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Suppression strategy for parametrically excited lateral vibration of mass-loaded string

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Abstract: This paper discusses a simple way to suppress the parametrically excited lateral vibration of a mass-loaded string. Supposing that the mass at the lower end of the string is subjected to a vertical harmonic excitation and neglecting the higher order vibration modes, the equation of motion for the mass-loaded string can be represented by a Mathieu's equation with cubic nonlinearity. According to the theory of the Mathieu's equation, in the mass-loaded string system, when the vertical vibration frequency of the mass approaches twice the natural frequency of the string lateral vibration, once the vertical vibration amplitude of the mass exceeds a critical value, the parametric resonance will occur in the string. To avoid the parametric resonance, a vibration absorber, composed of a thin beam and two mass blocks attached at both sides of the beam symmetrically, is proposed to install with the mass to reduce its vertical vibration, and ultimately suppress the lateral vibration of the string. Such a suppression strategy is finally validated by experiments.

Key words: mass-loaded string; parametric resonance; vibration absorber

A mass-loaded string model can be used to study the dynamic properties of practical systems such as elevator, crane, cable-stayed bridge and so on. Due to some horizontal perturbations, the string may vibrate laterally. Such lateral vibration usually decays gradually caused by the existence of damping. However, under some tuning conditions, the amplitude of the string lateral vibration may increase due to the parametric excitation caused by the coupling between the vertical and lateral directions. Large-amplitude lateral vibration of the string not only shortens string life, but also aggravates the vertical vibration of the mass.

The parametrically excited lateral vibration of a string-like model has been extensively studied. Mote[1-3] predicted the parametric stability-instability region boundaries for the axially moving string system by application of Hsu and Bolotin Methods. Huang, et al.[4] presented the dynamic stability of the moving string in 3-D vibration by using the modal analysis procedure and the method of multiple scales. Fung, et al.[5] used Hamiltonian formulation and an averaging method to examine the stability behavior of an axially moving string in the presence of parametric and combination resonance. Sun, et al.[6] developed a nonlinear dynamic model for investigating the parametrically excited vibration of stay cables caused by support motion in cable-stayed bridges. Also for a cable-stayed bridge, Wei, et al.[7] investigated the parametric vibration of one cable-stay using the finite element method. Zhang and Zu[8,9] investigated the dynamic response and stability of parametrically excited viscoelastic belts, which can be regarded as a mass-loaded string model in a horizontal direction. In the elevator field, the lateral vibration of the hoist rope has also been studied by some researchers[10-12]. Terumichi, et al.[10] simplified an elevator system to a physical model composed of a string with time-varying length, excited sinusoidally by a horizontal displacement at its upper end and a mass-spring system attached at the lower end. They paid attention to the influences of the axial velocity of the string on the peak amplitude of the string lateral vibration at the passage through resonance. Fung and Lin[11] considered the lateral vibration of an elevator string excited by the vibration of the rotor radius due to the winding of the string either on or off the rotor. Zhu and Teppo[12] developed a novel scaled model to simulate the linear lateral dynamics of a hoist cable with variable length in a high-rise, high-speed elevator.

1 Theoretical Analysis

The schematic diagram of the considered mass-loaded system is shown in Fig. 1, where the string is hung vertically and the mass is attached to it at the lower end; in addition, a harmonic excitation is applied to the mass in the vertical direction. Neglecting the higher order vibration modes of the string, the equations of motion for the mass-loaded string system can be established as follows[6]:

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$$\ddot{X}_1 + \frac{c_s}{\rho}\dot{X}_1 + \frac{\pi^2}{\rho L^2}\left[mg + \frac{EA_0}{L}\left(X_2 + \frac{\pi^2 X_1^2}{4L}\right)\right] X_1 = 0 \tag{1}$$

$$\ddot{X}_2 + \frac{c_m}{m}\dot{X}_2 + \frac{EA_0}{mL}\left(X_2 + \frac{\pi^2 X_1^2}{4L}\right) = \frac{F_0}{m}\cos\omega t \tag{2}$$

where X_1 is the lateral displacement at the midpoint of the string; X_2 is the vertical displacement of the mass; c_s is the damping coefficient for per unit length of the string; c_m is the damping coefficient for the mass; E, A_0, L and ρ are the Young's modulus, cross section area, length and linear density of the string, respectively; m is the mass; F_0 and ω are the excitation amplitude and frequency, respectively.

