$, $P_t = P_{22} = P_7{}^2 P_8$

$s + 5 = n = 24$, $P_t = P_{21} = P_7{}^3$

$s + 5 = n = 28$, $P_t = P_{23} = P_8{}^2 P_7$

$s + 5 = n = 32$, $P_t = P_{30} = P_7{}^2 P_8{}^2$

$s+5=n=36$, $P_t=P_{31}=P_8{}^3P_7$

$s+5=n=40$, $P_t=P_{38}=P_7{}^2P_8{}^3$

The above colorings are all $\{2,3,5,n\}$-consistent by lemma 1.

All the cases are discussed. Therefore, $\chi(2,3,5,n)=5$ if $n=8$ and 4 for the other cases.

In the above proof, the combination of two cardinal periodic 4-colorings P_7 and P_8 plays a very important role. By this method, three corollaries follow.

Corollary 1 $\chi(2,3,7,n)=\begin{cases}4 & \text{if } n=9,10\\ 3 & \text{otherwise}\end{cases}$

Corollary 2 $\chi(2,3,8,n)=\begin{cases}4 & \text{if } n=11\\ 3 & \text{otherwise}\end{cases}$

Corollary 3 $\chi(2,3,12,n)=\begin{cases}4 & \text{if } n=14,19\\ 3 & \text{otherwise}\end{cases}$

The next theorem determines the chromatic number $\chi(D)$ when $D=\{2,3,9,n\}$, $n>9$, $n\in\mathbf{N}$ with the similar method, but a small difficulty when $n=23$.

Theorem 3

$$\chi(2,3,9,n)=\begin{cases}4 & \text{if } n=11,12,17,23\\ 3 & \text{otherwise}\end{cases}$$

Proof Firstly we have $\chi(2,3,9,11)=4$ by lemma 5 and $\chi(2,3,9,12)=4$ by lemma 6 or by lemma 3 and lemma 1 ($P_7=aabbccd$ is $\{2,3,5\}$-consistent).

With accordance to that table which lists all the pairs (s,x) with $\chi(2,3,s,s+x)=4, s=1,4,5,\cdots,9$, we have $\chi(2,3,9,n)=3$ when $n=10,13,\cdots,18$ and $\chi(2,3,9,17)=4$. Let $P_5=aabcc$, $P_6=aabbcc$, $P_{11}=P_5P_6$, it is routine to check P_6, P_{11}, $P_6P_{11}, P_{11}P_6$ are all $\{2,3,9\}$-consistent. Furthermore, $\chi(2,3,9,n)\geqslant\chi(2,3)=3$ when $n>18$. Now it suffices to construct a proper 3-coloring of the distance graph $G(\mathbf{Z},\{2,3,9,n\})$ for $n>18$. Let $n=9+s$, in view of lemma 7 and lemma 1 there exist two non-negative integers l and m such that the periodic 3-coloring $P_t=P_{s+6}=P_6^lP_{11}^m$ is $\{2,3,9,n\}$-consistent for $s>43$, i.e. $n>52$.

While $10\leqslant s\leqslant 43$, i.e. $19\leqslant n\leqslant 52$, the proper periodic 3-colorings are obtained by the combination of two cardinal colorings P_6 and P_{11} except that we choose $P_{27}=aabbccaabccaabbccabbccaabbc$ when $n=29$. The details are omitted.

The next conclusion we will prove is $\chi(2,3,9,23)=4$. Note that $\chi(2,3,9,23)\leqslant 4$ by lemma 2 with $n=4$ and $\chi(2,3,9,23)\geqslant\chi(2,3)=3$. Now we will prove that $\chi(2,3,9,23)=3$ is impossible. Assume that the mapping $f:\mathbf{Z}\to C$ is a proper 3-coloring of the distance graph $G(\mathbf{Z},\{2,3,9,n\})$. Without the loss of generality we choose a 5-cycle C_5 which consists of the five vertices 1,3,4,5,7. Suppose $f(1)=a$, $f(3)=b$, then the coloring of C_5 is divided into five cases:

① $f(1)=a$, $f(3)=b$, $f(4)=c$, $f(5)=a$, $f(7)=b$;

② $f(1)=a$, $f(3)=b$, $f(4)=b$, $f(5)=a$, $f(7)=c$;

③ $f(1)=a$, $f(3)=b$, $f(4)=c$, $f(5)=c$, $f(7)=a$;

④ $f(1)=a$, $f(3)=b$, $f(4)=c$, $f(5)=c$, $f(7)=b$;

⑤ $f(1)=a$, $f(3)=b$, $f(4)=b$, $f(5)=a$, $f(7)=c$.

