

# Micro-displacement amplifying mechanism driven by piezoelectric actuator

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**Abstract:** Piezoelectric actuator has high stiffness, high frequency and infinite control precision, but a short output displacement which is often  $1/1\ 000$  of its length. In order to meet the requirements that tools feeding should be long-travel, high-frequency and high-precision in non-circular precision turning, a new one-freedom flexure hinge structure is put forward to amplify the output displacement of piezoelectric actuator. Theoretical analysis is done on the static and dynamic characteristics of the structure, differential equations are presented, and it is also verified by the finite element method. It's proved by experiments that the output displacement of the structure is  $293\ \mu\text{m}$  and its resonant frequency is  $312\ \text{Hz}$ .

**Key words:** piezoelectric actuator; flexure hinge; micro displacement; amplifying structure

A large number of industrial parts have a non-circular geometric feature, such as piston heads and camshafts. Traditionally, the non-circular features in these parts are manufactured by milling and subsequent grinding, which result in high cost and lower productivity. With the development of technology, non-circular turning has been employed as an alternative to the traditional manufacturing method for a shorter product life circle. The fast tools servo system (FTS) in non-circular turning should be high-frequency, long-travel and high-precision, for instance, the ellipticity of car piston heads is about  $1\ \text{mm}$ , and if they are turned at the speed of  $3\ 000\ \text{r/min}$ , the feeding frequency of tools should be higher than  $100\ \text{Hz}$ . The linear motor<sup>[1,2]</sup>, magnetostrictive<sup>[3]</sup>, hydraulic<sup>[4]</sup>, piezoelectric and voice-coil<sup>[5]</sup> actuators or their combinations<sup>[6-8]</sup> are used to achieve dynamic tools feeding.

Piezoelectric actuators with their infinite resolution, high stiffness and high frequency are widely used in many areas. Otherwise, because the maximum output displacement of stacked actuator is only  $1/1\ 000$  of its length, it has to be amplified to meet the long-travel requirements. The flexure hinge is such a simple and ingenious structure, which is often used in micro-motion stage<sup>[9]</sup>. For instance, Liu, et al.<sup>[10]</sup> presented a stage moving in two directions but it had no requirements for bearing dynamic force and offering a long distance. Flexure hinges can also

be used to amplify the output displacement of the actuators in non-circular turning.

A new amplifying structure, which is based on a right-circular flexure hinge, is put forward in the following sections. Theoretical analysis is done on its static and dynamic models, which are also verified by FEM and experiments.

## 1 Amplifying Structure

In non-circular turning, the tools installed on the amplifying structure are driven reciprocally by piezoelectric actuators, see Fig.1. The cutting forces

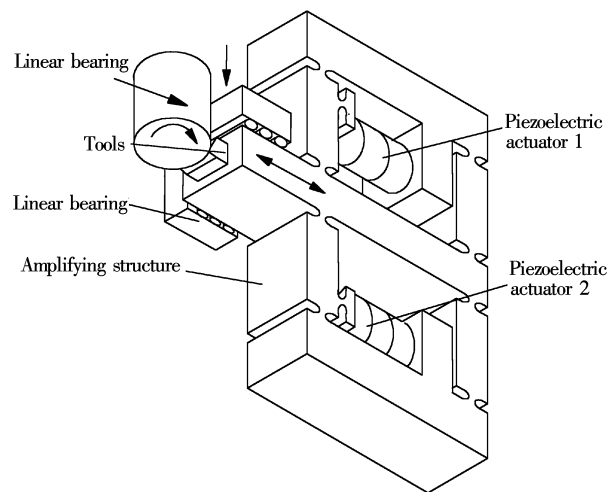


Fig.1 Feeding mechanism of tools

perpendicular to the feeding direction are borne by linear bearing, thus, the amplifying structure only bears the cutting force  $F_c$  which is in the direction of tools feeding, see Fig.2, and the amplifying ratio is  $N = l_1/l_2$ . As the output displacement of piezoelectric

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actuators is amplified, so do the forces act on piezo actuators; nevertheless, each actuator shares half of the forces that the actuator bears in only half of a symmetric part of the structure. Because of the symmetry of the mechanism, it has only one degree of freedom. The flexure hinge  $k_p$ , to which the output forces of piezoelectric actuators are applied, is compressed, meanwhile, it protects the actuators from destruction by shear stresses.

