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Efficient numerical analysis of guided wave structures by compact FDFD with PVL method

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Abstract: An efficient numerical simulation technique is introduced to extract the propagation characteristics of a millimeter guided wave structure. The method is based on the application of the Krylov subspace model order reduction technique (Padé via Lanczos) to the compact finite difference frequency domain (FDFD) method. This new technique speeds up the solution by decreasing the originally larger system matrix into one lower order system matrix. Numerical experiments from several millimeter guided wave structures demonstrate the efficiency and accuracy of this algorithm.

Key words: model order reduction; finite difference frequency domain (FDFD); guided wave structure; Padé via Lanczos (PVL)

Accurate modeling and simulation of millimeter guided wave structures are very important and indispensable in microwave engineering designs. A compact finite difference frequency domain (FDFD) algorithm, which uses a two-dimensional mesh to contain six field components for realistic three-dimensional wave guided structures, has been proven to be an efficient technique for performing such simulations[1]. Despite the fact that the compact FDFD method requires less central processing unit (CPU) time and dramatically reduced computer memory compared to the regular 3-D FDFD method, this approach still needs to solve a large sparse matrix many times if wide band frequency information is required.

To improve the computational efficiency of general wide band frequency structures, several papers have proposed using fitting modeling[1]. These techniques used efficient interpolation/extrapolation techniques to predict a wide band frequency response based on a reduced number of simulations. However, these works have not fully utilized the system equation that represents the system to be simulated. To take advantage of the information that is contained in the system equation, model order reduction (MOR) techniques have been developed. It was originally developed for very large scale integration (VLSI) type circuit analysis and has been recently extended to the electromagnetic (EM) simulation fields[2-9]. Among them, asymptotic waveform evaluation (AWE)[3], which uses moment matching and Padé approximation to approximate the transfer function of a large linear system, is straightforward and can be implemented in the EM field simulation efficiently[4]. Even though AWE can achieve good accuracy and efficiency, it suffers from numerical instability if higher order approximation is required[6]. In addition, for higher order systems, there is a large computational overhead to generate the moments to represent the original systems.

In order to overcome such issues associated with the AWE technique, Padé via Lanczos (PVL) algorithm, one of the Krylov subspace-based methods, was used in the electromagnetic simulation to reduce the original large linear equation matrix over a board frequency range[6-9]. In most of the works, PVL has been proven to be an efficient model order reduction technique due to its robustness and accuracy.

1 Theory and Formulae

1.1 Compact FDFD

Due to the invariance in the propagation direction of guided wave structures, the phases associated with electromagnetic fields have the form of $\boldsymbol{A}(x, y, z) = \boldsymbol{A}(x, y)\exp(-\mathrm{j}\beta z)$. For a propagation constant β in the z direction, it is easy to show that Maxwell's equation can be written as

$$\left.\begin{array}{ll}\mathrm{j}\omega \boldsymbol{E}_x = \dfrac{1}{\varepsilon}\left(\dfrac{\partial \boldsymbol{H}_z}{\partial y} + \mathrm{j}\beta \boldsymbol{H}_y\right) & \mathrm{j}\omega \boldsymbol{H}_x = -\dfrac{1}{\mu}\left(\dfrac{\partial \boldsymbol{E}_z}{\partial y} + \mathrm{j}\beta \boldsymbol{E}_y\right)\\ \mathrm{j}\omega \boldsymbol{E}_y = -\dfrac{1}{\varepsilon}\left(\mathrm{j}\beta \boldsymbol{H}_x + \dfrac{\partial \boldsymbol{H}_z}{\partial x}\right) & \mathrm{j}\omega \boldsymbol{H}_y = \dfrac{1}{\mu}\left(\mathrm{j}\beta \boldsymbol{E}_x + \dfrac{\partial \boldsymbol{E}_z}{\partial x}\right)\\ \mathrm{j}\omega \boldsymbol{E}_z = \dfrac{1}{\varepsilon}\left(\dfrac{\partial \boldsymbol{H}_y}{\partial x} - \dfrac{\partial \boldsymbol{H}_x}{\partial y}\right) & \mathrm{j}\omega \boldsymbol{H}_z = \dfrac{1}{\mu}\left(\dfrac{\partial \boldsymbol{E}_x}{\partial y} - \dfrac{\partial \boldsymbol{E}_y}{\partial x}\right)\end{array}\right\} \tag{1}$$

where ω is the angular frequency, ε is the permittivity

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and μ is the permeability. By using Eq.(1), electric fields ($\boldsymbol{E}_x, \boldsymbol{E}_y, \boldsymbol{E}_z$) and magnetic fields ($\boldsymbol{H}_x, \boldsymbol{H}_y, \boldsymbol{H}_z$) can be arranged in a Yee's cell as described in Ref. [3]. Using the finite difference method, we can represent the above equations by

