Journal of Southeast University (English Edition) Vol.19 No.4 Dec. 2003 ISSN 1003—7985

Global existence and blow up of a degenerate parabolic system

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Abstract: This paper deals with positive solutions of a degenerate parabolic system: $u_t = \Delta u^m + v^p \ln^\alpha(h+u)$, $v_t = \Delta v^n + u^q \ln^\beta(h+v)$ with homogeneous Dirichlet boundary conditions and positive initial conditions. This system describes the processes of diffusion of heat and burning in two-component continuous media with nonlinear conductivity and volume energy release. We obtain the global existence and blow up results of the solution relying on comparison with carefully constructed upper solutions and lower solutions.

Key words: parabolic system; degenerate; global existence; blow-up

1 Introduction and Main Results

In this paper, we consider the following degenerate parabolic system:

$$\left.\begin{array}{ll} u_t = \Delta u^m + v^p \ln^\alpha(h+u) & (x,t)\in\Omega\times(0,T) \\ v_t = \Delta v^n + u^q \ln^\beta(h+v) & (x,t)\in\Omega\times(0,T) \\ u(x,t) = v(x,t) = 0 & (x,t)\in\partial\Omega\times(0,T) \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x) & x\in\Omega \end{array}\right\} \tag{1}$$

where Ω is a bounded domain in $\mathbf{R}^N$, the parameters satisfy $\min\{m,n\}\geqslant 1$, $h\geqslant 1$, $\alpha\geqslant 0$, $\beta\geqslant 0$, $p>0$, $q>0$. The initial values $u_0(x)$ and $v_0(x)$ are nonnegative continuous functions.

This system describes the processes of diffusion of heat and burning in two-component continuous media with nonlinear conductivity and volume energy release. The functions u and v can thus be treated as temperatures of interacting components of a combustible mixture. And the nonlinear terms can be treated as the reaction sources, these require $h\geqslant 1$.

The local existence, uniqueness and comparison principle of the solution may be obtained by the method which is used in Ref.[1]. The aim of this paper is to get some conditions under which the solution of (1) blows up in finite time (or exists globally). In the next section, we deal with the blow-up phenomenon and prove our results. Let T^* be the maximal time of existence of the corresponding solution (u,v). If $T^*<\infty$, then $\lim\limits_{t\to T^*}\|u(t)\|_\infty + \|v(t)\|_\infty = \infty$ and the solution is said to blow up in finite time. On the other hand, if $T^*=\infty$, then the solution is said to be global.

In the past years, blow up problems for nonlinear parabolic systems have been widely studied[2-7].

In the next section, we prove the global existence and blow up results by the upper and lower solutions method. So we give the definition of the upper solutions and lower solutions.

Definition 1 Let $Q_T = \Omega\times(0,T)$, $\phi(u) = u^m$, $\psi(v) = v^n$, $f(u,v) = v^p\ln^\alpha(h+u)$, $g(u,v) = u^q\ln^\beta(h+v)$. $(u(x,t), v(x,t))$ defined on $\bar{Q}_T$ is called an upper solution(lower solution) of (1) if the following all hold.

1) $u,v\in L^\infty(Q_T)$;

2) $u(x,t)\geqslant 0$, $v(x,t)\geqslant 0$, $(x,t)\in\partial\Omega\times(0,T)$, $u(x,0)\geqslant u_0(x)$, $v(x,0)\geqslant v_0(x)$, $x\in\Omega$;

3) For all $t\in[0,T]$, $\xi,\zeta\in P = \{\xi(x,t)\in C(\bar{Q}_T)\cap C^{2,1}(Q_T), \xi\geqslant 0, \xi|_{\partial\Omega\times(0,T)} = 0\}$,

$$\int_\Omega u\xi\,\mathrm{d}x \geqslant (\leqslant)\int_0^t\int_\Omega [u\xi_s + \phi(u)\Delta\xi + f(u,v)\xi]\,\mathrm{d}x\mathrm{d}s + \int_\Omega u_0(x)\xi(x,0)\mathrm{d}x$$

$$\int_\Omega v\zeta\,\mathrm{d}x \geqslant (\leqslant)\int_0^t\int_\Omega [v\zeta_s + \psi(v)\Delta\zeta + g(u,v)\zeta]\,\mathrm{d}x\mathrm{d}s + \int_\Omega v_0(x)\zeta(x,0)\mathrm{d}x$$

Received 2003-05-05.

