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Anti-periodic solutions to a class of second-order evolution equations

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Abstract: In this paper we discuss the anti-periodic problem for a class of abstract nonlinear second-order evolution equations associated with maximal monotone operators in Hilbert spaces and give some new assumptions on operators. We establish the existence and uniqueness of anti-periodic solutions, which improve andgeneralize the results that have been obtained. Finally we illustrate the abstract theory by discussing a simple example of an anti-periodic problem for nonlinear partial differential equations.

Key words: maximal monotone operator; anti-periodic solution; Poincaré inequality; second-order evolution equations

Let H be a real Hilbert space of the inner product $\langle \cdot, \cdot \rangle$ with the norm $\| \cdot \|$, we consider the nonlinear second-order anti-periodic problems

$$\begin{cases} -u''(t) + au'(t) + A(t)u(t) \ni f(t) \qquad \text{a.e. } 0 \leqslant t \leqslant T & (1\text{a}) \\ u(T) = -u(0), \ u'(T) = -u'(0) & (1\text{b}) \end{cases}$$

for the H-valued $u(t)$, where $a \in \mathbf{R}$, and for each $t \in [0, T]$, $A(t)$ is a nonlinear (possibly multivalued and unbounded) maximal monotone operator.

Since anti-periodic problems have important applications in auto-control, partial differential equations and engineering, they have been studied extensively since the 1990's. First-order anti-periodic problems have been studied more in Refs. [1 – 3]. Subsequently, Aftabizadeh, et al. made use of maximal monotone theory or m-accretive theory to consider the anti-periodic solutions for second order equations in Hilbert space and general Banach space[4]. Aftabizadeh extended this problem to a higher order, but it requires that A be a linear self-adjoint (possibly unbounded) monotone operator. Later Aizicovici[5] proved the existence and uniqueness of problem (1) when $A(t) \equiv A \in \mathbf{R}$ and A is an odd, nonlinear, m-accretive multivalued operator. Aftabizadeh, et al.[6] considered the solutions to problem (1) under the condition that $A(t)x$ is continuous about t and x. This paper establishes the existence and uniqueness of solutions to problem (1) in the case that $A(t)x$ loses the continuity and the boundedness. Our technique employs some ideas from Refs. [1,7 – 9], where Cauchy problems regarding operator families $\{A(t)\}$ were considered.

1 Preliminaries

Definition 1 The nonlinear (possibly multivalued) operator $A: D(A) \subset H \rightarrow H$ is said to be monotone if

$$\langle x_1 - x_2, y_1 - y_2 \rangle \geqslant 0 \qquad \forall\, x_1, x_2 \in D(A);\ y_1 \in Ax_1; y_2 \in Ax_2$$

and is said to be maximal monotone if, in addition, $R(I + \lambda A) = H$, for all $\lambda > 0$ (or equivalently, for some $\lambda_0 > 0$).

Definition 2 If $u: [0, T] \rightarrow H$ is continuously differentiable, $u'(t)$ is absolutely continuous, $u''(t)$ exists for a.e. $t \in [0, T]$, $u(T) = -u(0)$, $u'(T) = -u'(0)$, and we can find $v(t) \in A(t)u(t)$ a.e. $t \in [0, T]$, such that $-u''(t) + au'(t) + v(t) = f(t)$ a.e. $t \in [0, T]$, then we may say that u is a strong solution to (1).

Let $\{A(t): 0 \leqslant t \leqslant T\}$ is a family of maximal monotone operators in H, then for $\lambda > 0$, we can define the Yosida approximation of $A(t)$:

$$J_\lambda(t) = (I + \lambda A(t))^{-1},\ A_\lambda(t) = \lambda^{-1}(I - J_\lambda(t))$$

The properties of $J_\lambda(t)$ and $A_\lambda(t)$ can be found in Ref. [10].

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Let $E = L^2(0,T;H)$ and $|u| = \left(\int_0^T \|u(t)\|^2 \mathrm{d}t\right)^{\frac{1}{2}}$, $\forall u \in E$, then $(E, |\cdot|)$ is a Hilbert space.

Lemma 1[10] Let $u:[0,T] \to H$ is differentiable, then so is $\|u(t)\|^2$ and $\frac{\mathrm{d}}{\mathrm{d}t}\|u(t)\|^2 = 2\langle u'(t), u(t)\rangle$.

