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# Performance analysis of complex wavelet packet based multicarrier CDMA system over Nakagami-$m$ fading channels

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**Abstract:** On the basis of analyzing the principle of multicarrier code division multiple access (MC-CDMA), an optimized complex wavelet packet based MC-CDMA system (CWP-MC-CDMA) is proposed. The system performances with equal gain combining (EGC) technique and maximum ratio combining (MRC) technique in Nakagami-$m$ fading channels are investigated, respectively; and the corresponding bit error rate (BER) expression is derived. The system can overcome the decrease of spectrum efficiency of discrete Fourier transform (DFT) based conventional MC-CDMA (DFT-MC-CDMA) due to inserting a cyclic prefix (CP). Theoretical analysis and simulation results show that the CWP-MC-CDMA system outperforms DFT-MC-CDMA as well as the real wavelet packet based MC-CDMA (RWP-MC-CDMA) system, it has superior ability to combat multi-path fading and multi-access interference (MAI). Moreover, the BER performance of the proposed system is also superior to that of the conventional MC-CDMA with CP.

**Key words:** complex wavelet packet; code division multiple access (CDMA); multicarrier CDMA; maximum ratio combining (MRC); equal gain combining (EGC)

Multicarrier code division multiple access (MC-CDMA) technique has received much attention among researchers, and it is one of the most promising techniques for future mobile radio communication systems beyond 3G[1]. It has the properties desirable for high data-rata wireless services such as insensitivity to frequency selective channels, frequency diversity, high spectral efficiency and flexibility to generate different data rates within a fixed bandwidth[2]. However, the conventional MC-CDMA is implemented by means of discrete Fourier transform (DFT) and inverse-DFT (IDFT) operators. In its frequency spectrum, the main lobe does not concentrate energy effectively and side lobe attenuates slowly; the multi-path fading or synchronization error will cause severe performance degradation due to inter-channel interference (ICI), inter-symbol interference (ISI) and multi-access interference (MAI). To overcome the above shortcomings, a number of improved MC-CDMA systems have been proposed. Among them, wavelet packet based MC-CDMA[3] have advantages of stronger ability to combat multi-path interference (MPI) and ISI than conventional MC-CDMA. Especially, the optimized complex wavelet packet[4] not only has good properties that the real wavelet packet possesses, such as shifting orthogonality, adaptability, time-frequency localization, but also matches the complex channel frequency spectrum and suits multi-carrier communications. Motivated by the above reason, we present an MC-CDMA system based on the optimized complex wavelet packet (CWP-MC-CDMA) in this paper, and the corresponding downlink performance is investigated over Nakagami-$m$ fading channels. By comparison, it performs better than DFT-MC-CDMA and the real wavelet packet based MC-CDMA (RWP-MC-CDMA) system; moreover, the CWP-MC-CDMA system with equal gain combining (EGC) outperforms that with maximum ratio combining (MRC).

## 1 CWP-MC-CDMA System Model

A synchronous downlink performance of CWP-MC-CDMA system with $K$ users is analyzed in this section. The transmitter and receiver of CWP-MC-CDMA are illustrated in Fig.1 and Fig.2, respectively. At the transmitter, the complex wavelet packet functions $g_i(t)$ are taken as the signature waveform, and the shifting orthogonality among $g_i(t)$ $(i=1, \cdots, M)$ can be given as $\langle g_i(t), g_l^*(t-nT_s)\rangle = \delta(i-l)\delta(n)$, where $\delta$ is the Kronecker function and superscript $*$ represents the complex conjugate. The transmitted signal of the $k$-th user is written as

$$S_k(t) = \sum_{l=1}^{M}\sum_{n=0}^{+\infty}\sqrt{\frac{2E_b}{M}}b_k(n)c_k(l)\cdot g_l(t-nT_s)\exp(jw_c t) \tag{1}$$