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{2}$$

where the matrix $\boldsymbol{A}$ is associated with the finite difference scheme, the vector $\boldsymbol{b}$ is determined by external excitations, and the vector $\boldsymbol{x}$ represents electric and magnetic fields at FDFD grid point. To extract the frequency dependent propagation characteristics, for each propagation constant value, Eq.(2) needs to be solved many times to extract modes' eigen-frequencies[1,2].

1.2 PVL method

Rather than repeatedly solve the system Eq.(2) at many distinct frequency points, the PVL technique allows us to approximate a system frequency response using several expansion points. Before applying this technique, Eq.(1) needs to be reformulated as

$$(s\boldsymbol{C} + \boldsymbol{G})\boldsymbol{x}(s) = \boldsymbol{e}(s) \tag{3}$$

where $\boldsymbol{x}(s) = \{\boldsymbol{H}_x(s), \boldsymbol{H}_y(s), \boldsymbol{H}_z(s), \boldsymbol{E}_x(s), \boldsymbol{E}_y(s), \boldsymbol{E}_z(s)\}^{\mathrm{T}}$, $\boldsymbol{C}$ is a diagonal matrix and $s = \mathrm{j}\omega$, $\boldsymbol{G}$ is also a sparse matrix whose element's values are associated with the finite difference scheme, and $\boldsymbol{e}(s)$ is related to external sources.

The first step in the PVL process is to expand $\boldsymbol{x}(s)$ around a point in the s-plane. Set $s = s_0 + \delta$, where s_0 is the expansion point in the s plane. Thus Eq.(3) can be converted into

$$\begin{aligned}\boldsymbol{x}(s) &= (\boldsymbol{G} + s_0\boldsymbol{C} + \delta\boldsymbol{C})^{-1}\boldsymbol{e} = \\ &[(\boldsymbol{G} + s_0\boldsymbol{C})(\boldsymbol{I} + \delta(\boldsymbol{G} + s_0\boldsymbol{C})^{-1}\boldsymbol{C})]^{-1}\boldsymbol{e} = \\ &[\boldsymbol{I} + \delta(\boldsymbol{G} + s_0\boldsymbol{C})^{-1}\boldsymbol{C}]^{-1}(\boldsymbol{G} + s_0\boldsymbol{C})^{-1}\boldsymbol{e}\end{aligned} \tag{4}$$

Define

$$\boldsymbol{A} = -(\boldsymbol{G} + s_0\boldsymbol{C})^{-1}\boldsymbol{C},\ \boldsymbol{r} = (\boldsymbol{G} + s_0\boldsymbol{C})^{-1}\boldsymbol{e} \tag{5}$$

We can rewrite $\boldsymbol{x}(s)$ as

$$\boldsymbol{x}(s_0 + \delta) = (\boldsymbol{I} - \delta\boldsymbol{A})^{-1}\boldsymbol{r} \tag{6}$$

The PVL algorithm uses the Krylov subspace based method to reduce the matrix $\boldsymbol{A}$ to a set of tri-diagonal matrices $\boldsymbol{T}_q$ and $\tilde{\boldsymbol{T}}_q$. After that, the system frequency response can be found by the Padé approximation of Refs. [7, 8]:

$$\boldsymbol{x}_q(s_0 + \delta) = \boldsymbol{r}\boldsymbol{e}_1^{\mathrm{T}}(\boldsymbol{I} - \delta\boldsymbol{T}_q)^{-1}\boldsymbol{e}_1 \tag{7}$$

which is the q-th order Padé approximation of $\boldsymbol{x}(s)$ and $\boldsymbol{e}_1 = \{1, 0, \cdots, 0\}_q^{\mathrm{T}}$. The tri-diagonal matrix $\boldsymbol{T}_q$ can be decomposed as

$$\boldsymbol{T}_q = \boldsymbol{S}_q\boldsymbol{\Lambda}_q\boldsymbol{S}_q^{-1}$$

where $\boldsymbol{\Lambda}_q$ is a diagonal matrix whose elements correspond to system eigenvalues and $\boldsymbol{S}_q$ is a matrix consisting of system eigenvectors. Therefore the equation can be rewritten as

$$\boldsymbol{x}_q(s_0 + \sigma) = \boldsymbol{r}\boldsymbol{e}_1^{\mathrm{T}}\,\boldsymbol{S}_q^{\mathrm{T}}(\boldsymbol{I} - \sigma\boldsymbol{\Lambda}_q)^{-1}\boldsymbol{S}_q^{-1}\,\boldsymbol{e}_1 \tag{8}$$