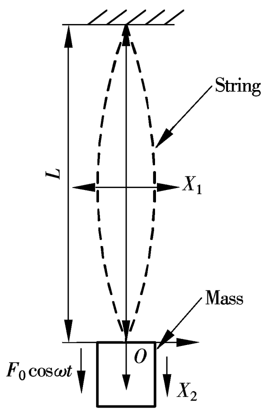

Fig.1 Schematic diagram of the mass-loaded string system

Because of the nonlinear terms in Eqs. (1) and (2), the dynamic behaviours of the string and the mass are coupled strongly. Now suppose that the amplitude of the string lateral vibration is so small that the nonlinear terms in Eqs. (1) and (2) can be neglected (initially, the lateral vibration of the string is usually very weak, therefore, such an assumption is reasonable), as a result, the following two equations are obtained.

$$\ddot{X}_1 + \frac{c_s}{\rho}\dot{X}_1 + \frac{\pi^2}{\rho L^2}\left(mg + \frac{EA_0}{L}X_2\right) X_1 = 0 \tag{3}$$

$$\ddot{X}_2 + \frac{c_m}{m}\dot{X}_2 + \frac{EA_0}{mL}X_2 = \frac{F_0}{m}\cos\omega t \tag{4}$$

In this case, the steady state response of the mass can be represented by a harmonic function, in which the frequency is the same as that of the external excitation and the amplitude is directly proportional to the excitation amplitude. Therefore, Eq. (3) is a Mathieu's equation. According to the theory of Mathieu's equation[13], when the excitation frequency ω is equal to about twice the natural frequency of the string lateral vibration, if the excitation amplitude is smaller than a certain critical value, the lateral vibration of the string will decay with time because of damping; if the excitation amplitude exceeds the critical value, the lateral vibration of the string will increase with time until it reaches a relatively large amplitude, and then the string will keep vibrating laterally with constant amplitude. The latter case means that parametric resonance occurs in the string, which should be avoided in practice.

According to the above analysis, in the mass-loaded string system, whether parametric resonance occurs in the string is greatly dependent on the steady state response of the mass. Even if the vertical vibration frequency of the mass approaches about twice the natural frequency of the string lateral vibration, once the vertical vibration amplitude of the mass is suppressed below the critical value, parametric resonance in the string should not occur surely. Next it will be analytically demonstrated that a dynamic vibration absorber installed with the mass can implement such an objective.

The schematic diagram of the mass-loaded string system with an vibration absorber is shown in Fig. 2, and the equations of motion for the system can be set up as

$$\ddot{X}_1 + \frac{c_s}{\rho}\dot{X}_1 + \frac{\pi^2}{\rho L^2}\cdot \left(mg + m_a g + \frac{EA_0}{L}X_2 + \frac{EA_0}{L}\frac{\pi^2 X_1^2}{4L}\right) X_1 = 0 \tag{5}$$

$$\ddot{X}_2 + \frac{c_m + c_a}{m}\dot{X}_2 - \frac{c_a}{m}\dot{X}_3 + \frac{k_s + k_a}{m}X_2 - \frac{k_a}{m}X_3 = \frac{F_0}{m}\cos\omega t - \frac{k_s}{m}\frac{\pi^2 X_1{}^2}{4L} \tag{6}$$

$$\ddot{X}_3 + \frac{c_a}{m_a}\dot{X}_3 - \frac{c_a}{m_a}\dot{X}_2 + \frac{k_a}{m_a}X_3 - \frac{k_a}{m_a}X_2 = 0 \tag{7}$$

where X_3, m_a, k_a and c_a denote the displacement, mass, stiffness and damping for the absorber, respectively; k_s represents the stiffness of the string and $k_s = EA_0/L$. Neglecting the nonlinear term in Eq. (6), the steady-state solutions to Eqs. (6) and (7) can be expressed as