Lemma 2[10] Suppose A and B are two maximal monotone operators in H, and $\{x_\lambda\}$ is the solution to the equation $y \in A_\lambda x_\lambda + Bx_\lambda + \varepsilon x_\lambda$ where $\varepsilon > 0$, A_λ is the Yosida approximation of A. If $\{x_\lambda\}$ and $\{A_\lambda x_\lambda\}$ are bounded as $\lambda \to 0^+$, then there is $x_\varepsilon \in D(A) \cap D(B)$, such that $\lim\limits_{\lambda \to 0^+} x_\lambda = x_\varepsilon$ and x_ε satisfies $y \in Ax_\varepsilon + Bx_\varepsilon + \varepsilon x_\varepsilon$. If $\lim\limits_{\varepsilon \to 0^+} x_\varepsilon = x$, then $y \in (A + B)x$.

Lemma 3[6] Assume that $u \in W^{1,2}(0,T;H)$ satisfies $u(0) = -u(T)$, then $\|u(t)\| \leqslant T^{\frac{1}{2}} |u'|$, $\forall t \in [0,T]$.

2 Main Result

We give the hypotheses on the t-dependence of $A(t)$ now.

1) For all $t \in [0,T]$, $A(t)$ is maximal monotone and satisfies $A(0)(-x) = -A(T)x$, $\forall x \in D(A(t)) \equiv D$.

2) For all $t \in [0,T]$, $0 \in A(t)(0)$.

3) ① There exists a continuous function $g:[0,T] \to H$, which is almost everywhere differential, $g' \in L^2(0,T;H)$; a bounded mapping L: $\mathbf{R}^+ \times \mathbf{R}^+ \to \mathbf{R}^+$; and $\lambda_0 > 0$, $h > 0$, such that

$$\langle A_\lambda(t)x - A_\lambda(s)y, x - y\rangle \geqslant -\|g(t) - g(s)\| \cdot \|x - y\| L(\|x\|, \|y\|)(1 + \|A_\lambda(s)y\|)|t - s|$$
$$\text{for all } x, y \in H;\ 0 < \lambda < \lambda_0;\ 0 \leqslant s, t \leqslant T;\ |t - s| < h$$

② $A_\lambda(t)u(t)$ is absolutely continuous on $[0,T]$ if $u \in W^{2,2}(0,T;H) \cap C^2(0,T;H)$.

3′) There exists $g:[0,T] \to \mathbf{R}$, which is absolutely continuous and $g' \in L^2(0,T)$; a bounded mapping L: $\mathbf{R}^+ \to \mathbf{R}^+$; and $\lambda_0 > 0$ and $h > 0$ such that

$$\|A_\lambda(t)x - A_\lambda(s)x\| \leqslant |g(t) - g(s)| L(\|x\|)(1 + \|A_\lambda(s)x\|)|t - s|$$
$$\text{for all } x \in H;\ 0 < \lambda < \lambda_0;\ 0 \leqslant s, t \leqslant T;\ |t - s| < h$$

Remark Hypothesis 3′) easily implies 3) when 1) is satisfied.

Now we give a simple example which satisfies all hypotheses.

Example 1 Let A be a maximal monotone operator in H, $0 \in A^{-1}(0)$, g: $[0,T] \to \mathbf{R}^+$ is absolutely continuous, $g' \in L^2(0,T)$, we set $A(t)y = Ay + g(t)y$, $\forall y \in D(A(t)) \equiv D(A)$. By calculations we have

$$\|J_\lambda(t)y - J_\lambda(s)y\| \leqslant \lambda|g(t) - g(s)|\,\|y\|/(1 - \lambda|g(t)|) \quad \text{for all } \lambda > 0;\ 0 \leqslant t, s \leqslant T;\ y \in H$$

Choose $\lambda_0 < \frac{1}{2M}$, $M = \sup\{|g(t)| : 0 \leqslant t \leqslant T\}$, then

$$\|A_\lambda(t)y - A_\lambda(s)y\| \leqslant 2|g(t) - g(s)|\,\|y\| \qquad \text{for all } 0 < \lambda \leqslant \lambda_0;\ 0 \leqslant t, s \leqslant T;\ y \in H$$

Thus clearly, $\{A(t): 0 \leqslant t \leqslant T\}$ satisfies 1), 2), 3′).

