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Vendor managed inventory and bullwhip effect

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Abstract: This paper studies that the bullwhip effect of order releases and the amplifications of safety stock arise within the supply chain even when the demand model is ARIMA(0, 1, 1) and the forecast method used is a simple exponentially weighted moving average. It also examines a vendor managed inventory (VMI) program to determine how it can help alleviate such negative effects, and gives the theoretical proofs and numerical illustrations. The results show that the effects with VMI are better than the effect without VMI in demand forecasting and safety stock levels, etc.

Key words: vendor managed inventory; bullwhip effect; supply chain; demand forecasting

Demand information in a supply chain is often altered when transferred from one part or node of the supply chain to another. Such changes as delays, amplification, and distortion of demand signals are known as the bullwhip effect. They lead to tremendous inefficiencies in a supply chain like excessive inventory investment, poor customer service, lost revenues, misguided capacity plans and ineffective transportation and production schedules. These inefficiencies can potentially incur significant costs to the enterprises. Hence, the bullwhip problem has more been noted and researched in recent years[1-12]. Some solutions for alleviating the effect have been brought forward. Essentially, among those solutions, the basic idea is supply chain coordination.

However, vendor managed inventory (VMI) and continuous replenishment planning (CRP) have been advocated by some as promising approaches to supply chain coordination[2,13]. These enable the seller to monitor inventory levels at the buyer's stock-keeping locations and assume responsibility for the requisite inventory replenishments needed to achieve specified inventory-turn targets and customer-service levels[14].

In addition, the autoregressive integrated moving average (ARIMA) model for time series has many good properties, and has been applied widely[15]. Hence, according to the demand model ARIMA(0, 1, 1)[6,12], and the basic ideas presented in Refs.[5, 11], this paper quantitatively studies the bullwhip effect and differences between implementing VMI before and after. The results show that the VMI program can lessen the impact of the bullwhip effect and reduction of stock levels on the supplier.

The research of this paper is different from previous work. Firstly, in comparison with the others', the paper uses distinct mathematical deduction methods, although the demand model is the same as that of Refs.[6, 12]. Secondly, the forecast method in this paper is also used by Refs.[5, 11], but their demand model is AR(1). This paper uses the demand model in Refs.[6, 12] as well as the ideas of Refs.[5, 11], and gets more useful outcomes.

1 Two-Level Supply Chain System

Consider a generic dyad consisting of only one retailer and one supplier that resembles a wide variety of transactional relationships including retailer-supplier. Now, suppose that the buyer retailer is the only customer supplier, which means that the retailer's purchase order becomes the supplier's observed demand, but the retailer's demand is non-stationary. The supplier's and retailer's order modes are all periodic review. Their replenishment lead-times are fixed and known, call them L and l, respectively. We will introduce additional assumptions as needed.

1.1 Retailer's demand and inventory control models

Suppose that the retailer's demand process is an ARIMA(0, 1, 1) process given as follows, which has been discussed in detail in Ref.[15]:

$$d_1 = \mu + \varepsilon_1 \text{ and } d_t = d_{t-1} - (1-\alpha)\varepsilon_{t-1} + \varepsilon_t \quad t = 2, 3, \cdots \tag{1}$$

where d_t is the observed demand in period t, α and μ are known parameters, and $\{\varepsilon_t\}$ is a time series of i.i.d. random variables. Assume that $0 \leqslant \alpha \leqslant 1$, and that ε_t is random noise with $E(\varepsilon_t) = 0$, $\text{var}(\varepsilon_t) = \sigma^2$.

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By varying α, we can model a range of demand processes. When $\alpha = 0$, the demand follows a stationary i.i.d process. For $0 < \alpha \leqslant 1$, the demand process is non-stationary, in which larger values of α result in a less stable or more transitory process. When $\alpha = 1$, the demand process is a random walk on a continuous state space.

To motivate this demand model, we expand (1) as

$$d_t = \varepsilon_t + \alpha\varepsilon_{t-1} + \alpha\varepsilon_{t-2} + \cdots + \alpha\varepsilon_1 + \mu \quad (2)$$

The retailer should forecast the future demand according to the past demand in order to control his inventory. A first-order exponential-weighted moving average provides the minimum mean square forecast for this demand process[15]. In fact, we define X_{t+1} to be the forecast, made after observing demand in time period t, for demand in period $t+1$:

$$X_1 = \mu \text{ and } X_{t+1} = \alpha d_t + (1-\alpha) X_t \quad t = 1, 2, \cdots \quad (3)$$

By subtracting (3) from (1), the forecast error is

$$d_t - X_t = \varepsilon_t \quad t = 1, 2, \cdots \quad (4)$$

Clearly, $E(d_t - X_t) = 0$, $\mathrm{var}(d_t - X_t) = \sigma^2$. Hence, there is no better forecast model for this demand process.

