Journal of Southeast University (English Edition) Vol.20 No.1 Mar. 2004 ISSN 1003—7985

Study on conveyor non-linear dynamics and its effect on dynamic behavior

Li Guangbu

(Department of Mechanical and Electric Engineering, Shanghai Teachers University, Shanghai 201400, China)

Abstract: A conveyor linear system assumption is based on an approximate description of belt mechanics behavior and constant elastic module. It produces analysis errors and improper dynamics simulation in large conveyors. The belt non-linear characteristics based on sag are described and the belt equivalent elastic module expression is deduced. The relationship between belt-equivalent elastic module and elastic module is studied, and their ratio varies from 0.1 to 1.0. The non-linear motion equation with a lumped element model is put forward. Its increment equation and numerical solution are built. A dynamics simulation on a conveyor is carried out, mainly to calculate and compare belt speed, acceleration, tension, displacement of gravity take-up and wave period with linear and non-linear models. It shows that the simulation errors between two models vary from 6% to 50%.

Key words: belt sag; equivalent elastic module; non-linear equation; dynamics simulation

Over the past 30 years, the belt conveyor transportation of high mass flows over long distances has been put into wide applications. At the same time, worldwide attention was paid to the dynamic analysis and simulation technology of belt conveyors. Engineers in many countries developed simulation software used in engineering[1-8]. Undoubtedly, they play an important role in dynamic prediction, analysis and design of belt conveyors. It is true that most of the attention goes to conveyors' linear dynamic system. Only a few papers deal with the non-linear characteristics of the belt[5], but not from the view of non-linear based geometry on belt sag. As a matter of fact, a belt has a sag between two idlers, which may produce additional longitudinal expansion and contraction of a belt during different conveyor operation stages. So the concept of belt equivalent elastic module is put forward[6]. But until now, no further researches have been carried out on this topic and dynamics calculation and a simulation of conveyors' non-linear system has not been made. Therefore, it's significant to study and simulate the belt conveyor dynamics of a non-linear system and compare them with those of a linear system.

1 Conveyor Dynamic Model

1.1 Conveyor linear system

There are many ways to set up a conveyor motion equation for linear system[1-6], such as continuous system model, lumped mass-spring model and beam element model, but the commonest model is a lumped mass-spring one. So its motion equation can be described by the following expression:

$$\boldsymbol{m}\ddot{\boldsymbol{x}}+\boldsymbol{c}\dot{\boldsymbol{x}}+\boldsymbol{k}\boldsymbol{x}=\boldsymbol{F} \tag{1}$$

where $\boldsymbol{m}$ is the mass matrix of the system, including rotation inertia of motors, reducers, hydraulic couplings and pulleys; $\boldsymbol{c}$ is the damping parameter matrix, $c_i = k_i\tau_i$, τ_i is the viscous constant; $\boldsymbol{k}$ is the stiffness constant matrix, $k_i = E_b B / L_i$, E_b is the belt elastic module per width, L_i is the belt length of any element, B is the belt width; $\ddot{\boldsymbol{x}}$, $\dot{\boldsymbol{x}}$, $\boldsymbol{x}$ are the row vectors of acceleration, velocity and displacement of a belt at any given element and moment, respectively; $\boldsymbol{F}$ is the row vector of force applied by motors or brakes at any time and running resistance, including vertical, horizontal and part resistance.

The above dynamics equation may be solved with known boundary conditions. They are displacement boundary conditions ($x=0$, or other given displacements) and velocity boundary conditions.

1.2 Conveyor non-linear characteristic

For the belt between two idlers, the sag expression is given by Ref.[7] in engineering unit. Its corresponding form in international units is

$$\Delta_h = \frac{1.23ql^2}{F_0}$$

The curve length of the belt between two idlers is

$$l_0 = l + \frac{4q^2l^3}{F_1{}^2}$$

Received 2003-05-14.

Foundation item: The Culture Exchange Program between China and Czech Governments (No.199899).

Biography: Li Guangbu (1962—), male, doctor, professor, li_guangbu@hotmail.com.

where Δ_h is the belt sag between two idlers, q is the unit mass of material and belt per meter, l is the idler spacing, l_0 is the belt curve length between two idlers, F_1 is the belt tension at position 1.