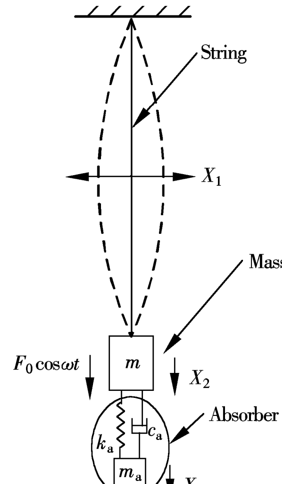

Fig.2 Mass-loaded string system with the vibration absorber

$$X_{2s}(t)=\frac{\sqrt{(k_a-m_a\omega^2)^2+(c_a\omega)^2}F_0\cos(\omega t-\varphi_2)}{\sqrt{[m_am\omega^4+k_sk_a-(k_am+k_am_a+k_sm_a+c_mc_a)\omega^2]^2+[c_m(k_a-m_a\omega^2)+c_a(k_s-m\omega^2)-c_am_a\omega^2]^2}} \tag{8}$$

$$X_{3s}(t)=\frac{\sqrt{k_a^2+(c_a\omega)^2}F_0\cos(\omega t-\varphi_3)}{\sqrt{[m_am\omega^4+k_sk_a-(k_am+k_am_a+k_sm_a+c_sc_a)\omega^2]^2+[c_m(k_a-m_a\omega^2)+c_a(k_s-m\omega^2)-c_am_a\omega^2]^2}} \tag{9}$$

According to Eq.(8), when $\sqrt{k_a/m_a}=\omega$ and $c_a=0$, $X_{2s}=0$. In other words, if the natural frequency f_a of the vibration absorber is equal to the excitation frequency f and the damping for the vibration absorber is equal to zero, the vertical vibration of the mass at the lower end of the string will be greatly suppressed, and thus the lateral vibration of the mass-loaded string is also suppressed. Since parametric resonance most likely occurs when f is close to $2f_s$, in order to avoid the parametric resonance in the string, f_a only requires to approach $2f_s$. Once the parametric resonance phenomena are eliminated, the transient lateral vibration of the string always decays gradually because of damping. Therefore, to install a vibration absorber with the mass is a feasible way to suppress the lateral vibration of the mass-loaded string.

2 Experimental Study

In this section, the description about the dynamic behavior of the mass-loaded string and the feasibility of the absorber in suppressing the string lateral vibration are to be validated by some experiments. Fig. 3 shows the schematic diagram of the experimental mass-loaded string system, which mainly consists of two vertical suspending strings and a frame attached at the lower end. To ensure that the frame vibrates in a

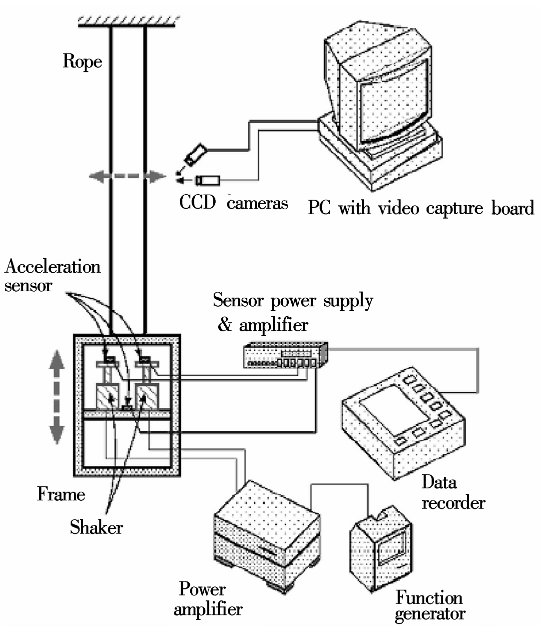

Fig.3 Schematic diagram of the experimental system without absorber

vertical direction, guide rails are arranged on both sides of the frame. In addition, two shakers are installed with the frame to serve as the vertical excitation, and a function generator supplies harmonic signals. Between the function generator and the shaker, there is a power amplifier. The motion images of the strings are captured by two CCD cameras, and the motion data in the CCD cameras are transmitted into a computer so that the dynamic behaviors of the string can be analyzed via Matlab software. Tab. 1 shows the basic information of the main devices in the experimental system.