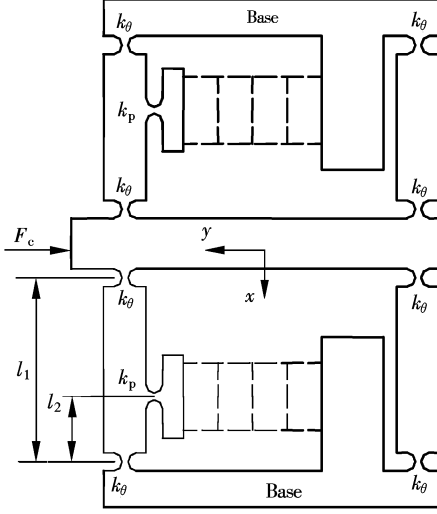


Fig.2 Schematic diagram of amplifying structure

### 1.1 Single-axis flexure hinge

Paros and Weisbord equations for single-axis flexure hinge<sup>[11]</sup> are used in the design of the structure to obtain an approximate estimate of the stiffness values for flexure hinges, see Fig.3. A finite

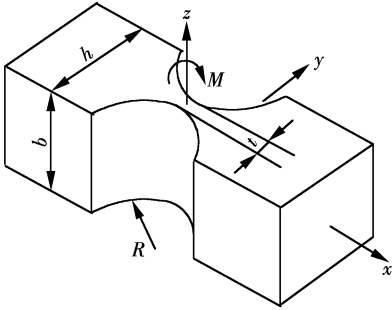


Fig.3 Single-axis flexure hinge

element method (FEM) can also be employed to obtain the more precise stiffness value of these flexure hinges in further verification. The bending stiffness around  $z$  axis can be calculated by

$$k_\theta = \frac{M}{\theta_z} = \frac{2Ebt^{\frac{5}{2}}}{9\pi R^{\frac{1}{2}}} \quad (1)$$

where  $E$  is Young's modulus of the material,  $\theta_z$  is the rotational angle around  $z$  axis, and  $M$  is the moment of

torsion. The maximum stress occurs at the outside surfaces of the thinnest part of the flexure hinge<sup>[12]</sup> and can be calculated by

$$\sigma_{\max} = K_t \frac{6M_{\max}}{t^2 b} \quad (2)$$

where  $K_t$  is the stress concentration coefficient, and  $M_{\max}$  is the maximum motion around the  $z$  axis.

$$K_t = \left(1 + \frac{t}{2R}\right)^{\frac{9}{20}} \quad (3)$$

### 1.2 Static model

It's shown in Fig.4 that  $F_c$  is the very force

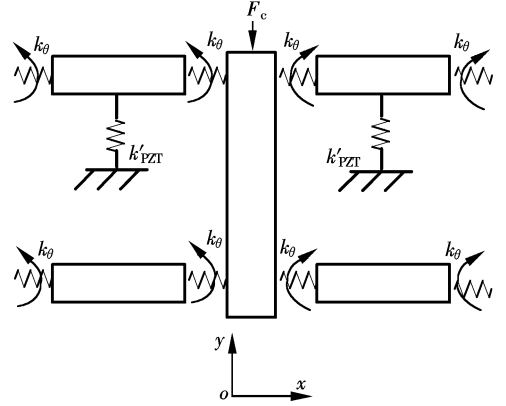


Fig.4 Static model of the structure

applied to the amplifying structure in the direction of the tools' feeding, and together with driving force, makes the tools feed in  $y$  direction. Therefore

$$F_c = 2k'_{PZT} \frac{y}{N^2} + 8k_\theta \frac{y}{l_1^2} \quad (4)$$

where  $k_\theta$  is the bending stiffness of a single flexure hinge, see Eq.(1),  $N$  is the amplifying coefficient,  $y$  is the tools' feeding displacement in  $y$  direction,  $k'_{PZT}$  is the effective stiffness of the piezoelectric actuators.