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where $k=1, \cdots, K$, $K$ is the active user number; $b_k(n)$ corresponding to the quadrature phase shift keying (QPSK) complex signal denotes the $n$-th data symbol of the $k$-th user, $\{b_k(n)\}$ are assumed to be independent, identically distributed (IID) random variables (RVs) taking value $\{\pm 2^{-\frac{1}{2}} \pm \mathrm{j}2^{-\frac{1}{2}}\}$ with equal probability. $E_{\mathrm{b}}$ is the mean energy of the transmitted signals over a bit. The symbols period $T_{\mathrm{s}}$ corresponds to the minimum orthogonal shifting defined in complex wavelet packet; $C_k=\{c_k(i), i=1, \cdots, M\}$ is Walsh-Hadamard code, which represents the $k$-th user-spreading code, the length of the code, that is the processing gain is equal to the number of subcarrier $M$. $w_{\mathrm{c}}$ is the carrier angle frequency, and $d_i$ represents the equalization gain for the subcarrier $i$.

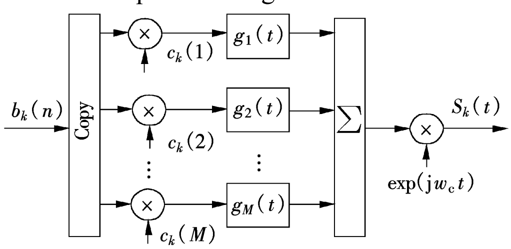


**Fig.1** CWP-MC-CDMA transmitter structure

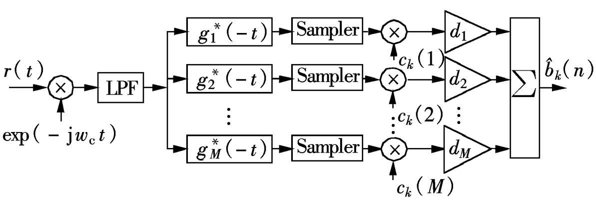


**Fig.2** CWP-MC-CDMA receiver structure

In this paper, the Nakagami-$m$ fading channels are considered. We suppose that each modulated subcarrier experiences independent fading channels and the channels remain constant over symbols period, so the lowpass impulse response of the $l$-th subcarrier channel for the $k$-th user can be represented as

$$h_{k,l}(t,\tau)=\alpha_{k,l}(t)\exp[\mathrm{j}\gamma_{k,l}(t)]\delta(\tau)=\beta_{k,l}(t)\delta(\tau) \tag{2}$$

where $\alpha_{k,l}$ and $\gamma_{k,l}$ denote the amplitude and phase of attenuation factors $\beta_{k,l}$, respectively[5]. $\{\alpha_{k,l}\}$ are Nakagami-$m$ RVs; the phase $\{\gamma_{k,l}\}$ are random variables uniformly distributed over $[0, 2\pi]$. Considering downlink transmission, all user signals pass through the same channel; hence, for $\forall l$, $\alpha_{k,l}(t)=\alpha_l(t)$, $\gamma_{k,l}(t)=\gamma_l(t)$. Therefore, at the receiver, the received signal is

$$r(t)=\sum_{k=1}^{K}\sum_{l=1}^{M}\sum_{n=0}^{+\infty}\sqrt{\frac{2E_{\mathrm{b}}}{M}}b_k(n)c_k(l)\cdot g_l(t-nT_{\mathrm{s}})\alpha_l(t)\exp\{\mathrm{j}w_{\mathrm{c}}[+\mathrm{j}\gamma_l(t)]\}+\eta(t) \tag{3}$$

where $\eta(t)$ is the additive white Gaussian noise (AWGN) with double sided power spectrum density $N_0/2$ and zero mean. Since the amplitude and phase remain constant during the symbols period, $\beta_l=\alpha_l\exp(\mathrm{j}\gamma_l)$. After passing through lowpass filter (LPF) and the complex wavelet packet matched filter in subchannel $i$, the output signal at the $uT_{\mathrm{s}}$ sampling interval is