Theorem Suppose $A(t)$ satisfies 1), 2), 3), then for every $f \in E$, Eq.(1) has one and only one solution $u \in W^{2,2}(0,T;H)$.

Proof of uniqueness Let u and v be two solutions to problem (1). Multiplying $(1\mathrm{a})_u - (1\mathrm{a})_v$ by $u(t) - v(t)$, then integrating it over $[0,T]$, one obtains

$$0 = -\int_0^T \langle u''(t) - v''(t), u(t) - v(t)\rangle \mathrm{d}t + a\int_0^T \langle u'(t) - v'(t), u(t) - v(t)\rangle \mathrm{d}t + \int_0^T \langle A(t)u(t) - A(t)v(t), u(t) - v(t)\rangle \mathrm{d}t$$

Via the monotonicity of $A(t)$, lemma 1 and (1b), we get

$$|u' - v'|^2 = \int_0^T \|u'(t) - v'(t)\|^2 \mathrm{d}t \leqslant 0$$

Using lemma 3, we deduce

$$\|u(t) - v(t)\| \leqslant 0 \qquad \forall t \in [0,T]$$

Hence $u(t) \equiv v(t)$, $\forall t \in [0,T]$.

Lemma 4[6] Define $B: D(B) \subset E \to E$, $Bu = -u'' + au'$, $u \in D(B)$, where

$$D(B) = \{u \in W^{2,2}(0,T;H): u(0) = -u(T), u'(T) = -u'(0)\}$$

Then B is maximal monotone in E.

We define $\mathscr{A} = \{[u,v] \in E \times E: u(t) \in D(A(t)), v(t) \in A(t)u(t) \text{ a.e. } t \in [0,T]\}$, then $\mathscr{A}$ is the realization of A in E and we have lemma 5.

Lemma 5 $\mathscr{A}$ is maximal monotone in E.

Proof Easily we can see that $\mathscr{A}$ is monotone. For $\bar{f} \in E$, we can define $\bar{g}(t) = (I + A(t))^{-1}\bar{f}(t)$, since $(I + A(t))^{-1}$ is nonexpansive and $0 \in A^{-1}(t)(0)$, $\forall t \in [0,T]$, then $\|\bar{g}(t)\| \leqslant \|\bar{f}(t)\|$ a.e. $t \in [0,T]$, i.e. $\bar{g} \in E$. Hence $R(I + \mathscr{A}) = E$.

We now return to the existence proof of the theorem.

Since B is maximal monotone in E, then $B + \mathscr{A}_\lambda + \varepsilon I$ is surjective for every $\varepsilon > 0, \lambda > 0$. Then, for every $\varepsilon > 0, \lambda > 0$ and $f \in E$, there is a unique $u_{\varepsilon,\lambda} \in D(B)$ satisfying

$$Bu_{\varepsilon,\lambda} + \mathscr{A}_\lambda u_{\varepsilon,\lambda} + \varepsilon u_{\varepsilon,\lambda} = f_\varepsilon \tag{2}$$

or

$$\begin{cases} -u''_{\varepsilon,\lambda}(t) + au'_{\varepsilon,\lambda}(t) + A_\lambda(t)u_{\varepsilon,\lambda}(t) + \varepsilon u_{\varepsilon,\lambda}(t) = f_\varepsilon(t) \qquad \text{a.e. } 0 \leqslant t \leqslant T & \text{(3a)} \\ u_{\varepsilon,\lambda}(T) = -u_{\varepsilon,\lambda}(0),\ u'_{\varepsilon,\lambda}(T) = -u'_{\varepsilon,\lambda}(0) & \text{(3b)} \end{cases}$$

where $f_\varepsilon \in W^{1,2}(0,T;H)$; $f_\varepsilon(T) = -f_\varepsilon(0)$; $f_\varepsilon \to f(\varepsilon \to 0^+)$(in E).