Set $\boldsymbol{\mu}^{\mathrm{T}} = \boldsymbol{e}_1^{\mathrm{T}}\,\boldsymbol{S}_q^{T}$, $\boldsymbol{v} = \boldsymbol{S}_q^{-1}\,\boldsymbol{e}_1$ and then the final reduced order model equation can be obtained:

$$\boldsymbol{x}_q(s_0 + \sigma) = \sum_{j=1}^{q}\frac{\boldsymbol{l}^{\mathrm{T}}\boldsymbol{r}\mu_j v_j}{1 - \sigma\lambda_j} = K_\infty + \sum_{\substack{j=1\\ \lambda_j\neq 0}}^{q}\frac{-\dfrac{\boldsymbol{l}^{\mathrm{T}}\boldsymbol{r}\mu_j v_j}{\lambda_j}}{\sigma - \dfrac{1}{\lambda_j}} \tag{9}$$

where the term K_∞ represents the value of $\boldsymbol{x}_q(s_0 + \sigma)$ when λ_j is zero. The whole PVL process is as follows:

1) Set $\rho_1 = \|\boldsymbol{r}\|_2$, $\eta_1 = \|\boldsymbol{l}\|_2$, $\boldsymbol{v}_1 = \dfrac{\boldsymbol{r}}{\rho_1}$, $\boldsymbol{w}_1 = \dfrac{\boldsymbol{l}}{\eta_1}$, $\boldsymbol{v}_0 = \boldsymbol{w}_0 = \boldsymbol{0}$ and $\delta_0 = 1$.

2) For $n = 1, 2, \cdots, q$, ① Compute $\delta_n = \boldsymbol{W}_n^{\mathrm{T}}\,\boldsymbol{V}_n$; ② Set $\alpha_n = \dfrac{\boldsymbol{w}_n^{\mathrm{T}}\boldsymbol{A}\boldsymbol{v}_n}{\delta_n}$, $\beta_n = \dfrac{\delta_n}{\delta_{n-1}}\eta_n$, $\gamma_n = \dfrac{\delta_n}{\delta_{n-1}}\rho_n$; ③ Set $\boldsymbol{V} = \boldsymbol{A}\boldsymbol{v}_n - \alpha_n\boldsymbol{v}_n - \beta_n\boldsymbol{v}_{n-1}$, $\boldsymbol{W} = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{w}_n - \alpha_n\boldsymbol{w}_n - \gamma_n\boldsymbol{w}_{n-1}$; ④ Set $\rho_{n+1} = \|\boldsymbol{V}\|_2$, $\eta_{n+1} = \|\boldsymbol{W}\|_2$, and then $\boldsymbol{v}_{n+1} = \dfrac{\boldsymbol{V}}{\rho_{n+1}}$, $\boldsymbol{w}_{n+1} = \dfrac{\boldsymbol{W}}{\eta_{n+1}}$.

3) Construct tri-diagonal matrix $\boldsymbol{T}_q$,

$$\boldsymbol{T}_q = \begin{bmatrix} \alpha_1 & \beta_2 & & & \\ \rho_2 & \alpha_2 & \beta_3 & & \\ & \rho_3 & \ddots & & \\ & & \ddots & \ddots & \\ & & & \ddots & \beta_q \\ & & & \rho_q & \alpha_q \end{bmatrix}$$

4) Carry on a decomposition of $\boldsymbol{T}_q$,

$$\boldsymbol{T}_q = \boldsymbol{S}_q\mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_q)\boldsymbol{S}_q^{-1}$$

and set $\boldsymbol{\mu} = \boldsymbol{S}_q^{\mathrm{T}}\boldsymbol{e}_1$, $\boldsymbol{v} = \boldsymbol{S}_q^{-1}\boldsymbol{e}_1$, where $\boldsymbol{e}_1 = \{1, 0, \cdots, 0\}_q^{\mathrm{T}}$..

5) Calculate the poles and residues of $\boldsymbol{x}_q$.