$$y_i(u)=\sum_{k=1}^{K}\sum_{l=1}^{M}\sum_{n=-\infty}^{+\infty}\sqrt{\frac{2E_{\mathrm{b}}}{M}}b_k(n+u)\cdot c_k(l)\beta_l R_g^{li}(nT_{\mathrm{s}})+\eta_i(u) \tag{4}$$

where

$$R_g^{li}(\tau)=\int_{T_{\mathrm{s}}}g_l(t)g_i^*(t+\tau)\mathrm{d}t$$

$$\eta_i(u)=\int_{T_{\mathrm{s}}}\eta(t)g_i^*(t-uT_{\mathrm{s}})\mathrm{d}t$$

## 2 Performance Analysis for CWP-MC-CDMA System

Without loss of generality, let user 1 be the desired user, then let the decision variable for the $u$-th data symbol of user 1 be

$$\begin{aligned}\hat{b}_1(u)&=\sum_{i=1}^{M}c_1(i)d_iy_i(u)=\sum_{i=1}^{M}\sqrt{\frac{2E_{\mathrm{b}}}{M}}b_1(u)\beta_id_i+\\&\sum_{i=1}^{M}\sum_{n=-\infty,\,n\neq 0}^{+\infty}\sqrt{\frac{2E_{\mathrm{b}}}{M}}b_1(n+u)\beta_iR_g^{ii}(nT_{\mathrm{s}})d_i+\\&\sum_{l=1}^{M}\sum_{i=1,\,i\neq l}^{M}\sum_{n=-\infty}^{+\infty}\sqrt{\frac{2E_{\mathrm{b}}}{M}}b_1(n+u)c_1(l)\beta_lR_g^{li}(nT_{\mathrm{s}})\cdot\\&c_1(i)d_i+\sum_{k=2}^{K}\sum_{l=1}^{M}\sum_{n=-\infty}^{+\infty}\sum_{i=1}^{M}\sqrt{\frac{2E_{\mathrm{b}}}{M}}b_k(n+u)\cdot\\&c_k(l)\beta_lR_g^{li}(nT_{\mathrm{s}})c_1(i)d_i+\sum_{i=1}^{M}c_1(i)d_i\eta_i(u)=\\&D+I_1+I_2+I_3+Z\end{aligned} \tag{5}$$

where the noise interference $Z$ is a Gaussian RV with zero mean, and $D$ is the desired output. There are three types of interference contained in (5): ① $I_1$ is the interference from the same subchannel $i$ and the same user $k=1$; ② $I_2$ is the interference from the other subchannels and the same user $k=1$; ③ $I_3$ is the interference from the other users $k\neq 1$. Since cross-correlation functions of optimized complex wavelet packets[4] satisfy the equation $R_g^{li}(nT_{\mathrm{s}})=\delta(i-l)\cdot\delta(n)$, then $I_1=I_2=0$, $I_3=\sqrt{\frac{2E_{\mathrm{b}}}{M}}\sum_{k=2}^{K}\sum_{i=1}^{M}b_k(u)c_k(i)c_1(i)\beta_id_i$.

### 2.1 System BER by applying EGC

For EGC, $d_i=\exp(-\mathrm{j}\gamma_i)$, so

$$D=\sum_{i=1}^{M}\sqrt{\frac{2E_{\mathrm{b}}}{M}}b_1(u)\beta_id_i=\sum_{i=1}^{M}\sqrt{\frac{2E_{\mathrm{b}}}{M}}b_1(u)\alpha_i$$

$$I_3 = \sqrt{\frac{2E_b}{M}} \sum_{k=2}^{K} \sum_{i=1}^{M} b_k(u) c_k(i) c_1(i) \alpha_i$$

$$Z = \sum_{i=1}^{M} c_1(i) \exp(-j\gamma_i) \int_{T_s} \eta(t) g_i^*(t - uT_s) \, dt$$