Tab.1 Basic information about the devices in the experimental system

Instrument name	Type
Shake	APS MODEL 113
CCD camera	SONY XC-75
Accelerometer	IC SENSORS MODEL 3140-002
Function generator	YOKOGAWA FC110
Data recorder	HIOKI MEMORY HICORDER 8855
Power amplifier (for the function generator)	TECHRON 5530
Power amplifier (for the accelerometer)	DEICY MA _ 101DC

The basic parameters of the experimental system are as follows: $m=296.5$ kg (here m includes the mass of the frame and the two shakers), $L=14.4$ m, $E=132$ GPa, $A_0=72.4\ \mathrm{mm}^2$, $\rho=0.68$ kg/m, $\zeta_m=0.046$ and $c_s=0.15$ Pa. According to these parameters, the natural frequencies of the string and mass can be calculated by the following two expressions, as a result, $f_s=2.27$ Hz, $f_m=7.53$ Hz.

$$\omega_s=\frac{\pi}{L}\sqrt{\frac{mg}{\rho}},\ \omega_m=\sqrt{\frac{EA_0}{mL}} \tag{10}$$

Make the function generator generate a harmonic function and tune its frequency to about 4.54 Hz. When the amplitude of the harmonic function is smaller than a critical value, the string vibrates slightly, shown in Fig. 4 (In this case, the output voltage from the function generator is 0.2 V). After the amplitude of the harmonic function is adjusted, it exceeds the critical value, the lateral vibration of the string takes on an emanative tendency, as shown in Fig. 5 (Here, the output voltage from the function generator is 0.4 V). In this case, the parametric resonance occurs in the string. However, the amplitude of the string lateral vibration does not increase infinitely, and the string eventually vibrates with a relatively large constant amplitude, as shown in Fig.6.

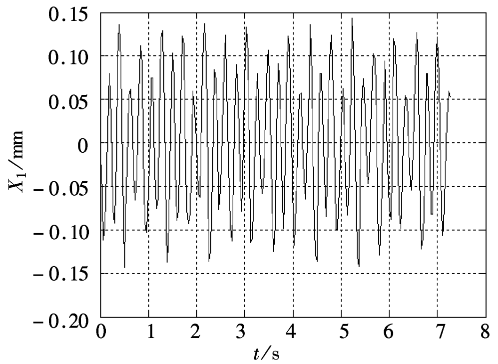

Fig.4 Slight lateral vibration of the string

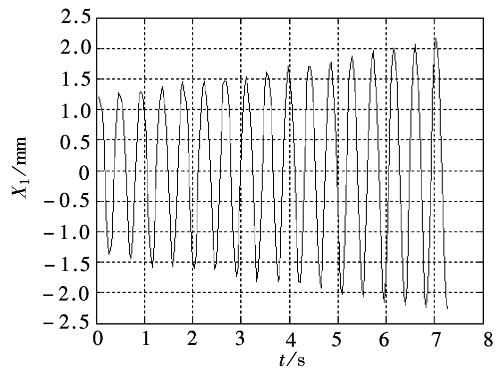

Fig.5 Emanative lateral vibration of the string

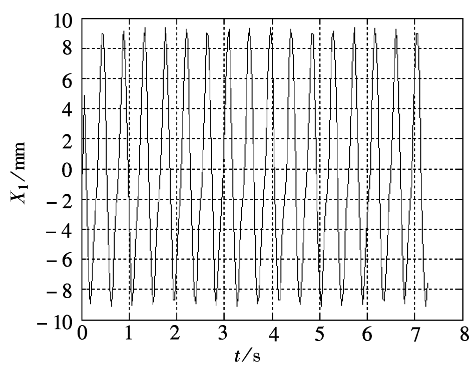

Fig.6 Large constant amplitude lateral vibration of the string

According to section 1, a vibration absorber installed with the frame at the lower ends of the strings can suppress the parametric resonance in the present experiment system. Theoretically, a spring-mass system can play a role of vibration absorber. However, to avoid the mass in the absorber from swaying in the horizontal direction, in this experiment, a beam-mass structure is adopted (see Fig. 7). The main parameters of the absorber are as follows: m_a = 1.5 kg, b_a = 70 mm, h_a = 2 mm, L_a = 0.64 m, ρ_a = 7.86 $\times 10^3$ kg/m^3, E_a = 209 GPa. Here, b_a, h_a, L_a, ρ_a and E_a are the width, thickness, length, density and the Young's modulus of the beam, respectively. Before installing the absorber with the frame, the position of the mass has been properly adjusted along the beam so that the natural frequency of the absorber is equal to twice the natural frequency of the string first order lateral vibration, i.e., $f_a = 2f_s = 4.54$ Hz.