Therefore, when this belt moves to position 2 along the conveyor, the belt tension increases from F_1 to F. Including elastic and curve length one, the total stretch of the belt is

$$\Delta l = \frac{F - F_1}{E_b B} l_0 + 4q^2 l^3 \left(\frac{1}{F_1{}^2} - \frac{1}{F^2} \right) \tag{2}$$

Its strain of this belt section is

$$\varepsilon = \frac{\Delta l}{l_0} = \frac{F - F_1}{E_b B} + \frac{4q^2 l^2}{l_0} \left(\frac{1}{F_1{}^2} - \frac{1}{F^2} \right) \tag{3}$$

Because $\Delta_h \leqslant 0.025l$, $l_0 \leqslant 1.002l$. Thus, it may be regarded as $l_0 \approx l$. Meanwhile,

$$F = F_1 + \frac{\partial F}{\partial x} \mathrm{d}x$$

So substituting these into Eq.(3) and operation yields

$$\varepsilon = \left(\frac{1}{E_b B} + \frac{8q^2 l^2}{F^3} \right) \frac{\partial F}{\partial x} \mathrm{d}x \tag{4}$$

Next an equivalent belt is analyzed. It is an ideal belt supposed not to sag between two idlers. This special belt only has elastic strain. With the same method, the strain of this equivalent belt is

$$\varepsilon = \frac{1}{EB} \frac{\partial F}{\partial x} \mathrm{d}x \tag{5}$$

Because Eqs.(4) and (5) are equivalent, there is

$$\frac{1}{E_b B} + \frac{8q^2 l^2}{F^3} = \frac{1}{EB}$$

That is

$$E = \frac{E_b}{1 + \dfrac{8q^2 l^2 E_b B}{F^3}} \tag{6}$$

where E is the belt equivalent elastic module per width.

From Eq.(6), it is easy to find out that belt equivalent elastic module E is not a constant. It is equal to belt elastic module when belt tension F is infinitely great, which means belt sag is zero. Also when belt tension F is infinitely small, which means belt sag is infinitely great, belt equivalent elastic module E is nearly zero. So the belt equivalent elastic module E is the function of its displacement x, for belt tension F relates to its displacement x. This kind of phenomenon is called a non-linear characteristic.

From the above studies, it is shown that the belt equivalent elastic module represents different characteristics under different operation conditions and at different positions on the conveyor.

Here is an example given to illustrate the non-linear characteristic of the belt. The belt specification is GX-1000, with a belt width of 1 200 mm. Its elastic module per width is 846.78 kN/cm. It transports material 750 t/h under full load conditions. Material and belt mass at unit length are 104 kg/m and 30 kg/m, respectively. Idler spacing is 1 m on the carry strand and 3 m on the return strand.

Therefore according to Eq.(6), the belt equivalent elastic module changes with the belt tension alteration as shown in Fig.1 and Fig.2. It can be seen that in the area of the belt tension 8 to 62.5 kN, the belt equivalent elastic module may be smaller than the belt elastic module, with their ratio about 10%. Also its non-linear property is very obvious, as shown in Fig. 1. This situation may often occur on the return strand of the belt conveyor. Similarly, when conveyor is fully loaded, the area of the belt tension is within 10 to 150 kN, the belt equivalent elastic module may also be smaller than the belt elastic module with their ratios about 10%. The non-linear property is also very complex in this tension area on the carry strand, as shown in Fig.2.

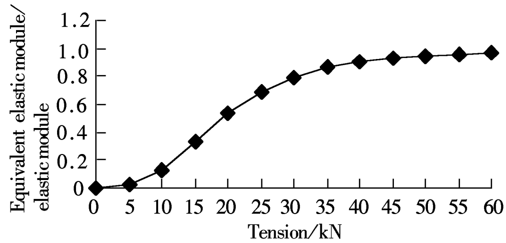

Fig.1 Relationship between belt force and its equivalent elastic module (return strand)