From (3) and (4), we can rewrite the forecast in terms of the noise terms:

$$X_{t+1} = X_t + \alpha\varepsilon_t = \alpha\varepsilon_t + \alpha\varepsilon_{t-1} + \cdots + \alpha\varepsilon_1 + \mu \quad (5)$$

Let q_t be the order placed in period t for delivery in period $t+l$. Then, the inventory balance equation for this system is

$$x_t = x_{t-1} - d_t + q_{t-l} \quad t = 1, 2, \cdots \quad (6)$$

where x_t denotes the on hand-inventory (or backorders) at the end of period t. We can set an initial inventory level x_0, and that $q_t = \mu$ for $t \leqslant 0$.

Suppose that the retailer operates with a safety stock policy, but adjusts the safety stock as his demand forecast changes. According to Ref. [6], we have

$$q_t = d_t + l(X_{t+1} - X_t) \quad (7)$$

Eq.(7) shows that there are two components to the order quantity q_t: the first component replenishes the demand for the immediate period, as with a typical safety stock policy; the second component adjusts the safety stock level to accommodate the change in the forecast, which changes the mean lead-time demand.

1.2 Supplier's demand and inventory control models

First consider the demand process of the supplier, namely order stream $\{q_t\}$ from the retailer. From (2), (5) and (7), we find

$$q_t = (1 + l\alpha)\varepsilon_t + \alpha\varepsilon_{t-1} + \cdots + \alpha\varepsilon_1 + \mu \quad t = 1, 2, \cdots \quad (8)$$

From (8), let $\zeta_t = (1 + l\alpha)\varepsilon_t$, and $\beta = \dfrac{\alpha}{1 + l\alpha}$, thus the time series $\{q_t\}$ also is a standard ARIMA(0, 1, 1) process, namely

$$q_1 = \mu + \zeta_1 \text{ and } q_{t+1} = q_t - (1-\beta)\zeta_t + \zeta_{t+1} \quad t = 1, 2, \cdots \quad (9)$$

From the assumptions for $\{\varepsilon_t\}$, we see that $\{\zeta_t\}$ is i.i.d. random noise with $E(\zeta_t) = 0$, $\mathrm{var}(\zeta_t) = \sigma'^2 = (1 + l\alpha)^2\sigma^2$. Similarly, we have

$$q_t = \zeta_t + \beta\zeta_{t-1} + \beta\zeta_{t-2} + \cdots + \beta\zeta_1 + \mu \quad (10)$$

The supplier should forecast the supplier's demand q_t to order from his upstream. The forecast process is

$$Y_1 = \mu \text{ and } Y_{t+1} = \beta q_t + (1-\beta) Y_t \quad t = 1, 2, \cdots \quad (11)$$

where Y_{t+1} denotes that the supplier forecasts demand in period $t+1$ by observing the demand q_t in period t, and β is the parameter for the inertia of the process. The supplier's demand process is more variable than the retailer's demand, and has more inertia. Additionally, because the supplier may suspect that the retailer's order cannot convey the basic variation of the real demand, his forecast result can't respond excessively to the retailer's order mode of the current changes, and $\beta \leqslant \alpha$ is reasonable.

The following expression is similar to (5):

$$Y_{t+1} = Y_t + \beta\zeta_t = \beta\zeta_t + \beta\zeta_{t-1} + \cdots + \beta\zeta_1 + \mu \quad (12)$$

We now examine the inventory requirement for the supplier. Let Q_t be the order placed in period t by the supplier to his supplier. The lead-time for replenishment to the upstream stage is L. The inventory balance equation for the supplier is

$$y_t = y_{t-1} - q_t + Q_{t-L} \quad t = 1, 2, \cdots \quad (13)$$

where y_t denotes the on-hand inventory (or backorders) at the end of period t. We can also set an initial inventory level y_0, and that $Q_t = \mu$ for $t \leqslant 0$.

Supposing that the supplier operates with a safety stock policy similar to (7) and (8), we get

$$Q_t = q_t + L(Y_{t+1} - Y_t) \quad (14)$$

and

$$Q_t = [1 + (l + L)\alpha]\varepsilon_t + \alpha\varepsilon_{t-1} + \cdots + \alpha\varepsilon_1 + \mu \quad t = 1, 2, \cdots \quad (15)$$

2 Bullwhip Effect

2.1 Properties of the random variables

We easily gain the following formulae by the definitions and properties of variance and correlation coefficient.

Proposition 1 There are the following formulae for the consumer demand d_t, the forecasts X_t and Y_t, the orders of the retailer and supplier q_t and Q_t, respectively.