From the hypotheses 3)② we know that $u''_{\varepsilon,\lambda}(t)$ is absolutely continuous on $[0,T]$ and differential almost everywhere. The hypothesis 1) implies that $A_\lambda(T)u_{\varepsilon,\lambda}(T) = -A_\lambda(0)u_{\varepsilon,\lambda}(0)$, hence

$$u''_{\varepsilon,\lambda}(T) = -u''_{\varepsilon,\lambda}(0) \tag{4}$$

Multiplying (3a) by $u_{\varepsilon,\lambda}(t)$ and integrating over $(0,T)$, we have

$$\int_0^T \langle f_\varepsilon(t), u_{\varepsilon,\lambda}(t)\rangle \mathrm{d}t = -\int_0^T \langle u''_{\varepsilon,\lambda}(t),\ u_{\varepsilon,\lambda}(t)\rangle \mathrm{d}t + a\int_0^T \langle u'_{\varepsilon,\lambda}(t),\ u_{\varepsilon,\lambda}(t)\rangle \mathrm{d}t + \int_0^T \langle A_\lambda(t)u_{\varepsilon,\lambda}(t),\ u_{\varepsilon,\lambda}(t)\rangle \mathrm{d}t + \varepsilon\int_0^T \|u_{\varepsilon,\lambda}(t)\|^2 \mathrm{d}t$$

Since $0 \in A_\lambda(t)(0)$, $A_\lambda(t)$ is maximal monotone, using lemma 1, lemma 3 and (3b), we get

$$|u'_{\varepsilon,\lambda}|^2 = \int_0^T \|u'_{\varepsilon,\lambda}(t)\|^2 \mathrm{d}t \leqslant |f_\varepsilon| \cdot |u_{\varepsilon,\lambda}| \leqslant T|f_\varepsilon| \cdot |u'_{\varepsilon,\lambda}|$$

so

$$|u'_{\varepsilon,\lambda}| \leqslant T|f_\varepsilon| \tag{5}$$

Invoking lemma 3, we obtain

$$\|u_{\varepsilon,\lambda}(t)\| \leqslant T^{\frac{1}{2}}|u'_{\varepsilon,\lambda}| \leqslant T^{\frac{3}{2}}|f_\varepsilon| \tag{6}$$

Differentiate (3a) with respect to t, multiply the resulting equation by $u'_{\varepsilon,\lambda}(t)$ and integrate over $(0,T)$, we have

$$\int_0^T \langle f'_\varepsilon(t), u'_{\varepsilon,\lambda}(t)\rangle \mathrm{d}t = -\int_0^T \langle u'''_{\varepsilon,\lambda}(t),\ u'_{\varepsilon,\lambda}(t)\rangle \mathrm{d}t + a\int_0^T \langle u''_{\varepsilon,\lambda}(t),\ u'_{\varepsilon,\lambda}(t)\rangle \mathrm{d}t + \int_0^T \langle \frac{\mathrm{d}}{\mathrm{d}t}(A_\lambda(t)u_{\varepsilon,\lambda}(t)),\ u'_{\varepsilon,\lambda}(t)\rangle \mathrm{d}t + \varepsilon\int_0^T \|u'_{\varepsilon,\lambda}(t)\|^2 \mathrm{d}t \tag{7}$$

From (6) and 3)① it follows that there exists a function g: $[0,T] \to H$, which is differential almost everywhere, $g' \in E, \lambda_0 > 0$, such that

$$\begin{aligned} &\langle A_\lambda(t+h)u_{\varepsilon,\lambda}(t+h) - A_\lambda(t)u_{\varepsilon,\lambda}(t),\ u_{\varepsilon,\lambda}(t+h) - u_{\varepsilon,\lambda}(t)\rangle \geqslant \\ &\quad -h\|(t+h) - g(t)\|\,\|u_{\varepsilon,\lambda}(t+h) - u_{\varepsilon,\lambda}(t)\|L(1 + \|A_\lambda(t)u_{\varepsilon,\lambda}(t)\|) \geqslant \\ &\quad -2hT^{\frac{2}{3}}L|f_\varepsilon|\,\|g(t+h) - g(t)\|(1 + \|A_\lambda(t)u_{\varepsilon,\lambda}(t)\|) \end{aligned} \tag{8}$$