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(u,v) is called a solution of (1) if it is both an upper solution and a lower solution of (1).

When $h>1$, we have the following results.

Theorem 1 Let $pq<mn$, all the solutions of (1) are global.

Theorem 2 Let $pq>mn$, ① The solutions of (1) are global provided that the initial functions are small enough; ② There exists a solution of (1) which blows up in finite time.

Theorem 3 Let $pq=mn$, ① If the diameter of Ω is sufficiently small, then all the solutions of (1) are global; ② If the diameter of Ω is sufficiently large, then every nontrivial solution of (1) blows up in finite time.

When $h=1$, we have the following results.

Theorem 4 ① If $pq<mn$, then all the solutions of (1) are global; ② If $pq\geqslant mn$ and the initial functions are small enough, then solution of (1) are global; ③ If $N=1,2,pq\geqslant mn$ and the initial functions are large enough, then there exists a solution of (1) which blows up in finite time.

2 Proof of Global Existence and Blow Up Results

Throughout this section, we assume that $u_0,v_0\in C^1(\bar{\Omega})$ and $u_0=v_0=0$ on $\partial\Omega$.

Lemma 1 Let φ be a solution of the equation,

$$\left.\begin{array}{ll}-\Delta\varphi(x)=1 & x\in\Omega\\ \varphi(x)=0 & x\in\partial\Omega\end{array}\right\} \tag{2}$$

It's obvious that $\varphi(x)>0,\forall x\in\Omega$. If Ω is "thin" at least in one direction, then $\sup\limits_{x\in\Omega}\varphi(x)$ must be small enough.

Proof Without the loss of generality we may assume that Ω is "thin" in x_1 direction. Then there exist $l>0$ ($l\ll 1$) and $k_j>0$, s.t. $\Omega\subset\subset[0,l]\times\prod\limits_{j=2}^{n}[0,k_j]=\mathbf{R}$. For $x_1\in[0,l]$ and $y\in\prod\limits_{j=2}^{n}[0,k_j]$, we set $\psi(x_1,y)=x_1(l-x_1)/2$, then $-\Delta\psi=1$ in $\mathbf{R}$ and $\psi\geqslant 0$ on $\partial\mathbf{R}$. It's obvious that $\psi>0$ in $\mathbf{R}$, $\Omega\subset\subset\mathbf{R}$, so $\psi(x)>0,\forall x\in\bar{\Omega}$. By the comparison principle, we have $\psi\geqslant\varphi,\forall\ x\in\bar{\Omega}$. But $\|\psi\|_{L^\infty(R)}=l^2/8$, therefore $\|\varphi\|_{L^\infty(\Omega)}\leqslant l^2/8$. Now we can see that if Ω is "thin" at least in one direction, then l is small enough, then $\sup\limits_{x\in\Omega}\varphi(x)$ must be small enough.

Lemma 2 Let φ be a solution of (2). Assume that there exist $a>0$, $b>0$ and $\delta>0$ s.t.

$$\left\{\begin{array}{l}a^m\geqslant b^p(\delta+\varphi)^{\frac{p}{n}}\ln^\alpha(h+a(\delta+\varphi)^{\frac{1}{m}}),\ b^n\geqslant a^q(\delta+\varphi)^{\frac{q}{m}}\ln^\beta[h+b(\delta+\varphi)^{\frac{1}{n}}]\\ a\varphi^{\frac{1}{m}}\geqslant u_0(x),\ b\varphi^{\frac{1}{n}}\geqslant v_0(x)\qquad x\in\Omega\end{array}\right\} \tag{3}$$

then the solutions of (1) are global.