① $\mathrm{var}(d_t)=[(t-1)\alpha^2+1]\sigma^2$;

② $\mathrm{var}(X_t)=(t-1)\alpha^2\sigma^2$;

③ $\mathrm{var}(Y_t)=(t-1)\beta^2\sigma'^2=(t-1)\alpha^2\sigma^2$;

④ $\mathrm{var}(q_t)=[(t-1)\beta^2+1]\sigma'^2=[(t-1)\alpha^2+(1+l\alpha)^2]\sigma^2$;

⑤ $\mathrm{var}(Q_t)=[(t-1)\alpha^2+(1+l\alpha+L\alpha)^2]\sigma^2$;

⑥ Let the correlation coefficient of X_t and d_t be $\rho_{X,d}$, thus

$$\rho_{X,d}=\sqrt{\frac{(t-1)\alpha^2}{(t-1)\alpha^2+1}}\qquad \rho_{X,d}\geqslant 0$$

⑦ Let the correlation coefficient of Y_t and q_t be $\rho_{Y,q}$, thus

$$\rho_{Y,q}=\sqrt{\frac{(t-1)\beta^2}{(t-1)\beta^2+1}}=\sqrt{\frac{(t-1)\alpha^2}{(t-1)\alpha^2+(1+l\alpha)^2}}\qquad \rho_{Y,q}\geqslant 0$$

Proposition 2 If $0<\alpha<1$ and $t>1$, then $\rho_{X,d}$ is the monotone increased function for α and t, respectively.

From proposition 2, we can see that when t and α are fixed, the larger α is, the more $\rho_{X,d}$ is. This indicates that the forecast X_t will be close to d_t linearly. In the following, we also have the proposition and explanation similar to proposition 2.

Proposition 3 If $0<\alpha<1$, then $\rho_{Y,q}$ is the monotone increased function for α and t, respectively.

Proposition 4 $\rho_{X,d}\geqslant\rho_{Y,q}$

Proposition 4 indicates that the retailer's forecast is better than the supplier's in linear relativity. This is due to the supplier's lead-time.

2.2 Bullwhip effect

Theorem 1 If the retailer and supplier both use safety stock systems coupled with simple exponentially weighted moving average forecasting systems and face non-stationary, serially correlated demand, then

$$\mathrm{var}(Q_t)\geqslant\mathrm{var}(q_t)\geqslant\mathrm{var}(d_t)\tag{16}$$

This is the so-called bullwhip effect, i.e., the variance of the supplier's order releases is higher than that of the retailer's, which in turn is higher than the variance of actual demand, although $\mathrm{var}(X_t)=\mathrm{var}(Y_t)=(t-1)\alpha^2\sigma^2$.

Theorem 2 If the retailer and supplier both use safety stock systems coupled with simple exponentially weighted moving average forecasting systems and face non-stationary, serially correlated demand, then

$$\mathrm{var}(Y_t-q_t)\geqslant\mathrm{var}(X_t-d_t)\tag{17}$$

We can know from theorem 2 that although $\beta\leqslant\alpha$, the forecast errors of the supplier have greater variance than those of the retailer.

3 VMI and Bullwhip Effect

Under VMI contract, the supplier manages the retailer's inventory, so he will forecast and order for both the retailer and himself by the end-consumer's demand, not by the retailer's order, then send his order to the upstream stage.

In the following, the variables with super "′" denote the case after VMI being implemented. Then we have

$$y'_t=y'_{t-1}-d_t+Q'_{t-L}\qquad t=1,2,\cdots\tag{18}$$

We can set an initial inventory level y'_0, and that $Q'_t=\mu$ for $t\leqslant 0$. Hence

$$Q'_t=d_t+L(X_{t+1}-X_t)\tag{19}$$

From (7), (14) and (19), we have

$$q_t=d_t+l\alpha\varepsilon_t$$
$$Q_t=d_t+(l+L)\alpha\varepsilon_t$$
$$Q'_t=d_t+L\alpha\varepsilon_t$$

This suggests that, in terms of reducing the supplier's safety stocks, VMI is always effective when the replenishment time is not zero and $\alpha\neq 0$.

Theorem 3 If the retailer and supplier both use safety stock inventory systems coupled with simple exponentially weighted moving average forecasting systems and face non-stationary, serially correlated demand, then

$$\mathrm{var}(Q_t-d_t)\geqslant\mathrm{var}(Q'_t-d_t)\tag{20}$$

Theorem 3 tells us that VMI program (where the supplier performs the forecasting and ordering activities for both parties using the retailer's smoothing constant) will result in lower order errors for the supplier. In addition, we have $\mathrm{var}(Q_t)\geqslant\mathrm{var}(Q'_t)$ under theorem 3 conditions.

