

Fuzzy programming approach solution for multi-objective solid transportation problem

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Abstract: Based on recent research developments in multi-objective solid transportation problem (MOSTP), this paper presents a fuzzy programming approach to determine the optimal compromise solution of MOSTP. The characteristic feature of the proposed approach is that various objectives are synthetically considered with marginal evaluation for individual objectives and global evaluation for all objectives. The decision-maker's preference is taken into account by his/her assigning weights to the objectives. With global evaluation for all objectives, a compromise programming model is formulated. As a generic aggregation operator is adopted, several solution methods proposed earlier become special cases of this approach, and the solution process also becomes more flexible and realistic. An illustrative numerical example is provided to demonstrate the approach.

Key words: solid transportation problem; multi-objective optimization; fuzzy compromise solution

The classical transportation problem (TP) refers to a special class of linear programming problems, which was originally developed by Hitchcock^[1]. In a typical problem a product is to be transported from m sources to n destinations. A variety of approaches have been developed by many authors^[2-4] for the linear multi-objective transportation problem (MOTP). Current, et al.^[5] have done a review work about the multi-objective design of transportation networks.

The solid transportation problem (STP) is a generalization of the classical transportation problem. An extension of the classical transportation problem to the solid transportation problem was stated by Schell. The necessity of considering this special type of transportation problem arises because many industrial problems are shaped in this special form. The STP can be converted to a classical transportation problem by considering a single type of conveyance. Haley^[6] described the solution procedure of a solid transportation problem. His method of solution is an extension of the modified distribution method. He presented a comparison of the solid transportation problem and the classical transportation problem. He discussed the degeneracy case in STP. He also defined the multi-index transportation problem and presented the solution procedure for the multi-index transportation problem.

Fuzzy set theory was proposed by Zadeh^[7] and has been found to be extensive in various fields. Bit, et al.^[8,9] advocated the application of Zimmermann's fuzzy programming^[10] and presented an additive fuzzy programming model for the multi-objective solid transportation problem (MOSTP)^[11]. The most important aspect in the fuzzy approach is the compensatory or non-compensatory nature of the aggregate operator. Several investigators^[12-14] have discussed this aspect. Due to the ease of computation, the most frequently used aggregation operator in Zimmermann's fuzzy programming approach is the "min" operator which has also been employed by Bit, et al.^[8,9]. The biggest disadvantage of the aggregation operator "min" is that it is non-compensatory in the sense of Yager^[13]. In other words, the results obtained by the "min" operator represent the worst situation and cannot be compensated by other members which may be very good. Lee, et al.^[14] pointed out that the aggregation operator "min" didn't guarantee non-dominated solutions for multiple objective programming problems.

To overcome the drawbacks mentioned above, we propose a fuzzy compromise programming approach to MOSTP. In the fuzzy compromise programming approach proposed, various objectives are synthetically considered by marginally evaluating individual objectives and globally evaluating all objectives. The decision-makers' preferences among the objectives are reflected in their global subjective evaluation, taking into consideration the various

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respective objectives. It is shown that the approach proposed here can give a compromise solution that is not only non-dominated but also optimal in the sense that the decision-maker's global subjective evaluation value is maximized. It is also shown that the approach proposed covers a wide spectrum of methods with Bit's approach^[5] essentially equivalent to one of its **special cases under certain conditions.**

1 Multi-objective Solid Transportation Problem

The well-known traditional TP is concerned with the distribution of goods (products) from several sources (supply points) to several destinations (demand points) at minimal total transportation cost. The multi-objective linear transportation problem (MOLTP), on the other hand, deals with the distribution of goods with the consideration of several objectives, such as transportation cost, delivery time and quantity of goods delivered, simultaneously. MOSTP is a generalization of the MOLTP in which three item properties (source, destination and mode of) are taken into account in the constraint set instead of two (source and destination).

Consider m sources $O_i (i = 1, \dots, m)$ and n destinations $D_j (j = 1, \dots, n)$. At each source O_i , let a_i be the amount of a homogeneous product which we want to transport to n destinations D_j to satisfy the demand for b_j units of the product there. Let $e_k (k = 1, \dots, K)$ be the units of this product which can be carried by K different modes of transportation, such as trucks, air freight, freight trains, ship, etc. A penalty C_{ijk}^p is associated with transportation of a unit of the product from source i to destination j by means of the k -th conveyance for the p -th criterion. The penalty can represent transportation cost, delivery time, quantity of goods delivered, etc. A variable x_{ijk} represents the unknown quantity to be transported from source O_i to destination D_j by means of the k -th conveyance. In the real world, however, solid transportation problems are not all single objective type. There may be more than one objective in a solid transportation problem.