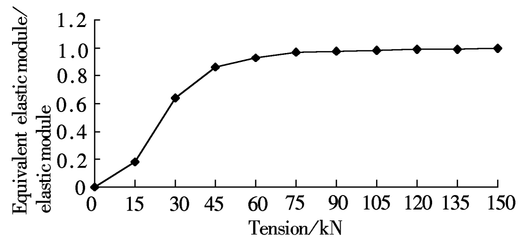

Fig.2 Relationship between belt force and its equivalent elastic module (full load and carry strand)

According to the above analysis, this non-linear influence on conveyor parameters, such as belt equivalent elastic module, is obviously big. Any neglect of this non-linear influence may result in great errors in calculation of conveyor dynamics behavior. Therefore it should be fully considered in design and simulation, especially on the return strand of conveyor.

On the other hand, in a belt-lumped, mass-spring model, belt stiffness coefficient k_i in any section length may be expressed as

$$k_i = \frac{EB}{L_i} \tag{7}$$

where L_i is the length of any belt section of a lumped mass-spring model.

Thus the belt stiffness coefficient k_i also has non-linear characteristics similar to that of the belt equivalent stiffness. It changes from time to time and from position to position.

1.3 Numerical solution for non-linear system

Because a large belt conveyor is over hundreds or thousands of meters in length, the number of lumped elements is in the hundreds. So the analytics are very difficult to achieve the purpose of solution of dynamics response for the conveyor. The fast development in modern computer technology puts forward many digital calculation methods for a complex non-linear lumped-element system[8]. These methods use numerical step-by-step integration for system motion equation. They divide time history of response into many time sections or steps Δt and build response results at the beginning and end of Δt. In each time step Δt, a response is carried out according to the calculation principle of the linear system. The calculated results are used to modify the system data including displacement, stress and strain in this Δt, which are regarded as system characteristics for the next step Δt. Thus the analysis of a non-linear system is taken approximately as the analysis of a series of linear systems, which vary one by one. Therefore it is possible for the response solution of non-linear dynamics of a conveyor system.

The characteristic of a non-linear system is that each element in $\boldsymbol{m}$, $\boldsymbol{c}$, $\boldsymbol{k}$ of motion equation (1) relates to its acceleration, velocity and displacement. Therefore it relates to time. If system motion equation (1) is regarded as a balance of system inertial force $\boldsymbol{F}_t^{\mathrm{I}}$, viscous force $\boldsymbol{F}_t^{\mathrm{D}}$, elastic force $\boldsymbol{F}_t^{\mathrm{S}}$ and active force including running resistances $\boldsymbol{F}_t$, then the balance equations at moment t and $t + \Delta t$ are respectively

$$\boldsymbol{F}_t^{\mathrm{I}} + \boldsymbol{F}_t^{\mathrm{D}} + \boldsymbol{F}_t^{\mathrm{S}} = \boldsymbol{F}_t$$

$$\boldsymbol{F}_t^{\mathrm{I}} + \Delta\boldsymbol{F}_t^{\mathrm{I}} + \boldsymbol{F}_t^{\mathrm{D}} + \Delta\boldsymbol{F}_t^{\mathrm{D}} + \boldsymbol{F}_t^{\mathrm{S}} + \Delta\boldsymbol{F}_t^{\mathrm{S}} = \boldsymbol{F}_{t+\Delta t}$$

where the force increments are

$$\Delta F_t^{\mathrm{I}} = F_{t+\Delta t}^{\mathrm{I}} - F_t^{\mathrm{I}} = \boldsymbol{M}_t \Delta\ddot{\boldsymbol{x}}$$

$$\Delta F_t^{\mathrm{D}} = F_{t+\Delta t}^{\mathrm{D}} - F_t^{\mathrm{D}} = \boldsymbol{C}_t \Delta\dot{\boldsymbol{x}}$$

$$\Delta F_t^{\mathrm{S}} = F_{t+\Delta t}^{\mathrm{S}} - F_t^{\mathrm{S}} = \boldsymbol{K}_t \Delta \mathrm{x}$$