Definition 1 Let $\Delta V_{\mathrm{q}}=\dfrac{\mathrm{var}(Q_t)-\mathrm{var}(q_t)}{\mathrm{var}(q_t)}$, $\Delta V_{\mathrm{d}}=\dfrac{\mathrm{var}(Q'_t)-\mathrm{var}(d_t)}{\mathrm{var}(d_t)}$. We call ΔV_{q} and ΔV_{d} as the magnification rate of the supplier's demand variance before and after VMI has been implemented, respectively.

ΔV_{q} and ΔV_{d} can be rewritten as

$$\Delta V_{\mathrm{q}}=\frac{(L\alpha)^2+2L\alpha(1+l\alpha)}{(t-1)\alpha^2+(1+l\alpha)^2}$$

$$\Delta V_{\mathrm{d}}=\frac{(L\alpha)^2+2L\alpha}{(t-1)\alpha^2+1}$$

Now we compare ΔV_q with ΔV_d. When α, l and L are fixed, and $\Delta V_q / \Delta V_d > 1$, $t \geqslant t_0$ can be got. So, $\Delta V_q > \Delta V_d$ under $t > t_0$, while $\Delta V_q < \Delta V_d$ under $t < t_0$ (see Fig. 1), where t_0 is the maximum integer no excess $\frac{(1 + l\alpha + L\alpha)^2[(1 + l\alpha)^2 - 2] + 1}{L\alpha^3(2 + 2l\alpha + L\alpha)} + 1$. This means that as long as t is big enough ($t \geqslant t_0$), VMI is always effective. We can also discuss the effects of α, l, and L on the magnification rates, which is omitted.

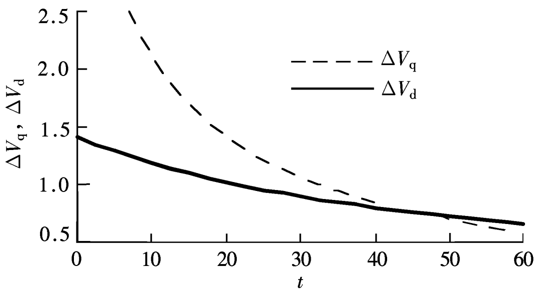

Fig.1 Relation between V_q, ΔV_d and t when $\alpha = 0.3$, $l = 2$ and $L = 3$

Definition 2 Let $\Delta Q = \frac{\operatorname{var}(Q_t) - \operatorname{var}(Q_t')}{\operatorname{var}(Q_t)}$.

We call ΔQ the increased percent of the supplier's order variance with VMI.

Suppose $t = 3$, $L = 2$ and $l = 3$, ΔQ is listed in Tab.1 when α varies. From Tab. 1, we conclude that the bigger α is, the bigger the effect of VMI program on ΔQ is. Because the forecast results depend more on recently observed data, with α increasing, the accuracy becomes higher.

Tab.1 VMI effect on variances of order

α	β	ΔQ/%
0.1	0.077	35.68
0.3	0.158	57.39
0.5	0.200	64.71
0.7	0.226	68.25
0.9	0.243	70.32

4 Conclusions

In this paper, we analytically examine the improvement of VMI by effective information sharing and consistent forecasting. It shows that independent actions taken by members of a conventional supply chain typically have the negative impact on order release volatility and forecast error volatility. Such increases in variation are argued to pose the bullwhip effect. However, VMI program can help alleviate such negative effects. The results are summarized as follows:

1) When the supplier's forecasting errors are greater than those of the retailer's without VMI, VMI will be effective on reducing the supplier's safety stocks. VMI usually reduces fluctuations in order releases as well.

2) The smoothing constants adopted by the retailer and supplier determine the extent of the effect of VMI on reducing safety stock levels, the variances of order releases, and the others such as the magnification rate of the supplier's demand variance, etc.

3) VMI is effective in the case of stationary demand, but not so effective in the case of non-stationary demand.

There are many unanswered questions or open issues worthy of further research. In this paper, we study the simple non-stationary processes in the simple supply chain. It would certainly be of interest to enrich either the demand model or supply chain model or both. For example, what happens if the replenishment lead-times are stochastic What happens if the number of retailers is more than one?

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供应商管理库存和牛鞭效应

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摘要: 研究了发生在供应链中订货的牛鞭效应和安全库存放大问题, 其需求模型是 ARIMA(0, 1, 1), 而所用的预测方法是简单的指数平滑.在此基础上, 还研究了供应商管理库存计划如何确定减轻这样的负面效应, 并且给出了理论证明和数值解释, 结果表明在需求预测、安全库存水平等方面使用供应商管理库存的效果比没用供应商管理库存的要好.

关键词: 供应商管理库存; 牛鞭效应; 供应链; 需求预测

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