2 Numerical Results

The first example we considered here is an air-filled rectangular waveguide with a width of 19.05 mm and a height of 9.525 mm. For each β value in Fig. 1, we extract the eigen-frequencies for the first three propagation modes. In Fig. 1, the propagation constant as a function of frequency is shown. For comparison, the analytical resolution is also given. We can see that the results from PVL/compact FDFD method agree very well with analytical solution.

The second structure analyzed is the partially filled inhomogeneous dielectric loaded waveguide as shown in Fig. 2. The dimensions of this structure are $a = 10.16$ mm, $b = 5.588$ mm, $c = 3.048$ mm, $d = 5.08$ mm. The relative permittivity of the slab is $\varepsilon_r = 8$. The dispersion curves of the first two modes, shown in Fig.3, are compared to the results in Ref. [1] and a good agreement can be observed.

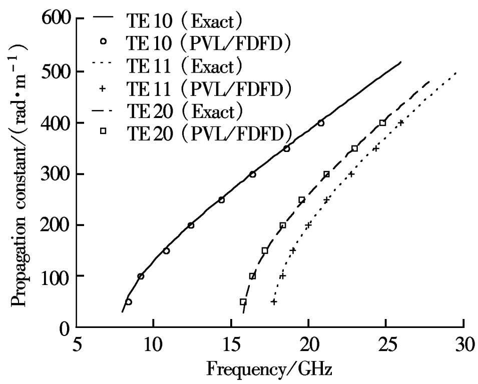

Fig.1 Extracted propagation constant for the first three modes of a waveguide structure

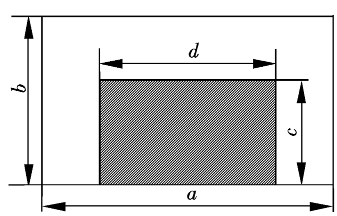

Fig.2 Cross section of a partially filled inhomogeneous dielectric loaded waveguide

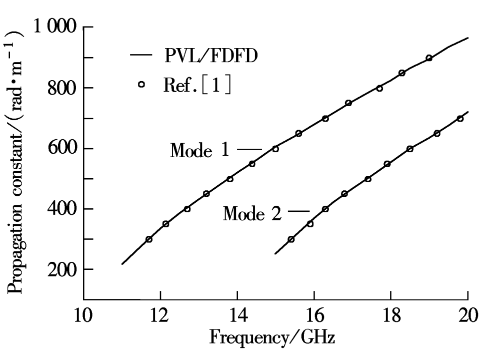

Fig.3 Dispersion characteristics of a partially filled waveguide

The final example considered here is a dual-plane triple microstrip structure as shown in Fig.4. For this particular example, the excitation point is located under the left microstrip line of the lower layer in order to excite all three propagation modes. The substrate here is anisotropic and has a relative permittivity matrix as

$$\bar{\bar{\boldsymbol{\varepsilon}}}_{r_1} = \bar{\bar{\boldsymbol{\varepsilon}}}_{r_2} = \begin{bmatrix} 9.4 & 0 & 0 \\ 0 & 11.6 & 0 \\ 0 & 0 & 9.4 \end{bmatrix}$$

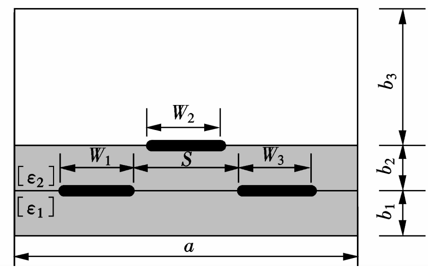

Fig.4 Illustration of a dual-plane triple microstrip structure

The space between the lower two striplines $S = 2.0$ mm. Other dimensions defined in Fig.4 are: $a = 10.0$ mm, $b_1 = b_2 = 1.0$ mm, $b_3 = 4.0$ mm, and $W_1 = W_2 = W_3 = 1.0$ mm. The normalized propagation constants of the first three propagation modes are shown in Fig. 5. Again, we can see a good agreement between the results by PVL/compact FDFD technique and those from Ref. [10].