Proof It's sufficient to take

$$\bar{u}=a(\delta+\varphi)^{\frac{1}{m}},\bar{v}=b(\delta+\varphi)^{\frac{1}{n}}$$

then we have

$$\left.\begin{array}{l}\Delta\bar{u}^m+\bar{v}^p\ln^\alpha(h+\bar{u})-\bar{u}_t=-a^m+b^p(\delta+\varphi)^{\frac{p}{n}}\ln^\alpha[h+a(\delta+\varphi)^{\frac{1}{m}}]\leqslant 0\\ \Delta\bar{v}^n+\bar{u}^q\ln^\beta(h+\bar{v})-\bar{v}_t=-b^n+a^q(\delta+\varphi)^{\frac{q}{m}}\ln^\beta[h+b(\delta+\varphi)^{\frac{1}{n}}]\leqslant 0\end{array}\right\} \tag{4}$$

therefore $(\bar{u},\bar{v})$ is an upper solution of (1), by the comparison principle it follows that the solutions of (1) are global.

Proof of theorem 1 and theorem 4① If we can find two positive constants a,b such that (3) holds, then we can obtain theorem 1 and theorem 4① by lemma 2. Set $K=\sup\limits_{x\in\Omega}[\delta+\varphi(x)]$, if we have

$$a^m\geqslant b^pK^{\frac{p}{n}}\ln^\alpha(h+aK^{\frac{1}{m}}),\ b^n\geqslant a^qK^{\frac{q}{m}}\ln^\beta(h+b\,K^{\frac{1}{n}}) \tag{5}$$

then the first formula in (3) holds. Put $b=a^{\frac{m}{p}}K^{\frac{1}{n}}\ln^{-\frac{\alpha}{p}}(h+aK^{\frac{1}{m}})$, we obtain the inequality for a:

$$a^{mn-pq}\geqslant K^{p+\frac{pq}{m}}\ln^{\alpha n}(h+aK^{\frac{1}{m}})\ln^{p\beta}[h+a^{\frac{m}{p}}\ln^{-\frac{\alpha}{p}}(h+aK^{\frac{1}{m}})]$$

Choose a sufficiently large s.t. $\ln^{-\frac{\alpha}{p}}(h+aK^{\frac{1}{m}})\leqslant 1$, then we only have to check $a^{mn-pq}\geqslant K^{p+\frac{pq}{m}}\ln^{\alpha n}(h+aK^{\frac{1}{m}})\ln^{p\beta}(h+a^{\frac{m}{p}})$. Since $pq<mn$, this inequality holds if we choose a sufficiently large. Since $u_0,v_0\in C^1(\bar{\Omega})$, we have $u_0\leqslant a\varphi^{\frac{1}{m}},v_0\leqslant b\varphi^{\frac{1}{n}}$ in $\bar{\Omega}$ if we choose a,b sufficiently large.

Proof of theorem 2①, theorem 3① and theorem 4② It's obvious that there exist constants $k,\theta,\eta>0$

s.t.

$$\ln^{\alpha}(h+u) \leqslant \ln^{\alpha}h + ku^{\theta},\ \ln^{\beta}(h+v) \leqslant \ln^{\beta}h + kv^{\eta} \tag{6}$$

Let

$$\bar{u} = a(\delta+\varphi)^{\frac{1}{m}},\ \bar{v} = b(\delta+\varphi)^{\frac{1}{n}}$$

where $\delta > 0$, φ is a solution of (2). Let $K = \sup\limits_{x\in\Omega}[\delta+\varphi(x)]$.