where $L = \sup\{L(\|u_{\varepsilon,\lambda}(t+h)\|, \|u_{\varepsilon,\lambda}(t)\|): t, t+h \in [0,T]\}$, denote $L_1 = 2T^{\frac{2}{3}}L|f_\varepsilon|$. From 3)② we know that $A_\lambda(t)u_{\varepsilon,\lambda}(t)$ is differential almost everywhere. Hence we divide (8) by h^2, let $h \to 0^+$, then yields

$$\langle \frac{\mathrm{d}}{\mathrm{d}t}(A_\lambda(t)u_{\varepsilon,\lambda}(t)), u'_{\varepsilon,\lambda}(t)\rangle \geqslant -L_1\|g'(t)\|(1 + \|f_\varepsilon(t)\| + \varepsilon\|u_{\varepsilon,\lambda}(t)\| + |a|\,\|u'_{\varepsilon,\lambda}\| + \|u''_{\varepsilon,\lambda}(t)\|) \tag{9}$$

Integrating (9) over $(0,T)$, by (5), (6) and Hölder inequality, we have

$$\int_0^T \langle \frac{\mathrm{d}}{\mathrm{d}t}(A_\lambda(t)u_{\varepsilon,\lambda}(t), u'_{\varepsilon,\lambda}(t)\rangle \mathrm{d}t \geqslant -L_1T^{\frac{1}{2}}|g'| - L_1|g'|\,|f_\varepsilon| -$$

$$\varepsilon L_1 T^2 |g'||f_\varepsilon| - |a| L_1 T |g'||f_\varepsilon| - L_1 |g'||u''_{\varepsilon,\lambda}| \tag{10}$$

Putting (10) into (7), using lemma 1, (3b), (4) and (5), we find

$$|u''_{\varepsilon,\lambda}|^2 = \int_0^T \|u''_{\varepsilon,\lambda}(t)\|^2 \mathrm{d}t \leqslant L_1 T^{\frac{1}{2}} |g'| + L_1 |g'||f_\varepsilon| + \varepsilon L_1 T^2 |g'||f_\varepsilon| + L_1 T |a||g'||f_\varepsilon| + L_1 |g'||u''_{\varepsilon,\lambda}| + T|f_\varepsilon||f'_\varepsilon|$$

Noting (2) it follows that $\{A_\lambda u_{\varepsilon,\lambda}\}$ is bounded in E, hence by lemma 2, we can let $\lambda \to 0^+$. Specifically, let $u_\varepsilon = \lim\limits_{\lambda \to 0^+} u_{\varepsilon,\lambda}$ (in E), where $u_\varepsilon \in D(A) \cap D(B)$ satisfies

$$Bu_\varepsilon + \mathscr{A} u_\varepsilon + \varepsilon u_\varepsilon \ni f_\varepsilon \tag{11}$$

or equivalently

$$\begin{cases} -u''_\varepsilon(t) + a u'_\varepsilon(t) + A(t) u_\varepsilon(t) + \varepsilon u_\varepsilon(t) \ni f_\varepsilon(t) \qquad \text{a.e. } 0 \leqslant t \leqslant T & (12\text{a}) \\ u_\varepsilon(T) = -u_\varepsilon(0),\ u'_\varepsilon(T) = -u'_\varepsilon(0) & (12\text{b}) \end{cases}$$

Multiplying (12a) by $u_\varepsilon(t)$ and integrating over $(0, T)$, using the maximal monotonicity of $A(t)$, similarly to (5), we also have $|u'_\varepsilon| \leqslant T|f_\varepsilon|$. This and lemma 3 leads to

$$|u_\varepsilon| \leqslant T^2 |f_\varepsilon|,\ \|u_\varepsilon(t)\| \leqslant T^{\frac{3}{2}} |f_\varepsilon| \qquad \forall t \in [0, T]$$