A multi-objective solid transportation problem can be formulated as a standard linear programming problem in the following way

$$\min Z_p = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K C_{ijk}^p x_{ijk} \quad p = 1, 2, \dots, P \quad (1)$$

subject to

$$\left. \begin{aligned} \sum_{j=1}^n \sum_{k=1}^K x_{ijk} &= a_i & i = 1, 2, \dots, m \\ \sum_{i=1}^m \sum_{k=1}^K x_{ijk} &= b_j & j = 1, 2, \dots, n \\ \sum_{i=1}^m \sum_{j=1}^n x_{ijk} &= e_k & k = 1, 2, \dots, K \\ x_{ijk} &\geq 0 & i = 1, 2, \dots, m; j = 1, 2, \dots, n; \\ & & k = 1, 2, \dots, K \end{aligned} \right\} \quad (2)$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = \sum_{k=1}^K e_k \text{ (balanced condition)}$$

Notice that the balanced condition is treated as a necessary and sufficient condition for the existence of a feasible solution to (1)-(2). We denote by X the set of all feasible solutions of the MOSTP, which is **formulated via Eq.(2).**

2 Fuzzy Compromise Programming

Consider the multi-objective programming problem

$$\min Z(x) = \{Z_1(x), Z_2(x), \dots, Z_p(x)\}^T \quad (3)$$

subject to $x \in X$

where X is the set of feasible solutions (feasible solution space).

Notice that the multi-objective programming problem (3) often consists of a set of conflicting goals that cannot be achieved simultaneously. Instead of trying to find such an optimal solution that every objective is optimal (usually this is impossible), we try to find an optimal compromise solution at which the global evaluation of the synthetic membership degree of optimum for all objectives is maximized. The global evaluation employed here reflects the decision maker's consideration of all criteria contained in the multi-objective functions.

In this section, we present an approach to obtain the marginal evaluation for each objective and to aggregate these marginal evaluations into the global evaluation of the synthetic membership degree of optimum for all objectives. Based on the global evaluation obtained, we can formulate a fuzzy compromise programming approach to multi-objective **solid programming problems.**

2.1 Marginal evaluation for a single objective

To define the membership function of MOSTP problem, let L_p and U_p be the lower and upper bounds of the objective function. These values are determined as follows: calculate the individual minimum of each objective function as a single objective transportation problem subject to the given set of constraints. Let $x_1,$

x_2, \dots, x_p be the respective optimal solutions for the P different transportation problems and evaluate each objective function at all these P optimal solutions. It is assumed here that at least two of these solutions are different for which the p -th objective function has different bounded values. For each objective function, find the lower bound (minimum value) L_p and the upper bound (maximum value) U_p .

Once the highest acceptable level U_p and the aspired level L_p have been specified, the following formula can be employed to define the marginal evaluation mapping $\phi_p \in [0, 1]$ for the objective Z_p ($p = 1, 2, \dots, P$)

$$\phi_p(Z_p) = \begin{cases} 1 & Z_p \leq L_p \\ \frac{U_p - Z_p}{U_p - L_p} & L_p < Z_p < U_p \\ 0 & Z_p \geq U_p \end{cases} \quad (4)$$

In most practical applications, U_p and L_p can be determined from the ideal solution^[6] of problem (3) in the following way.

Let $(x^{1*}, x^{2*}, \dots, x^{P*})$ be the ideal solution of problem (3), i.e. x^{P*} is the optimal solution of the single objective programming problem:

$$\min_{x \in X} Z_p(x) \quad p = 1, 2, \dots, P$$

Then the values of all the P objective functions can be calculated at all these P optimal solution x^{p*} ($p = 1, 2, \dots, P$) to form a payoff matrix

$$\begin{bmatrix} Z_1(x^{1*}) & Z_1(x^{2*}) & \dots & Z_1(x^{P*}) \\ Z_2(x^{1*}) & Z_2(x^{2*}) & \dots & Z_3(x^{P*}) \\ \vdots & \vdots & & \vdots \\ Z_p(x^{1*}) & Z_p(x^{2*}) & \dots & Z_p(x^{P*}) \end{bmatrix} \quad (5)$$

From the payoff matrix (5), U_p and L_p can be determined for each objective Z_p ($p = 1, 2, \dots, P$) by the following

$$\left. \begin{aligned} U_p &= \max_{1 \leq j \leq P} \{Z_p(x^{j*})\} \\ L_p &= Z_p(x^{p*}) \end{aligned} \right\} \quad (6)$$

where $p = 1, 2, \dots, P$.