Considering that $M$ and $K$ are large in general, MAI term $I_3$ can be approximately a zero-mean Gaussian RV. By computation and making use of superior orthogonality of Walsh-Hadamard code applied, the variance of $I_3$ and $Z$ are $\mathrm{var}(I_3) = 2E_b(K-1)\sigma^2$ and $\mathrm{var}(Z) = MN_0/2$, respectively, where $\sigma^2$ is the variance of $\alpha_i$. Hence, the bit error probability conditioned on $\{\alpha_i\}$ can be written as

$$P(e \mid \{\alpha_i\}) = Q\left(\sqrt{\frac{|D|^2}{\mathrm{var}(I_3) + \mathrm{var}(Z)}}\right) = Q(\sqrt{\lambda X^2}) = P(e \mid X) \tag{6}$$

where $\lambda = (2E_b/M)/[2E_b(K-1)\sigma^2 + MN_0/2]$, $X = \alpha_1 + \cdots + \alpha_M$.

According to Eq. (2) in Ref. [6], $Q(t) = \frac{1}{\pi} \cdot \int_0^{\frac{\pi}{2}} \exp\left(-\frac{t^2}{2\sin^2\theta}\right) d\theta$, $t \geqslant 0$, then BER can be obtained via averaging $P(e \mid X)$ over $X$ as

$$\mathrm{BER} = \int_0^{\infty} P(e \mid X = x) p_X(x) \, dx = \int_0^{\infty} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \exp\left(-\frac{\lambda x^2}{2\sin^2\theta}\right) d\theta \, p_X(x) \, dx \tag{7}$$

Considering that $p_X(x)$ is the inverse Fourier transform of its characteristic function $\psi_X(jv)$, and IID assumption on $\{\alpha_i, i = 1, 2, \cdots, M\}$, $p_X(x)$ can be rewritten as

$$p_X(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_X(jv) \exp(-jvx) \, dv = \frac{1}{2\pi} \int_{-\infty}^{\infty} [\psi_{\alpha_i}(jv)]^M \exp(-jvx) \, dv \tag{8}$$

Substituting (8) into (7), BER becomes

$$\mathrm{BER} = \frac{1}{2\pi^2} \int_0^{\frac{\pi}{2}} \int_{-\infty}^{\infty} [\psi_{\alpha_i}(jv)]^M \cdot \left[\int_0^{\infty} \exp\left(-\frac{\lambda x^2}{2\sin^2\theta} - jvx\right) dx\right] dv \, d\theta \tag{9}$$

### 2.2 System BER by applying MRC

For MRC, $d_i = \alpha_i \exp(-j\gamma_i)$, so

$$D = \sum_{i=1}^{M} \sqrt{\frac{2E_b}{M}} b_1(u) \beta_i d_i = \sum_{i=1}^{M} \sqrt{\frac{2E_b}{M}} b_1(u) \alpha_i^2$$

$$I_3 = \sqrt{\frac{2E_b}{M}} \sum_{k=2}^{K} \sum_{i=1}^{M} b_k(u) c_k(i) c_1(i) \alpha_i^2$$

$$Z = \sum_{i=1}^{M} c_1(i) \alpha_i \exp(-j\gamma_i) \int_{T_s} \eta(t) g_i^*(t - uT_s) dt$$

Because $\{\alpha_i, i = 1, 2, \cdots, M\}$ are independent random variables, all $M(K-1)$ terms in the summation of $I_3$ are uncorrelated with zero-mean. Considering $M$ is large in practice, $I_3$ can be approximated by a Gaussian RV conditioned on the fading channel amplitudes with zero-mean and variance $\mathrm{var}(I_3) = (2E_b/M)(K-1)\sum_{i=1}^{M} \alpha_i^2$.

Similarly, we can evaluate the variance of $Z$ by $\mathrm{var}(Z) = \frac{N_0}{2} \sum_{i=1}^{M} \alpha_i^2$.