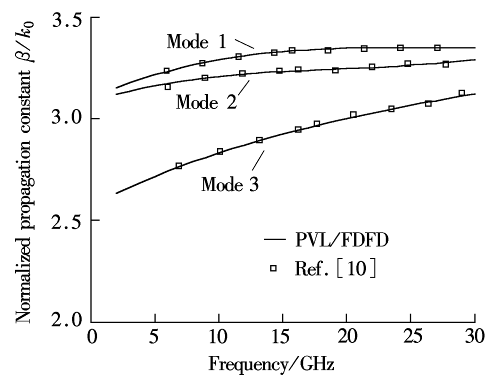

Fig.5 Normalized propagation constant for the three propagation modes of the dual-plane triple microstrip structure

Also the CPU times of different methods for the above guided wave structures are compared and shown in Tab.1. These different methods yield almost identical results. Therefore the comparison of the accuracies is not given here for the sake of clarity.

Tab.1 CPU time comparison between different methods

Method	CPU time/s		
	Air filled waveguide	Inhomogeneous waveguide	Dual plane triple microstrip
FDFD	50	126	612
FDFD/AWE	38	63	238
FDFD/PVL	31	47	126

3 Conclusion

In this paper, the PVL method has been implemented in the compact FDFD algorithm. This new approach uses the Krylov subspace technique to reduce the original large system equation matrix generated by the FDFD to a lower order small matrix and therefore improves the efficiency of the FDFD simulation. Compared to the AWE method, PVL overcomes the bad-condition problem and at the same time preserves stability and accuracy. However, the valid frequency expansion region of the PVL technique is narrower compared to the AWE technique. An objective in the future is to use the propagation constant expansion to overcome such limitations. This efficient propagation expansion will make the combination of compact FDFD and PVL more efficient for general millimeter guide wave simulations.

References

[1] Zhao Y J, Wu K L, Cheng K M. A compact 2-D full-wave finite-difference frequency-domain method for general guided wave structures [J]. *IEEE Trans Microwave Theory Tech*, **2002**, **50**(7): 1844 - 1848.

[2] Lui M L, Chen Z. A direct computation of propagation constant using compact 2-D full-wave eigen-based finite-difference frequency-domain technique [A]. In: *Int Conference on Computational Electromagnetics and Its Applications* [C]. Beijing, China, 1999. 78 - 81.

[3] Pillage L T, Rohrer R A. Asymptotic waveform evaluation for timing analysis [J]. *IEEE Trans Computer-Aided Design*, **1990**, **9**(4): 352 - 366.

[4] Li M, Zhang Q L, Nakhla M S. Solution of EM fields by asymptotic waveform techniques [A]. In: *URSI International Symp Signals, Systems and Electronics* [C]. San Francisco, CA, 1995. 393 - 396.

[5] Odabasioglu A, Celik M, Pillage L T. PRIMA: passive reduce-order interconnect macromodeling algorithm [J]. *IEEE Trans Computer-Aided Design*, **1998**, **17**(8): 645 - 653.

[6] Feldmann P, Freund R W. Efficient linear circuit analysis by Padé approximation via the Lanczos process [J]. *IEEE Trans Computer-Aided Design*, **1995**, **14**(5): 639 - 649.

[7] Grimme E J. Krylov projection methods for model reduction [D]. USA: University of Illinois at Urbana-Champaign, 1997.

[8] Zunoubi M R, Jin J M, Donepudi K C, et al. A spectral Lanczos decomposition method for solving 3-D low-frequency electromagnetic diffusion by the finite-element method [J]. *IEEE Trans Antennas and Propagation*, **1999**, **47**(2): 242 - 248.

[9] Celik M, Cangellaris A C. Simulation of dispersive multiconductor transmission lines by Padé approximation via the Lanczos process [J]. *IEEE Trans Microwave Theory Tech*, **1996**, **44**(12): 2525 - 2535.

[10] Liu F, Schutt-Aine J E, Chen J. Full-wave analysis and modeling of multiconductor transmission lines via 2-D-FDTD and signal-processing techniques [J]. *IEEE Trans Microwave Theory Tech*, **2002**, **50**(2): 570 - 577.

用紧凑格式频域有限差分法和 PVL 技术分析导波结构的有效方法

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摘要: 提出了一种十分有效的提取导波结构传播特性的数值方法. 它的基本思想是把 Krylov 子空间的模式缩减技术(Padé逼近/Lanczos 分解)使用在紧凑格式频域有限差分法上, 把大的系统矩阵降阶为很小的系统矩阵,从而加速了矩阵的运算求解.通过对几种导波结构的分析及和其他方法的比较, 证明了这种新方法的准确性和高效性.

关键词: 降阶模型方法; 频域有限差分; 导波结构; Padé 逼近/Lanczos 分解

中图分类号: TN455