In the case of $h > 1$, we would find two positive constants a, b s.t.

$$a^{m} \geqslant b^{p} K^{\frac{p}{n}}(\ln^{\alpha}h + ka^{\theta}K^{\frac{\theta}{m}}),\ b^{n} \geqslant a^{q}K^{\frac{q}{m}}(\ln^{\beta}h + kb^{\eta}K^{\frac{\eta}{n}}) \tag{7}$$

Put $b = a^{\frac{m}{p}}K^{-\frac{1}{n}}(\ln^{\alpha}h + k\,a^{\theta}K^{\frac{\theta}{m}})^{-\frac{1}{p}}$, then we obtain the inequality for a:

$$a^{mn-pq} \geqslant K^{p+\frac{pq}{m}}(\ln^{\alpha}h + ka^{\theta}K^{\frac{\theta}{m}})^{n}(\ln^{\beta}h + ka^{\frac{m\eta}{p}}[\ln^{\alpha}h + ka^{\theta}K^{\frac{\theta}{m}})^{-\frac{\eta}{p}}]^{p} \tag{8}$$

In the proof of theorem 2①, note that $pq > mn$, we can choose a sufficiently small to satisfy (8). At the same time, let the initial values u_0, v_0 satisfy $u_0 \leqslant a\varphi^{\frac{1}{m}}, v_0 \leqslant b\varphi^{\frac{1}{n}}$, then we obtain theorem 2① by lemma 2. In the proof of theorem 3①, because Ω is "thin" at least in one direction, we see from lemma 1 that $\sup\limits_{x\in\Omega}\varphi(x)$ is sufficiently small. Let δ sufficiently small, namely K is sufficiently small. Combining this and $pq = mn$, we conclude that (8) holds. As the same as the proof of theorem 1, choose a, b sufficiently large s.t. $u_0 \leqslant a\varphi^{\frac{1}{m}}, v_0 \leqslant b\varphi^{\frac{1}{n}}$. From lemma 2, it follows theorem 3①.

In the case of $h = 1$, we can find two positive constants a, b s.t.

$$a^{m} \geqslant kb^{p}K^{\frac{p}{n}}a^{\theta}K^{\theta/m},\ b^{n} \geqslant ka^{q}K^{\frac{q}{m}}b^{\eta}K^{\frac{\eta}{n}} \tag{9}$$

Let $kK^{\frac{p}{n}+\frac{\theta}{m}} = C_1$, $kK^{\frac{q}{m}+\frac{\eta}{n}} = C_2$, $b = C_1^{-\frac{1}{p}}a^{\frac{m-\theta}{p}}$, it's sufficient to verify

$$C_1^{\eta-n} \geqslant C_2\,a^{pq-(n-\eta)(m-\theta)} \tag{10}$$

Note that $pq \geqslant mn$, then we have $pq > (n-\eta)(m-\theta)$. So we can choose a sufficiently small to satisfy (10). Finally we choose the initial functions small enough s.t. $u_0 \leqslant a\varphi^{\frac{1}{m}}, v_0 \leqslant b\varphi^{\frac{1}{n}}$. From lemma 2, we obtain theorem 4②.

Proof of theorem 2② and theorem 3② Let $\lambda = \min\{\ln^{\alpha}h, \ln^{\beta}h\} > 0$, then we have

$$\left.\begin{aligned} u_t &= \Delta u^{m} + v^{p}\ln^{\alpha}(h+u) \geqslant \Delta u^{m} + \lambda v^{p} \\ v_t &= \Delta v^{n} + u^{q}\ln^{\beta}(h+v) \geqslant \Delta v^{n} + \lambda u^{q} \end{aligned}\right\} \tag{11}$$

Now we consider the following problem:

$$\left.\begin{aligned} &u_t = \Delta u^{m} + \lambda v^{p},\ v_t = \Delta v^{n} + \lambda u^{q} && (x,t) \in \Omega\times(0,T) \\ &u(x,t) = v(x,t) = 0 && (x,t) \in \partial\Omega\times(0,T) \\ &u(x,0) = u_0(x),\ v(x,0) = v_0(x) && x \in \Omega \end{aligned}\right\} \tag{12}$$