2.2 Global evaluation for multiple objectives

Having all the marginal evaluations $\phi_1(x), \phi_2(x), \dots, \phi_p(x)$ for a given decision $x \in X$, the problem is then to determine a global evaluation of x with respect to all the objectives. Usually the global evaluation method to all the objectives is to adopt an aggregation operator that can combine these objectives into a single one that can represent the decision maker's preferences. An aggregation operator has the following form

$$\mu(x) = \Phi(\phi_1(x), \phi_2(x), \dots, \phi_p(x)) \quad (7)$$

There have been many aggregation operators up till now. Generally, an aggregation operator is a form of union or intersection operator. Fuzzy set theory provides an attractive aggregation connective for integrating membership values representing uncertain information. Here we will adopt the weighted root-power mean operator to aggregate these objectives.

A weighted root-power mean operator M_w^α has the following form

$$M_w^\alpha(a_1, a_2, \dots, a_p) = \left(\sum_{i=1}^p w_i a_i^\alpha \right)^{\frac{1}{\alpha}} \quad 0 < |\alpha| < \infty \quad (8)$$

where a_i is the objective to be aggregated, and w_i is the weight of objective i , which represents the relative importance of objective i to the decision maker. The bigger the weight is, the more important it is to the decision maker, generally the weights are normalized to satisfy the following form

$$W = \{w_1, w_2, \dots, w_p\} \quad w_p \geq 0, \sum_{p=1}^P w_p = 1 \quad (9)$$

Weighted root-power mean operator covers a wide spectrum of aggregation operators which are often used in the areas of multi-criteria decision making and multiple objective programming problems. From a practical standpoint, the most important parameters α is 1, 2 and $-\infty$.

1) The weighted arithmetic mean ($\alpha = 1$):

$$M_w^1(a_1, a_2, \dots, a_p) = \sum_{i=1}^p w_i a_i \quad (10)$$

2) The weighted quadratic mean ($\alpha = 2$):

$$M_w^2(a_1, a_2, \dots, a_p) = \left(\sum_{i=1}^p w_i a_i^2 \right)^{\frac{1}{2}} \quad (11)$$

3) The conjunctive mean ($\alpha \rightarrow -\infty$):

$$\left. \begin{aligned} M_w^{-\infty}(a_1, a_2, \dots, a_p) &= \min_{1 \leq i \leq p} a_i \\ w_1 = w_2 = \dots = w_p &= \frac{1}{P} \end{aligned} \right\} \quad (12)$$

We note that M_w^1 produces the weighted additive model used in goal programming problems, whereas $M_w^{-\infty}$ is the most frequently used aggregation operator in fuzzy multi-criteria decision-making and Zimmermann's fuzzy programming problems^[7-10].

Having chosen a suitable aggregating operator Φ_w , we can convert problem (3) into the following fuzzy compromise programming problem:

$$\max \mu(x) = \Phi_w(\phi_1(x), \phi_2(x), \dots, \phi_p(x)) \quad (13)$$

subject to $x \in X$

The fuzzy compromise programming problem (13) is a single objective programming problem. We

can employ the ordinary optimization technique to solve it.

3 Fuzzy Compromise Programming for MOSTP

The MOSTP consisting of Eqs.(1) and (2) is a multi-objective linear programming problem. Therefore, it can be solved with the fuzzy compromise programming approach mentioned above. The details of the foregoing steps may be presented as follows:

Step 1 Pick the first objective function and solve it as a single objective transportation problem subject to the constraints (2).

$$\min_{\{x_{ijk}\} \in X} Z_p(\{x_{ijk}\}) = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^K C_{ijk}^p x_{ijk} \quad (14)$$

Continue this process P times for P different objective functions. Let $x^{1*} = \{x_{ijk}^1\}$, $x^{2*} = \{x_{ijk}^2\}$, ..., $x^{P*} = \{x_{ijk}^P\}$ be the optimal solutions for P different solid transportation problems. If all the solutions (i.e. $x^{1*} = x^{2*} = \dots = x^{P*} = \{x_{ijk}\}$; $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$) are the same, then one of them is the optimal compromise solution and go to step 6. Otherwise, go to step 2.