$$K'_{PZT} = \frac{k_{PZT}k_a}{k_{PZT} + k_a} \quad (5)$$

where  $k_a$  is the effective stiffness due to the influence of flexure hinge  $k_p$  and the contact stiffness between actuators and amplifying structure. Because the deformation of flexure hinge  $k_p$ , which is caused by the compression of the output force of actuators, is also amplified, it cannot be neglected. The stiffness of the actuator is

$$k_{PZT} = \frac{A}{s_{33}^E L_0} \quad (6)$$

where  $A$  is the cross-section area of the actuator,  $s_{33}^E$  is the elastic compliance in longitudinal direction when subjected to a constant electrical field, and  $L_0$  is the actuator length. The effective stiffness of the structure is

$$K = \frac{2k'_{PZT}}{N^2} + \frac{8k_\theta}{l_1^2} \quad (7)$$

### 1.3 Dynamic model

The micro-displacement amplifying structure can be simplified as a spring-mass system, as shown in Fig.5. Because each of the flexure hinges in the struc-

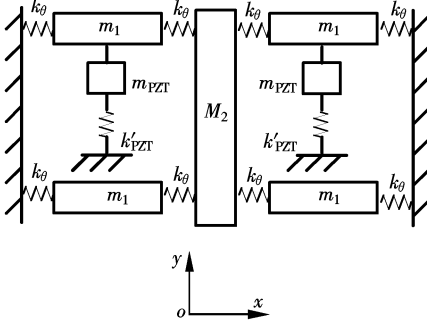


Fig.5 Dynamic model of the structure

ture has one degree of freedom and due to the symmetry of the structure, the structure can provide output displacement only in  $y$  direction. Respectively, the kinetic energy and potential energy of the structure are expressed as

$$E_k = \frac{1}{2}m_2\dot{y}^2 + 4\left[\frac{1}{2}m_1\left(\frac{\dot{y}}{2}\right)^2 + \frac{1}{2}I_1\left(\frac{\dot{y}}{l_1}\right)^2\right] + 2\left[\frac{1}{2}m_{PZT}\left(\frac{\dot{y}}{N}\right)^2\right] \quad (8)$$

$$E_p = 8\left[\frac{1}{2}k_\theta\left(\frac{y}{l_1}\right)^2\right] + 2\left[\frac{1}{2}k'_{PZT}\left(\Delta L - \frac{y}{N}\right)^2\right] \quad (9)$$

where  $l_1$  and  $I_1$  are the length and moment of inertia of mass  $m_1$ , respectively,  $m_{PZT}$  is the effective mass of the piezoelectric actuator, and  $\Delta L$  is the output displacement of the actuator subjected to no external forces. Substituting Eqs. (8) and (9) into Lagrange function, then

$$\left(m_1 + m_2 + \frac{4I_1}{l_1^2} + \frac{2m_{PZT}}{N^2}\right)\ddot{y} + \left(\frac{8k_\theta}{l_1^2} + \frac{2k'_{PZT}}{N^2}\right)y = \frac{2k'_{PZT}}{N}\Delta L \quad (10)$$

Because  $\Delta L$  in Eq. (10) can be expressed as a function of applied voltage,  $\Delta L = f(V)$ , Eq. (10) can be written as

$$m\ddot{y} + Ky = \frac{2k'_{PZT}}{N}f(V) \quad (11)$$

where  $m$  and  $K$  are the effective mass and effective stiffness of the amplifying structure, respectively. The undamped natural frequency can be obtained from

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad (12)$$

When the amplifying ratio  $N = 3$ , effective stiff-

ness  $K = 11.87 \text{ N}/\mu\text{m}$  and effective mass  $m = 3.13 \text{ kg}$ , the undamped natural frequency is 320.5 Hz.

## 2 FEM Verification

In this section, analytical verification is done on the static and dynamic model of the amplifying structure by commercial finite software ANSYS. Due to the symmetry of the structure on constraints, forces and geometric feature, the symmetric boundary conditions are applied to the structure, and the friction of linear bearing can be neglected; thus, this simplification requires less calculation. The finite mesh graph of the structure to which symmetric boundary conditions are applied is shown in Fig.6.