$$\Delta F_t = F_{t+\Delta t} - F_t$$

$\boldsymbol{M}_t$, $\boldsymbol{C}_t$, $\boldsymbol{K}_t$ in the above expressions are system mass, damping, stiffness matrices which vary with time. In order to calculate them approximately, time is divided into n very small time intervals Δt, during which each element in $\boldsymbol{M}_t$, $\boldsymbol{C}_t$, $\boldsymbol{K}_t$ is regarded as a constant. So the previous motion equation in each integral step Δt is expressed as

$$\boldsymbol{M}_t \Delta\ddot{\boldsymbol{x}} + \boldsymbol{C}_t \Delta\dot{\boldsymbol{x}} + \boldsymbol{K}_t \Delta\boldsymbol{x} = \Delta\boldsymbol{F}_t \tag{8}$$

So the displacement $\boldsymbol{X}_t$, velocity $\dot{\boldsymbol{X}}_t$, acceleration $\ddot{\boldsymbol{X}}_t$ are taken from Eq.(8) in each integral step Δt. Then $\boldsymbol{M}_t$, $\boldsymbol{C}_t$, $\boldsymbol{K}_t$ are required to be modified so as to calculate in the next integral step. Therefore triangle treatment for $\boldsymbol{K}_t$ should be taken in each integral step, that is, $\hat{\boldsymbol{K}}_t = \boldsymbol{LDL}^{\mathrm{T}}$; do likewise with $\boldsymbol{M}_t$ and $\boldsymbol{C}_t$.

For a belt conveyor, its mass and damping matrix are approximately regarded as constants. Non-linear force only depends on displacement. Therefore the simpler expression of the motion equation and increment equation at the moment t are

$$\boldsymbol{M}\Delta\ddot{\boldsymbol{x}} + \boldsymbol{C}\Delta\dot{\boldsymbol{x}} + \boldsymbol{F}_t^{\mathrm{S}} = \boldsymbol{F}$$

$$\boldsymbol{M}\Delta\ddot{\boldsymbol{x}} + \boldsymbol{C}\Delta\dot{\boldsymbol{x}} + \boldsymbol{K}_t \Delta\boldsymbol{x} = \Delta\boldsymbol{F}_t \tag{9}$$

Thus the displacement $\boldsymbol{X}_t$, velocity $\dot{\boldsymbol{X}}_t$, acceleration $\ddot{\boldsymbol{X}}_t$ and belt tension are taken from Eq.(9) in each integral step Δt. Then $\boldsymbol{K}_t$ is calculated again according to Eq.(7) for the next integral step. Other procedures are the same as previously stated.

2 Case Study

In order to illustrate the influence of a non-linear system or model on conveyor dynamics behavior, one conveyor is calculated and analyzed.

A horizontal belt conveyor is 1 200 m in length with a speed of 2.5 m/s. It transports material of 1 080 t/h in full load condition. The belt specification is GX-1000, with a belt width of 1 m. Its elasticity is 641 kN/cm. The belt viscous constant is 0.02 s. Two motors and hydraulic couplings are mounted together on the head of the conveyor. The rated power of a motor is 110 kW. The speed ratio of a speed reducer is 31.5. The diameter of a drive pulley is 1 m. The gravity take-up weighs 40 kN, mounted on the head of the conveyor too. Material and belt mass at unit length are respectively 104 kg/m and 30 kg/m. Idler spacing are 1.2 m on the carry strand and 2.5 m on the return strand.

With belt conveyor dynamics simulation software BDS, the operation process of this conveyor is calculated with linear and non-linear system model. Belt speed, acceleration, belt tension and displacement of gravity take-up is taken from the simulation. The whole process includes starting stage (about 25 s), operation stage and stopping stage (about 9 s) as shown in Fig.3 to Fig.6. From these four charts, the vibration period of a linear model is 4.17 s and that of non-linear model is 4.67 s, with their difference about 12%. As analyzed previously, the belt equivalent elastic module of a non-linear model is less than the belt elastic module of a linear model, and system vibration period is in inverse proportion to the system stiffness or elastic module, therefore the vibration period of a linear model is less than that of a non-linear model. Fig.3 shows that the conveyor models exert great influence on the variation magnitude of belt speed. At about the 10th second after starting, the difference of variation magnitude of belt speed between linear and non-linear model is 28%. From Fig.4, a non-linear model, compared to a linear model, has certain influence on belt tension as well. During the 5th to 10th second after starting, the difference in belt tension of both is 6% to 8%. Fig.5 illustrates that a non-linear model also exerts certain influence on acceleration but not much on their variation magnitude. From Fig.6, a non-linear model, compared to a linear model, influences greatly on displacement of gravity take-up. During the 5th to 10th second after starting, both difference in displacement of gravity take-up is about 50%. Actually the non-linear variation in belt length resulting from belt sag will directly lead to the variation of displacement of gravity take-up.