Forming the inner product of $(12\text{a})_\varepsilon - (12\text{a})_\eta$ with $u_\varepsilon(t) - u_\eta(t)$, and integrating the result over $(0, T)$, making use of the maximal monotonicity of $A(t)$ and lemma 1, we obtain

$$\int_0^T \langle u''_\varepsilon(t) - u''_\eta(t),\ u_\varepsilon(t) - u_\eta(t)\rangle \mathrm{d}t + \int_0^T \langle \varepsilon u_\varepsilon(t) - \eta u_\eta(t), u_\varepsilon(t) - u_\eta(t)\rangle \mathrm{d}t \leqslant \int_0^T \langle f_\varepsilon(t) - f_\eta(t),\ u_\varepsilon(t) - u_\eta(t)\rangle \mathrm{d}t$$

hence

$$|u'_\varepsilon - u'_\eta|^2 \leqslant \int_0^T \|\varepsilon u_\varepsilon(t) - \eta u_\eta(t)\| \|u_\varepsilon(t) - u_\eta(t)\| \mathrm{d}t + \int_0^t \|f_\varepsilon(t) - f_\eta(t)\| \|u_\varepsilon(t) - u_\eta(t)\| \mathrm{d}t \leqslant c_1(\varepsilon + \eta) + c_2 |f_\varepsilon - f_\eta|$$

where $c_1, c_2 > 0$ is independent of ε, η. For $f_\varepsilon \to f(\varepsilon \to 0^+)$ (in E), so $\{u'_\varepsilon\}$ is a Cauchy sequence in E. Recalling lemma 3, we get

$$\|u_\varepsilon(t) - u_\eta(t)\| \leqslant T^{\frac{1}{2}} |u'_\varepsilon - u'_\eta| \qquad \forall t \in [0, T]$$

hence $\{u_\varepsilon\}$ is a Cauchy sequence in $C([0, T]; H)$. Let $u_\varepsilon \to u(\varepsilon \to 0^+)$ (in $C([0, T]; H)$), the closedness of $\mathscr{A} + B$ in E enables us to pass to the limit in (12) as $\varepsilon \to 0^+$, and conclude that $(B + \mathscr{A})u \ni f$ as desired.

3 Application

In this section we give a simple example in a partial differential equation. Let $\gamma \subset \mathbf{R} \times \mathbf{R}$ be a maximal monotone set, $0 \in \gamma(0)$, let $\Omega \subset \mathbf{R}^n$ be a bounded domain with smooth boundary $\partial\Omega$. Now we consider:

$$\left.\begin{aligned} &\frac{\partial^2}{\partial t^2} u(t, x) = -\Delta u(t, x) + g(t) u(t, x) + f(t, x) && (t, x) \in (0, T) \times \Omega \\ &\frac{\partial u}{\partial n} = -\gamma(u) && (t, x) \in (0, T) \times \partial\Omega \\ &u(0, x) = -u(T, x),\ u'(0, x) = -u'(T, x) && \text{in } \Omega \end{aligned}\right\} \tag{13}$$

Here we assume $H = L^2(\Omega)$, $g: [0, T] \to \mathbf{R}^+$ is continuous, $g' \in L^2(0, T)$, $f \in L^2(0, T; H)$, we define

$$A(t) u = -\Delta u + g(t) u \qquad \forall u \in D$$

$$D = \{u \in L^2(\Omega): -\Delta u \in L^2(\Omega), gu \in L^2(\Omega), \frac{\partial u}{\partial n} = -\gamma(u)\}$$

From example 1 we know that $\{A(t)\}_{0 \leqslant t \leqslant T}$ are maximal monotone operators in $L^2(\Omega)$ and satisfy hypotheses 1), 2) and 3'), thus (13) is equivalent to the abstract second order equation in $L^2(\Omega)$.

$$\begin{cases} u''(t) = A(t) u(t) + f(t) \qquad \text{a.e. } t \in [0, T] & (14\text{a}) \\ u(0) = -u(T),\ u'(0) = -u'(T) & (14\text{b}) \end{cases}$$

So by the theorem we know that problem (14) has a unique solution $u \in H^2(\Omega)$.

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关于一类二阶发展方程的反周期解

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摘　要　本文研究了在 Hilbert 空间中与极大单调算子族相联系的抽象的二阶发展方程的反周期问题, 给出了关于算子族 $\{A(t): 0 \leqslant t \leqslant T\}$ 的新的假设, 并在此假设下证明了反周期解的存在性与惟一性, 推广了已有的结果. 最后给出一个例子说明抽象的反周期问题在非线性偏微分方程中的简单应用.

关键词　极大单调算子; 反周期解; Poincaré 不等式; 二阶发展方程

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