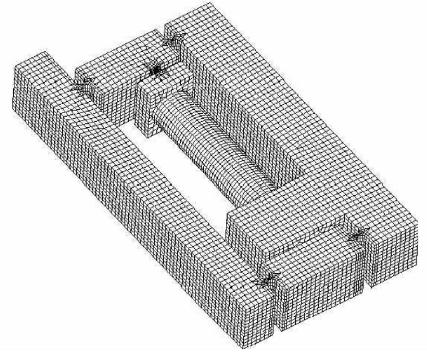


Fig.6 Finite model of the structure

The amplifying structure and piezoelectric actuator are meshed with hexahedral element Solid 45 and Solid 5, respectively. The piezoelectric actuator is comprised of ceramic stacks, which are  $d_{33}$  design; the coefficient  $d_{33}$  dominates the other piezoelectric constants in the constitutive equations. The electrodes of the ceramic discs are connected in parallel and the piezo ceramic discs in series mechanically, and the maximum voltage applied to piezo actuator is 150 V. According to empirical data, the maximum radial cutting force applied to tools is 300 N. Respectively, when maximum radial cutting force and voltage are applied to the structure and piezo ceramic discs, the maximum output displacement of the structure is 298  $\mu\text{m}$ . Because voltage DOF (degree of freedom) has no mass, the electrodes of ceramics should be short-circuit in modal analysis. The first modal frequency of the structure is 331.4 Hz. The results of the FEM show the agreement with the static and dynamic models.

## 3 Experiments

The amplifying structure is made from stainless steel (1Cr13), and machined by EDM (electrical dis-

charge machining) linear cutting machine. After this, it was heat-treated at 1 000 °C for about 40 min, oil quenching, and finally tempering for about 3 h at 400 °C. The mechanism was driven by two piezoelectric actuators, PAHL100/20 (Piezosystem Jena Co.), the maximum output displacement of them is 100  $\mu\text{m}$ , maximum output force is 3 500 N and the control voltage is from 0 to 150 V. Ultra precision laser displacement meter, LC2440 (Keyence Co.), was used to measure the output displacement, its measurement range is  $(30 \pm 0.5)\text{mm}$  and the resolution is 0.4  $\mu\text{m}$ .

The stiffness of the structure was obtained by placing weights where the tools were mounted, and the structure was placed vertically while the actuator was running in open loop. The stiffness of the structure was measured to be 11.2 N/ $\mu\text{m}$ . The output displacement was 293  $\mu\text{m}$ , that is to say that the ellipticity of parts can be up to 0.58 mm. In order to get the harmonic frequency, a sinusoidal wave signal was applied to the piezoelectric actuators in the frequency range of 10 to 500 Hz, and the output displacement measured by laser displacement meter is shown in Fig. 7. It's shown that the lowest harmonic frequency of the amplifying structure is about 312 Hz, and the rotating speed of the lathe spindle can be raised to over 2 000 r/min. It's good for improving the surface quality and geometric precision of parts.

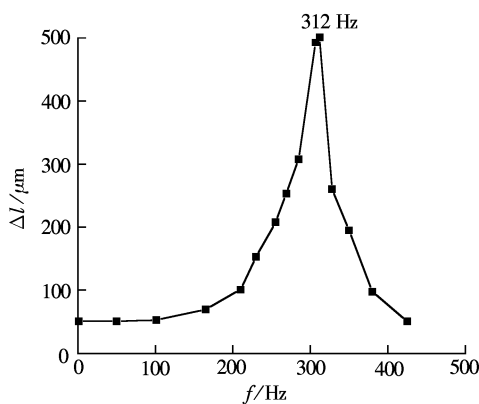


Fig.7 Frequency response of the structure

## 4 Conclusion

The design and modeling of a flexure hinge structure driven by piezoelectric actuator is put forward. A new one-freedom tools' feeding mechanism used in non-circular turning, is also presented. Experiments show that the mechanism can meet the basic requirements in non-circular turning.

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# 基于压电陶瓷驱动的微位移放大机构

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**摘要:** 压电陶瓷驱动器具有刚性好、频响高和控制精度高的特点, 但是其输出位移却较小, 通常是其长度的 1/1 000. 在非圆回转曲面精密车削中, 为了实现刀具大位移、高频响、高精度的进给要求, 提出了新型单自由度柔性铰链机构对压电陶瓷驱动器的输出位移进行放大. 同时对机构进行了静态和动态特性理论分析, 提出了机构静态和动态计算解析式, 并用有限元方法对机构动态和静态特性进行了验证. 实验证明机构的最大输出位移为 293  $\mu\text{m}$ , 谐振频率为 312 Hz.

**关键词:** 压电陶瓷驱动器; 柔性铰链; 微位移; 放大机构

**中图分类号:** TH134