By using the Gaussian approximation, the probability of bit error conditioned on $\{\alpha_i\}$ is given by

$$P(e \mid \{\alpha_i\}) = Q\sqrt{\frac{|D|^2}{[\mathrm{var}(I_3) + \mathrm{var}(Z)]}} = Q\sqrt{\sum_{i=1}^{M} s_i} = P(e \mid \{s_i\}) \tag{10}$$

where $s_i = \rho\alpha_i^2$, $\rho = 4E_b/[4E_b(K-1) + MN_0]$.

Considering that $\alpha_i$ is Nakagami-$m$ random variable, so the probability density function (PDF) of $\alpha_i$ is written as $p(\alpha_i) = [2/\Gamma(m)](m/\Omega)^m \alpha_i^{2m-1} \cdot \exp(-m\alpha_i^2/\Omega)$, where $\Omega = E(\alpha_i^2)$.

By transformation, we can calculate the PDF of $s_i$ by

$$p(s_i) = \frac{s_i^{m-1}}{\Gamma(m)} \left(\frac{m}{\bar{s}_i}\right)^m \exp\left(-\frac{m s_i}{\bar{s}_i}\right) \tag{11}$$

where $\bar{s}_i = E(\rho\alpha_i^2) = \rho\Omega$.

Thus, we can obtain BER via averaging $P(e \mid s_i)$ over $s_i$ as

$$\mathrm{BER} = \int_0^{\infty} \left[\frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{i=1}^{M} \exp\left(-\frac{s_i}{2\sin^2\theta}\right) d\theta\right] \cdot \left[\frac{s_i^{m-1}}{\Gamma(m)} \left(\frac{m}{\bar{s}_i}\right)^m \exp\left(-\frac{m s_i}{\bar{s}_i}\right)\right] ds_i = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{i=1}^{M} \left(1 + \frac{\bar{s}}{2m\sin^2\theta}\right)^{-m} d\theta \tag{12}$$

According to the BER expression from (9) and (12), we can conclude that the proposed system obtain $M$ diversity order, which comes from the frequency diversity gain.

## 3 Simulation Results

In this section, the performance of CWP-MC-CDMA system with EGC and MRC technique is simulated in the Nakagami-$m$ fading channel. For simplicity of the channel model, $m = 1$, i.e., the Rayleigh fading channel is employed in the simulation, and it is modeled by the Jakes modified model. It is assumed that different subcarriers experience independent identically distributed fading channels. The simulation results are shown in Fig. 3 and Fig. 4, respectively. In the simulation, we assume that the carrier synchronization, clock synchronization and spreading code synchronization are achieved correctly; all users receive the same power and the channel estimation is perfect. The parameters are set as carrier frequency $f_c = 2$ GHz, the sampling rate $f_s = 3.84$ MHz, the vehicle velocity $v = 50$ km/h, the bit rate per user is 384 kbit/s. $M = 32$, $K = 16$, EGC, optimized complex wavelet packet with the fifth-level of the binary wavelet packet tree[4] and corresponding real-valued Daubechies wavelet packet are considered in Fig. 3. Meanwhile, $M = 16$, EGC, MRC, as well as the complex wavelet packet corresponding to the fourth-level of a binary wavelet packet tree[4] are shown in Fig. 4.