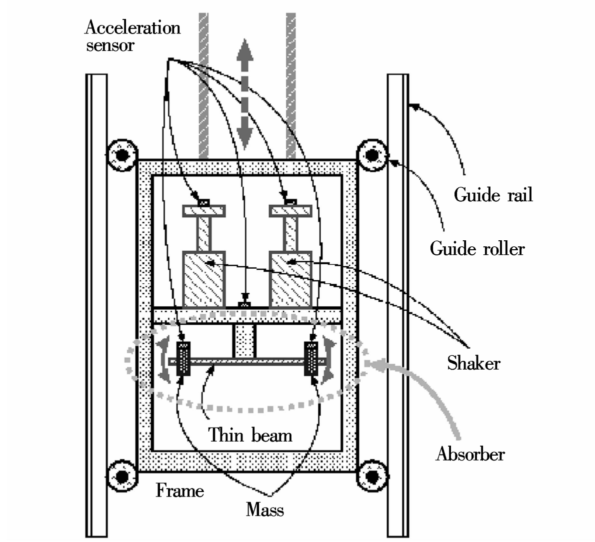

Fig.7 Schematic diagram of the experimental frame with the absorber

Fig.8 shows the displacement at the midpoint of the string before and after the absorber is installed within the frame. The two curves in Fig.8 are obtained under the same harmonic excitation, i.e., the frequency of the harmonic function is about 4.54 Hz and the output voltage from the function generator is 0.4 V. It is seen from Fig.8 that parametric resonance occurs in the case of no absorber and parametric resonance does not occur when the absorber is installed with the frame. In fact, the positive effect of the absorber in suppressing the lateral vibration of the string has been validated by many experiments. However, all the other experimental results have neglected to save space.

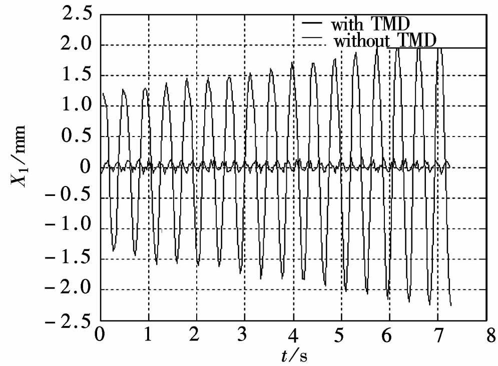

Fig.8 Lateral vibration of the string before and after the absorber is installed

3 Conclusion

In the mass-loaded string system, due to the coupling between the vertical vibration of the mass and the lateral vibration of the string, when the vertical vibration frequency of the mass approaches about twice the natural frequency of the string lateral vibration, once the vertical vibration amplitude of the mass exceeds a critical value, parametric resonance will occur in the string, which induces the string to vibrate with large amplitude in the lateral direction. To avoid such harmful phenomena, it is proposed that a vibration absorber should be installed with the mass at the lower end of the string, and the natural frequency of the absorber should be equal to twice the natural frequency of the string lateral vibration. Such a suppression strategy has been validated theoretically and experimentally.

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载重绳索参数激励横向振动的减振策略

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摘要: 论述了一种抑制载重绳索参数激励横向振动的简便方法. 假设绳索底端的质量受到一个垂直简谐激励并忽略载重绳索高阶振型的影响, 载重绳索的运动方程可以用一个带有立方非线性项的 Mathieu 方程来描述. 根据 Mathieu 方程的有关理论, 在载重绳索系统中, 当索端质量垂直振动的频率接近绳索横向振动固有频率 2 倍时, 一旦索端质量垂直振动的幅度超过某个临界数值时, 绳索将产生参数共振. 为了避免这种现象, 建议在索端质量上加装一个减振器以削减索端质量的垂直振动, 进而抑制绳索的横向振动. 实验验证了该减振方案的有效性.

关键词: 载重绳索; 参数共振; 减振器

中图分类号: O322; TM726.4