Step 2 Evaluate the p -th objective function at the P optimal solutions ($p = 1, 2, \dots, P$). For each objective function, determine its lower and upper bounds (L_p and U_p) according to the set of optimal solutions.

$$\left. \begin{aligned} U_p &= \max_{1 \leq j \leq P} \{Z_p(x^{j*})\} \\ L_p &= Z_p(x^{P*}) \end{aligned} \right\} \quad (15)$$

where $p = 1, 2, \dots, P$.

Step 3 Determine the membership function as mentioned in Eq.(4), that is

$$\phi_p(x_{ijk}) = \begin{cases} 1 & Z_p(\{x_{ijk}\}) \leq L_p \\ \frac{U_p - Z_p(\{x_{ijk}\})}{U_p - L_p} & L_p < Z_p(\{x_{ijk}\}) < U_p \\ 0 & Z_p(\{x_{ijk}\}) \geq U_p \end{cases} \quad (16)$$

Step 4 Determine the weight $W = \{w_1, w_2, \dots, w_p\}$ and choose a suitable weighted root-power mean M_w^α to formulate a fuzzy compromise programming problem as follows:

$$\max_{\{x_{ijk}\} \in X} \mu(\{x_{ijk}\}) = M_w^\alpha(\phi_1(\{x_{ijk}\}), \phi_2(\{x_{ijk}\}), \dots, \phi_p(\{x_{ijk}\})) \quad (17)$$

Step 5 Solve the fuzzy compromise programming problem (17) using the ordinary optimization technique to obtain the optimal compromise solution of the MOSTP.

Step 6 Stop.

The parameter α in (17) can take 1, 2 and $-\infty$, respectively, and the fuzzy programming approach to MOSTP developed by Bit, et al. is a special case of the fuzzy compromise programming approach proposed in this paper for $\alpha = -\infty$.

4 Numerical Example

To illustrate the fuzzy programming approach discussed above, we consider the following two-objective standard solid transportation problem:^[9]

Supplies: $a_1 = 24, a_2 = 8, a_3 = 18, a_4 = 10$

Demands: $b_1 = 11, b_2 = 19, b_3 = 21, b_4 = 9$

Conveyance capacities: $e_1 = 17, e_2 = 31, e_3 = 12$

Destinations: $D_1 \ D_2 \ D_3 \ D_4$

Conveyances: 1 2 3 1 2 3 1 2 3 1 2 3

Penalties:

	D_1			D_2			D_3			D_4			
	1	2	3	1	2	3	1	2	3	1	2	3	
$C^1 =$	o_1	15	18	17	12	22	13	10	4	12	8	11	13
	o_2	17	20	19	21	21	22	21	19	18	30	10	23
	o_3	14	11	12	25	34	33	20	16	15	21	23	22
	o_4	22	18	13	24	35	32	18	21	14	13	23	20
$C^2 =$	o_1	6	7	8	10	6	5	11	3	7	10	9	6
	o_2	13	8	11	12	2	9	20	15	13	17	15	13
	o_3	5	6	7	11	9	7	10	5	2	15	14	18
	o_4	13	6	6	17	11	18	12	16	12	18	14	7

These penalties can be expressed in a three dimensional table. This problem can be modeled as follows:

$$\min Z_p = \sum_{i=1}^4 \sum_{j=1}^4 \sum_{k=1}^3 C_{ijk}^p x_{ijk} \quad p = 1, 2$$

subject to

$$\begin{aligned} \sum_{j=1}^4 \sum_{k=1}^3 x_{1jk} &= 24, \quad \sum_{j=1}^4 \sum_{k=1}^3 x_{2jk} = 8, \\ \sum_{j=1}^4 \sum_{k=1}^3 x_{3jk} &= 18, \quad \sum_{j=1}^4 \sum_{k=1}^3 x_{4jk} = 10, \\ \sum_{i=1}^4 \sum_{k=1}^3 x_{i1k} &= 11, \quad \sum_{i=1}^4 \sum_{k=1}^3 x_{i2k} = 19, \\ \sum_{i=1}^4 \sum_{k=1}^3 x_{i3k} &= 21, \quad \sum_{i=1}^4 \sum_{k=1}^3 x_{i4k} = 9, \\ \sum_{i=1}^4 \sum_{j=1}^4 x_{ij1} &= 17, \quad \sum_{i=1}^4 \sum_{j=1}^4 x_{ij2} = 31, \\ \sum_{i=1}^4 \sum_{j=1}^4 x_{ij3} &= 12 \end{aligned}$$