Let $u(x,t) = w(\sqrt{\lambda}x, \lambda t)$, $v(x,t) = z(\sqrt{\lambda}x, \lambda t)$, we have

$$\left.\begin{aligned} &w_t = \Delta w^{m} + z^{p},\ z_t = \Delta z^{n} + w^{q} && (x,t) \in \Omega^{*}\times(0,T) \\ &w(x,t) = z(x,t) = 0 && (x,t) \in \partial\Omega^{*}\times(0,T) \\ &w(x,0) = u_0(x/\sqrt{\lambda}),\ z(x,0) = v_0(x/\sqrt{\lambda}) && x \in \Omega^{*} \end{aligned}\right\} \tag{13}$$

When $pq > mn$, from the conclusion in Ref.[4], it follows that there is no global solution of (13) if the initial data is sufficiently large. Then the solutions of (1) blow up in finite time.

When $pq = mn$, if Ω is sufficiently large, then Ω^{*} is sufficiently large. From the conclusion in Ref.[5], it follows that the solutions of (13) blow up in finite time. Then the solutions of (1) blow up in finite time.

Proof of theorem 4③ We consider the following problem:

$$\left.\begin{aligned} &-\Delta\varphi = \varphi^{\sigma} && x \in \Omega \\ &\varphi(x) = 0 && x \in \partial\Omega \end{aligned}\right\} \tag{14}$$

where Ω is a bounded domain in $\mathbf{R}^{N}$, $N = 1,2$, σ is an even. From the results in Ref.[8], (14) has a solution $\varphi(x)$ in $W_0^{1,2}(\Omega)$ for all positive even σ. Since $N = 1,2$, φ is a classic solution by the regularity theory, then φ is bounded. It's obvious that $\varphi(x) > 0, \forall x \in \Omega$. Let $w_1(x) = a\varphi^{\frac{1}{m}}(x), z_1(x) = b\varphi^{\frac{1}{n}}(x)$. In the following we will find two positive constants a, b s.t.

$$\left.\begin{aligned} &\Delta w_1^{m} + z_1^{p}\ln^{\alpha}(1+w_1) \geqslant 0 && x \in \bar{\Omega} \\ &\Delta z_1^{n} + w_1^{q}\ln^{\beta}(1+z_1) \geqslant 0 && x \in \bar{\Omega} \end{aligned}\right\} \tag{15}$$

namely

$$\left.\begin{aligned} & b^p\varphi^{\frac{p}{n}}\ln^{\alpha}(1+a\varphi^{\frac{1}{m}})\geqslant a^m\varphi^{\sigma} \qquad x\in\bar{\Omega} \\ & a^q\varphi^{\frac{q}{m}}\ln^{\beta}(1+b\varphi^{\frac{1}{n}})\geqslant b^n\varphi^{\sigma} \qquad x\in\bar{\Omega}\end{aligned}\right\} \tag{16}$$

It's obvious that if $\varphi(x)=0$ on $\partial\Omega$ then (16) holds. Now we consider the case $\varphi>0$. First consider the inequality for S:

$$b^pS^{\frac{p}{n}}\ln^{\alpha}(1+aS^{\frac{1}{m}})\geqslant a^mS^{\sigma},\ a^q\,S^{\frac{q}{m}}\ln^{\beta}(1+bS^{\frac{1}{n}})\geqslant b^nS^{\sigma} \qquad 0<S\leqslant S_1 \tag{17}$$