We will next consider the first case. There is a similar discussion for the remaining case, but it is different in some details.

For convenience, $f(i)=j$ is denoted by $i\to j$ in the following proof. By the $\{2,3\}$-consistence, we can obtain $2\to b$, $6\to a$, $8\to b$ or c. If $8\to c$, then $9\to c$, but 10 cannot be colored by the three colors a,b,c since 10 is adjacent to 1,7 and 8, which contradicts the proper 3-coloring f of $G(\mathbf{Z},\{2,3,9,n\})$.

If $8\to b$, then $9\to c$, $10\to c$, $11\to a$, $12\to a$, $13\to b$, $14\to b$ or c.

Case 1 $14\to c$, $15\to c$, $16\to a$, $17\to a$, $18\to b$, $19\to b$, $20\to c$, $21\to c$, $22\to a$, then $23\to b$ or a.

1) $23\to b$, $24\to b$, $25\to c$, $26\to c$, $27\to a$, so 28 cannot be colored by a,b,c because 28 is adjacent to 5, 19, 25 and 26.

2) $23\to a$, $24\to b$, $25\to c$, $26\to c$, $27\to a$, so 28 cannot be colored by a,b,c because of the same fact in 1).

Case 2 $14\to b$, $15\to c$, then we have $16\to a$ or c.

1) $16\to a$, $17\to a$, $18\to b$, $19\to b$, $20\to c$, $21\to c$, $22\to a$, $23\to a$, $24\to b$, $25\to c$, $26\to a$, $27\to a$, the same contradiction occurs in 28.

2) $16\to c$, $17\to a$, then $18\to b$ or a.

① $18\to b$, $19\to b$, $20\to c$, $21\to c$, $22\to a$, $23\to a$, $24\to b$.

② $18\to a$, $19\to b$, it follows that:

a) $20\to c$, $21\to c$, $22\to a$, $23\to a$, $24\to b$.

b) $20\to b$, $21\to c$, then $22\to a$ or c.

(i) $22\to a$, $23\to a$, $24\to b$; (ii) $22\to c$, $23\to a$, $24\to b$.

In the above cases the vertex 25 is not colorable by a,b,c.

The above are all possible cases in which contradictions occur. Therefore $\chi(2,3,9,23) \neq 3$. It follows that $\chi(2,3,9,23) = 4$. So theorem 3 holds.

3 The Case When the Distance Set D Is a 5-Element Set Containing $\{2,3,5\}$

The following theorem was proved in Ref. [3] by the combinatorial method similar to that used in the above section.

Theorem 4 If an integer $s \geqslant 10$ and the other one $x \geqslant s^2 - 6s + 3$, then $\chi(2,3,x,x+s) = 3$.

When the distance set $D = \{2,3,5,s,s+u\}$, we have the following theorem about the chromatic number $\chi(D)$.

Theorem 5 For any $u \geqslant 20$ and the other one $s \geqslant u^2 - 5s - 2$, it holds that $\chi(2,3,s,s+u) = 4$.

The proof of theorem 5, similar to that of theorem 4, is omitted, but in the process we introduce a new proper periodic 4-coloring $P_{20} = aabccddabbccdaabbcdd$ in addition to P_7 and P_8.

Note that there are many chromatic numbers $\chi(D)$ not determined by theorem 5 when $D = \{2,3,5,s,s+u\}$, as a complement, we can obtain the following two theorems.

Theorem 6 If $D = \{2,3,5,s,s+u\}, s > 5, u = 6,7,8$, it follows that

$$\chi(D) = \begin{cases} 5 & \text{if } n = 8 \\ 4 & \text{otherwise} \end{cases}$$

Proof When $s = 7$, a routine check shows that $P_7 = aabbccd, P_8 = aabbccdd, P_7P_8, P_8P_7$ are all $\{2,3,4,5\}$-consistent. There exists a pair of non-negative integers n and m such that $P_t = P_{s+i} = P_7^n P_8^m$ is $\{2,3,5,s,s+7\}$-consistent ($i = 2$ or 5) when $s + i > 7 \times 8 - 7 - 8$, i.e. $s > 36$ or 39 due to lemma 7.