where $x_{ijk} \geq 0$; $i = 1, 2, 3, 4$; $j = 1, 2, 3, 4$; $k = 1, 2, 3$; $C^1 = [c_{ijk}^1]$; $C^2 = [c_{ijk}^2]$.

Applying the fuzzy programming method to the problem of this example, we obtain

$$U_1 = 866, L_1 = 703, U_2 = 537, L_2 = 293$$

And by using the max-min operator of fuzzy linear programming problem, Bit, et al. obtained the optimal compromise value (749.285 3, 362.286 0).

Applying the fuzzy compromise programming with weighted root-power mean M_w^α as an aggregation operator and taking L_p and U_p for each objective Z_p as its ideal value and anti-ideal value, respectively, we obtain the results of this example with a calculation program based on MATLAB 6.0 platform. Various weights assigned and the corresponding optimal compromise objective values for $\alpha = 1, 2$ and $-\infty$ are shown in Tab.1. The corresponding fuzzy compromise solutions $\{x_{ijk}^*\}$ are omitted for reasons of space limitations.

Tab.1 The optimal compromise objective values of the example

Weights (w_1, w_2)	Optimal fuzzy compromise objective values (Z_1, Z_2)		
	$\alpha = 1$	$\alpha = 2$	$\alpha = -\infty$
(0.0, 1.0)	(887, 293)	(887, 293)	
(0.1, 0.9)	(852, 295)	(866, 293)	
(0.2, 0.8)	(826, 302)	(866, 293)	
(0.3, 0.7)	(826, 302)	(846, 296)	
(0.4, 0.6)	(733, 376)	(826, 302)	
(0.5, 0.5)	(715, 394)	(826, 302)	(749.285 3, 362.286 0)
(0.6, 0.4)	(715, 394)	(715, 394)	
(0.7, 0.3)	(715, 394)	(715, 394)	
(0.8, 0.2)	(710, 418)	(715, 394)	
(0.9, 0.1)	(710, 418)	(710, 418)	
(1.0, 0.0)	(703, 537)	(703, 537)	

5 Conclusions

In the present work, a fuzzy programming approach is used to find an optimal compromise solution for a multi-objective solid transportation problem. The membership function is defined and a linear compromise optimization model is developed using the fuzzy compromise programming approach proposed in this paper. The approach has the following features:

- 1) It provides the analyst with a simple and easy mathematical programming problem.
- 2) It can be easily implemented to solve all types of multi-objective solid transportation problems, multi-objective transportation problems, multi-objective multi-index transportation problems, linear or non-linear vector minimum problems and vector maximum problems.
- 3) It gives a preferred compromise solution which is not only non-dominated but also optimal in the sense that the decision-maker's global subjective evaluation value, taking into consideration the various respective objectives, is maximized.
- 4) It synthetically considers the property of

multi-objective decision making by marginally evaluating the individual objectives and globally evaluating all the objectives, and can always get the compromise solution by the weights that reflect the decision maker's preference attention with the fuzzy compromise programming approach.

On the whole, the fuzzy compromise programming approach proposed here is a suitable method for the multi-objective solid transportation problem.

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多目标多模式运输问题的模糊规划方法解

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摘要: 本文在现有研究进展的基础上, 提出了一种用于求解多目标多模式运输问题(MOSTP)的最优折衷解的模糊规划方法, 该方法的特征是综合考虑了每个目标的边缘评价和所有目标的整体评价因素. 通过分配每个目标的权重将决策者的偏好充分体现到决策过程中, 并通过相应的折衷规划模型, 在对所有目标整体评价的基础上得到决策者所期望的折衷解. 由于采用广义的模糊目标集成算子, 该方法不仅对现有求解方法进行了扩展, 而且在求解方式上也更加灵活和切合实际. 最后采用实例论证了该方法的求解过程.

关键词: 多模式运输问题; 多目标优化; 模糊折衷方法

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