When $pq>mn$, it's easy to see that there exist positive constants a_0, b_0 s.t. $b_0{}^p=a_0{}^m$, $a_0{}^q\geqslant b_0{}^n$. Let $\sigma=2[\max\{p/n+\alpha/m, q/m+\beta/n\}+1]$(here $[\cdot]$ represents the integer part of a real number), then

$$\lim_{S\to 0}\frac{S^{\sigma-\frac{p}{n}}}{\ln^{\alpha}(1+a_0S^{\frac{1}{m}})}=\lim_{S\to 0}\frac{S^{\sigma-\frac{q}{m}}}{\ln^{\beta}(1+b_0S^{\frac{1}{n}})}=0$$

on the other hand

$$\lim_{S\to 0}\frac{S_1{}^{\frac{\sigma}{p}}}{S^{\frac{1}{n}}\ln^{\frac{\alpha}{p}}(1+a_0S^{\frac{1}{m}})}=\lim_{S\to 0}\frac{S_1{}^{\sigma}}{S^{\frac{q}{m}}\ln^{\beta}(1+b_0S^{\frac{1}{n}})}=+\infty$$

therefore there exists $S_0=S_0(a_0,b_0)<S_1$ such that when $0<S\leqslant S_0$, we have

$$\frac{S^{\sigma-\frac{p}{n}}}{\ln^{\alpha}(1+a_0S^{\frac{1}{m}})}\leqslant 1,\ \frac{S^{\sigma-\frac{q}{m}}}{\ln^{\beta}(1+b_0S^{\frac{1}{n}})}\leqslant 1$$

when $S=S_0$, we have

$$\frac{S_1{}^{\frac{\sigma}{p}}}{S_0{}^{\frac{1}{n}}\ln^{\frac{\alpha}{p}}(1+a_0S_0^{\frac{1}{m}})}\geqslant 1 \tag{18}$$

$$\frac{S_1{}^{\sigma}}{S_0^{\frac{q}{m}}\ln^{\beta}(1+b_0S_0{}^{\frac{1}{n}})}\geqslant 1 \tag{19}$$

Next we will find two positive constants a, b satisfying:

$$\left.\begin{aligned} & b^pS_0{}^{\frac{p}{n}}\ln^{\alpha}(1+a_0S_0{}^{\frac{1}{m}})\geqslant a^mS_1^{\sigma} \\ & a^qS_0{}^{\frac{q}{m}}\ln^{\beta}(1+b_0S_0{}^{\frac{1}{n}})\geqslant b^n\,S_1{}^{\sigma} \\ & a\geqslant a_0,\ b\geqslant b_0 \\ & \frac{b^p}{a^m}\geqslant 1,\ \frac{a^q}{b^n}\geqslant 1\end{aligned}\right\} \tag{20}$$

Set

$$b=a^{\frac{m}{p}}S_0{}^{-\frac{1}{n}}S_1{}^{\frac{\sigma}{p}}\ln^{-\frac{\alpha}{p}}(1+a_0S_0{}^{\frac{1}{m}}) \tag{21}$$

then we obtain the inequality for a:

$$a^{q-\frac{mn}{p}}\ln^{\frac{\alpha n}{p}}(1+a_0S_0{}^{\frac{1}{m}})\ln^{\beta}(1+b_0S_0{}^{\frac{1}{n}})\geqslant S_1{}^{\sigma(1+\frac{n}{p})}S_0{}^{-(1+\frac{q}{m})}$$

Since $pq>mn$, the above inequality holds if we choose a sufficiently large. Let $a\geqslant a_0$, from (18) and (21), we have $b\geqslant a^{\frac{m}{p}}\geqslant a_0{}^{\frac{m}{p}}=b_0$. From (19) and the second formula in (20) we obtain $a^q\geqslant b^n$. So when $0<S\leqslant S_0$, we have

$$\frac{b^p}{a^m}\geqslant 1\geqslant\frac{S^{\sigma-\frac{p}{n}}}{\ln^{\alpha}(1+a_0S^{\frac{1}{m}})}\geqslant\frac{S^{\sigma-\frac{p}{n}}}{\ln^{\alpha}(1+aS^{\frac{1}{m}})}$$