By the application of lemma 2 with $n = 4$, it is easily obtained that $\chi(2,3,5,s,s+7) \leqslant 4$ if $s \neq 4k$ or $4k+1$ and $6 \leqslant s \leqslant 36$. The proper periodic 4-colorings for $s = 4k$ or $4k+1$ and $6 \leqslant s \leqslant 36$ except $s = 8$ can be constructed by the combination of P_7 and P_8 whose D-consistence is obtained by lemma 1. The process is omitted.

On the other hand, $\chi(2,3,5,s,s+7) \geqslant \chi(2,3,5) = 4$ by lemma 4. Therefore, $\chi(2,3,5,s,s+7) = 4, s \neq 8$.

If $s = 8$, it follows that $\chi(2,3,5,8,15) \geqslant \chi(2,3,5,8) = 5$ in view of lemma 6. In the next step, we choose $P_{13} = aabcdeeabbcde$ which is checked to be $\{2,3,5,8,15\}$-consistent. This implies that $\chi(2,3,5,8,15) \leqslant 5$, so $\chi(2,3,5,8,15) = 5$.

The proof for $u = 6,8$ is similar to that of $s = 7$, except that $P_{17} = abcddbaaccdbbadce$ is chosen as a $\{2,3,5,8,14\}$-consistent coloring for $u = 6$ and $s = 8$. It is omitted.

Theorem 7

$$\chi(2,3,5,s,s+1) = \begin{cases} 5 & \text{if } n = 7,8 \\ 4 & \text{otherwise} \end{cases}$$

In the proof of theorem 7, P_{17} is also introduced.

Using this combinatorial method, the following theorem of which form is similar to that of theorem 2 is more apparent as a counterpart of theorem 6.

Theorem 8 If $D = \{2,3,5,s,n\}, s < n, s = 10,11,12$, it holds that $\chi(D) = 4$.

4 Open Problems

If $D = \{1,2,3,\cdots,r\}$, then $\chi(D) = r+1$ (see Ref. [4]). What is the minimum distance set $\bar{D}$ of cardinality r such that $\chi(\bar{D}) = r+1$ and $1 \notin \bar{D}$. If $r = 2$, then $\bar{D} = \{2,3\}$, if $r = 3$, then $\bar{D} = \{2,3,5\}$, and if $r = 4$, then $\bar{D} = \{2,3,5,8\}$. To our disappointment, the answer for $r \geqslant 5$ is unknown to us. Observing the results for $r = 2,3,4$ one may conjecture that the minimum distance set for $r = 5$ is $D = \{2,3,5,8,n\}$ where n is a given natural number. However, this is not true.

Theorem 9 $\chi(2,3,5,8,n) = 5$.

It can be proved by the combinatorial method in which P_{10} is introduced in addition to P_{13}, P_{17} used in the fourth section.

So far no more information about the minimum distance set $\bar{D}$ for $r \geqslant 5$ is known to the best of our knowledge, even regarding its form.

In contrast with the chromatic numbers $\chi(2,3,5,n)$ for all $n \in \mathbf{N}$, it seems to be difficult to determine the chromatic number $\chi(D)$ for general set $D = \{a, b, a+b, n\}, n > a+b$ where a, b are all given natural numbers; at the very least the combinatoric method doesn't seem to work. In addition, the circular chromatic number $\chi_c(D)$ and the fractional chromatic number $\chi_f(D)$ (their definitions can be seen in Ref. [15]) are well worth researching, up to now there are few significant results.

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某些整数距离图的染色

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摘　要　整数距离图是这样一类图 $G(\mathbf{Z},D)$,其中,$V(G)=\mathbf{Z}$,两点 u,v 之间存在一条边,当且仅当 $|u-v|\in D$,这里 D 是由自然数组成的一个集合.利用组合数学中的一个相关定理和距离图染色中我们已知的一些结论,通过几种周期染色组合的方法,本文确定了 $|D|=4$ 且 D 中包含 $\{2,3\}$ 和 $|D|=5$ 且包含 $\{2,3,5\}$ 时某些距离图 $G(\mathbf{Z},D)$ 的点色数 $\chi(D)$.

关键词　距离图；点色数；组合

中图分类号　O157.5