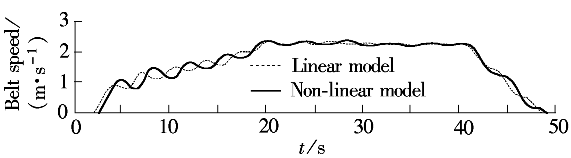

Fig.3 Belt speed at upper point of tail pulley

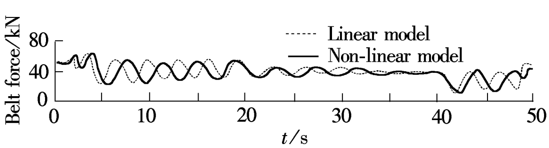

Fig.4 Belt force at upper point of tail pulley

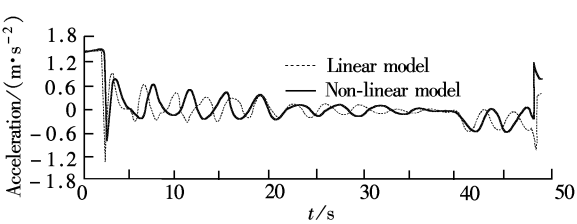

Fig.5 Belt acceleration at upper point of tail pulley

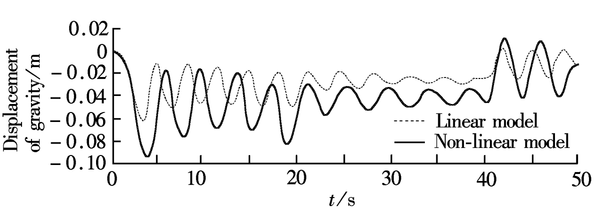

Fig.6 Displacement of gravity take-up

3 Conclusion

This paper studies the non-linear characteristics of belt conveyor based on sag, deduces a belt-equivalent elastic module and finds that it is not constant but variable data with the belt tension changing. In operation conditions, the belt-equivalent elastic module may be 10% of the belt elastic module. This paper also explains the principle of numerical, step-by-step integrals for a non-linear system and the solution. The motion equation and a non-linear model of belt conveyor are set up. Dynamics calculation on a belt conveyor is carried out with a linear and a non-linear model. The results show that non-linear model, compared to the linear model, influences displacement of gravity take-up greatly. At some moment, the results got from both models differ by about 50%. Also it makes their vibration periods differ by about 12%. In some cases, a non-linear model, compared to a linear model makes the variation magnitude of belt speed differ by 28% and that of belt tension differ by 6% to 8%. Finally, a non-linear model exerts certain influence on conveyor acceleration.

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输送机非线性动力学的研究及其对动态性能的影响

李光布

(上海师范大学机电学院, 上海 201400)

摘要: 输送机线性系统的假设是在胶带力学特性和弹性模量不变的情况下近似提出的.这种假设会使大型输送机产生理论误差和不正确的动力学仿真结果.本文分析了基于胶带悬垂度的非线性特性, 推导了等效弹性模量的表达式, 研究了胶带等效弹性模量和弹性模量之间的关系, 并发现两者的比值在 0.1 ~1.0 之间变化.提出了输送机有限单元模型的非线性运动方程及其增量方程和数值解法.通过对一输送机动态特性的仿真分析, 发现用线性和非线性 2 种模型, 它们动态参数(如带速, 胶带加速度, 张力以及重锤位移和张力波速)的计算误差介于 6% ~50% 之间.

关键词: 胶带悬垂度; 等效弹性模量; 非线性运动方程; 动力学仿真

中图分类号: TH113; O322