$$b^pS^{\frac{p}{n}}\ln^{\alpha}(1+a\,S^{\frac{1}{m}})\geqslant a^mS^{\sigma}$$

namely

$$b^p\,S^{\frac{p}{n}}\ln^{\alpha}(1+aS^{\frac{1}{m}})\geqslant a^mS^{\sigma}$$

as the same as the above, we also have

$$a^qS^{\frac{q}{m}}\ln^{\beta}(1+bS^{\frac{1}{n}})\geqslant b^nS^{\sigma}$$

When $S_0\leqslant S\leqslant S_1$, we have

$$b^p\,S^{\frac{p}{n}}\ln^{\alpha}(1+aS^{\frac{1}{m}})\geqslant b^pS_0{}^{\frac{p}{n}}\ln^{\alpha}(1+a_0S_0{}^{\frac{1}{m}})\geqslant a^mS_1{}^{\sigma}\geqslant a^mS^{\sigma}$$

and also have

$$a^q S^{\frac{q}{m}} \ln^\beta (1 + bS^{\frac{1}{n}}) \geqslant b^n S^\sigma$$

When $pq = mn$, let " = " hold in the first inequality in (20), then solve the equation with respect to b. Combining this and the second inequality in (20), we can guarantee the second inequality in (20) holds by choosing a, σ large enough. Thus the desired a, b, σ are found, that is, these positive numbers a, b, σ guarantee that (16) is true for all $\varphi(x)$.

Let (u_1, v_1) be a solution of (1) with the initial value (w_1, z_1). From the upper and lower solutions method, it follows that (u_1, v_1) increases in t. Let $u_0(x) \geqslant w_1(x), v_0(x) \geqslant z_1(x)$, then $u(x,t) \geqslant u_1(x,t), v(x,t) \geqslant v_1(x,t)$. Because $u_1(x,t) \geqslant w_1(x,t) > 0, v_1(x,t) \geqslant z_1(x,t) > 0, \forall x \in \Omega$, $t \geqslant 0$, there exist $\Omega_1 \subset\subset \Omega$ and $\delta > 0$, such that

$$u(x,t) \geqslant u_1(x,t) \geqslant w_1(x,t) \geqslant \delta,\ v(x,t) \geqslant v_1(x,t) \geqslant z_1(x,t) \geqslant \delta \qquad x \in \Omega_1; t \geqslant 0$$

Denote $\lambda = \min\{\ln^\alpha(1+\delta), \ln^\beta(1+\delta)\}$, and consider (1) in Ω_1. Similar to the proof of theorem 2②, we conclude that our statement is valid when $pq > mn$.

In the case that $pq = mn$, fix $\Omega_1 \subset\subset \Omega$, then there exists a constant $d > 0$ such that $\varphi(x) \geqslant d, \forall x \in \bar{\Omega}_1$. Choose a sufficiently large, then δ and λ are also sufficiently large. The transformation $(x,t) \to (\sqrt{\lambda}x, \lambda t)$, $x \in \bar{\Omega}_1, \sqrt{\lambda}x \in \bar{\Omega}^*$ yields that Ω^* is large enough. Hence as in the proof of theorem 3②, our conclusion is valid when $pq = mn$.

Acknowledgements I would like to thank my supervisor Prof. M.X.Wang, for his help and encouragement.

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一个退化抛物型方程组解的整体存在性与爆破

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摘　要　研究了一个具有齐次 Dirichlet 边界条件以及正的初值条件的退化抛物型方程组：$u_t = \Delta u^m + v^p \ln^\alpha(h+u), v_t = \Delta v^n + u^q \ln^\beta(h+v)$. 该方程组描述了一个具有 2 种连续介质的燃烧过程及热扩散过程. 本文利用上、下解方法获得了方程组解的整体存在性和爆破的条件.

关键词　抛物型方程组；退化；整体存在性；爆破

中图分类号　O175.26