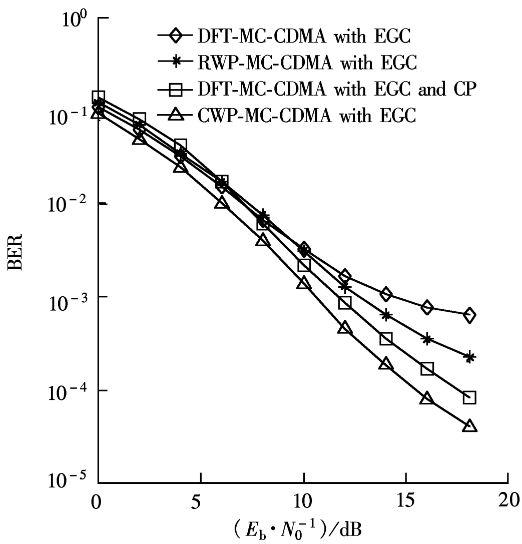


**Fig.3** BER against SNR for different systems

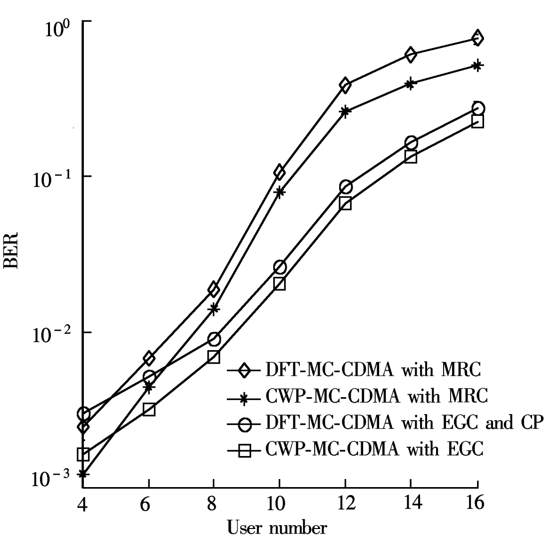


**Fig.4** BER vs. numbers of users for different systems

The simulation results show that the system with different combing methods has different performances. From Fig. 3, the performance of the CWP-MC-CDMA outperforms that of DFT based MC-CDMA systems as well as the RWP-MC-CDMA system, and it is close or superior to that of conventional MC-CDMA system with CP (CP = 6), where "CP = $x$" denotes that $x$ cyclic prefix symbols are inserted. Especially, the CWP-MC-CDMA system with EGC technique can decrease the BER effectively over Nakagami-$m$ ($m = 1$) fading channel. We also give the average BER of different multi-carrier systems against different numbers of users in Fig. 4, where the SNR is fixed at 8 dB. As a result, the presented CWP-MC-CDMA system with EGC technique has the best performance; it outperforms the CWP-MC-CDMA system with MRC technique and DFT-MC-CDMA system with CP = 3; it can get lower BER than the other comparative systems conditioned on the same number of users. Moreover, with the increase of sub-carrier numbers, the performance of the proposed system is improved further due to the frequency diversity gain from multi-carrier modulation.

## 4 Conclusion

In this paper, an orthogonal MC-CDMA scheme based on a complex wavelet packet is presented, and the corresponding performance analyses have been undertaken in Nakagami-$m$ fading environments. The error rate probability expressions for CWP-MC-CDMA system with EGC and MRC are derived in detail, respectively. The system can overcome the decrease of spectrum efficiency and energy of conventional MC-CDMA system due to the insertion of a cyclic prefix. The analytical and simulation results show that the proposed CWP-MC-CDMA system performs better than the DFT-MC-CDMA system, and is close or superior to the DFT-MC-CDMA system with CP. For EGC and MRC considered in our system, the system performance with EGC technique shows obvious performance improvement, and compared with DFT-MC-CDMA and RWP-MC-CDMA system, the proposed CWP-MC-CDMA system with EGC technique can effectively strengthen the system's ability to cope with multi-path fading and MAI.

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# 基于复小波包的多载波 CDMA 系统在 Nakagami-*m* 衰落信道下的性能

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**摘要**: 在分析多载波码分多址(MC-CDMA)的原理基础上, 提出了一种基于复小波包的 MC-CDMA 系统. 同时, 研究了该系统分别采用等增益合并和最大比合并时在 Nakagami-*m* 衰落信道下的性能; 并推导了相应的误码率表达式. 该系统能克服通常 MC-CDMA 系统由于插入循环前缀(CP)所带来的频谱效率的下降. 理论分析和仿真结果表明:该系统要优于通常 MC-CDMA 系统和基于实小波包的 MC-CDMA 系统, 有着优越的抗多径衰落和多址干扰能力,而且该系统也优于采用 CP 的通常 MC-CDMA 系统.

**关键词**: 复小波包; CDMA; 多载波 CDMA; 最大比合并; 等增益合并

**中图分类号**: TN911.5