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Constructing G^1 interpolation curves on surfaces with Coons surface

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Abstract: Using a bi-cubic Coons surface, a new method for G^1-continuous interpolation of an arbitrary sequence of points on an implicitly or parametrically defined surface with a specified tangent direction at every point was presented. Firstly construct a G^1-continuous composite Coons surface patch, then compute the intersection of the surface patch and the given surface. The desired interpolation curve is the intersection curve. Unlike some existing methods, introduced into the method were several free parameters that can be used in subsequent interactive modification so that the last curve's shape meets our demand. Experiments demonstrate the method is simple, feasible and applicable to computer aided design and computer graphics, etc.

Key words: interpolation; Coons surface; G^1 continuity; surface-to-surface intersection

The curve interpolation, a classical and basic issue in geometric modeling, means finding a curve that passes through given spatial or planar points and possesses some geometric or parametric continuity at those points or meets additional constraints. Curve interpolation remains a very dynamic area of research. Many remarkable achievements have been made in research on the issue. However, a special case, in which the interpolation process is confined on a surface, i.e., data points and the resulting curve must be contained on a surface, appears more and more significant in surface trimming[1], path design of numerical control machining[2] and robot[3] and other research and industrial fields. As yet, several efforts have been made towards dealing with the issue. Pobegailo proposed an approach for G^1 interpolation and blending on a sphere[4]. Shoemake also discussed interpolation on a sphere[5]. Dietz, et al. solved G^0 interpolation problem on quadrics for prescribed pairs of points and parameters with the help of rational Bézier curves[6]. Hartmann developed a method for geodesic curvature-continuous interpolation of an arbitrary sequence of points on a surface (implicit or parametric) with specified tangent and geodesic curvature at every point[7]. The approach is applicable to implicit or parametric surface. The resulting interpolation curve possesses a relatively low degree. However it is based on the techniques of functional blending[8] and finding intersection between an implicit and the given surface. Other research includes those reported in Refs. [9, 10]. Apparently, the method[9] is good for the display of the resulting interpolation curve. However the resulting interpolation curve is a composed curve. A curve got by composition might have a very high degree that might be prohibited in some CAD system. In addition, it is applicable to the surface that can be parameterized. In this paper, we present a new and relatively simple technique for G^1 interpolation of an arbitrary sequence of points on surfaces with prescribed tangent directions at any points. It has more flexible controllable means for the user to conduct interactive shape modification and is applicable to both implicitly and parametrically defined surfaces.

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1 Problem Statement and Interpolation Method

Assume that $\boldsymbol{P}_i$ ($i=1, 2, \cdots, k$) is a sequence of points on a regular and smooth surface S, $\boldsymbol{t}_i$ is the unit tangent vector of S at point $\boldsymbol{P}_i$. We want to find G^1 continuous curve on surface S such that the curve passes the given points with prescribed tangent direction $\boldsymbol{t}_i$ at every point $\boldsymbol{P}_i$. The main steps of our method are as follows:

- Construct a G^1 continuous Coons surface;
- Compute the intersection of the resulting surface and the given surface.

1.1 Bi-cubic Coons surface

We only consider a special case, i.e., define a surface patch by such information as position vectors, derivative vectors in u and v directions, and cross derivative vectors at four corner points.

Let four corner points be $\boldsymbol{P}(0,0)$, $\boldsymbol{P}(0,1)$, $\boldsymbol{P}(1,0)$, $\boldsymbol{P}(1,1)$. Write their corresponding derivative vectors in u and v directions and twist vectors respectively as follows: $\boldsymbol{P}_u(0,0)$, $\boldsymbol{P}_v(0,0)$, $\boldsymbol{P}_u(0,1)$, $\boldsymbol{P}_v(0,1)$, $\boldsymbol{P}_u(1,0)$, $\boldsymbol{P}_v(1,0)$, $\boldsymbol{P}_u(1,1)$, $\boldsymbol{P}_v(1,1)$, $\boldsymbol{P}_{uv}(0,0)$, $\boldsymbol{P}_{uv}(0,1)$, $\boldsymbol{P}_{uv}(1,0)$, $\boldsymbol{P}_{uv}(1,1)$.

Take $F_0(u)$, $F_1(u)$, $G_0(u)$, $G_1(u)$ and $F_0(v)$, $F_1(v)$, $G_0(v)$, $G_1(v)$ as two groups of cubic blending functions, then the equation of the resulting Coons patch (also called Ferguson surface) is

$$C(u,v)=\{F_0(u),F_1(u),G_0(u),G_1(u)\}\cdot\begin{bmatrix}\boldsymbol{P}(0,0) & \boldsymbol{P}(0,1) & \boldsymbol{P}_v(0,0) & \boldsymbol{P}_v(0,1)\\ \boldsymbol{P}(1,0) & \boldsymbol{P}(1,1) & \boldsymbol{P}_v(1,0) & \boldsymbol{P}_v(1,1)\\ \boldsymbol{P}_u(0,0) & \boldsymbol{P}_u(0,1) & \boldsymbol{P}_{uv}(0,0) & \boldsymbol{P}_{uv}(0,1)\\ \boldsymbol{P}_u(1,0) & \boldsymbol{P}_u(1,1) & \boldsymbol{P}_{uv}(1,0) & \boldsymbol{P}_{uv}(1,1)\end{bmatrix}\begin{Bmatrix}F_0(v)\\ F_1(v)\\ G_0(v)\\ G_1(v)\end{Bmatrix} \tag{1}$$

Obviously, it follows from Eq.(1) that:

Proposition If we take the four twist vectors as zero vectors respectively and set $\boldsymbol{P}_u(0,0)=\boldsymbol{P}_u(0,1)$ and $\boldsymbol{P}_u(1,0)=\boldsymbol{P}_u(1,1)$, then the cross boundary tangent vectors $\boldsymbol{P}_u(0,v)$ and $\boldsymbol{P}_u(1,v)$ are constant vectors, respectively.

We will use the proposition to construct the G^1 interpolation curve.

1.2 G^1 interpolation

Since our tactic is piecewise interpolation, it suffices to consider the interpolation curve defined by only a pair of points, such as $\boldsymbol{P}_i$, $\boldsymbol{P}_{i+1}$ with the prescribed unit vectors $\boldsymbol{t}_i$, $\boldsymbol{t}_{i+1}$ at corresponding points on surface S.

It is given that every point on a regular surface corresponds to a unique normal vector of the surface. Assume that the normal vectors of the surface S at $\boldsymbol{P}_i$, $\boldsymbol{P}_{i+1}$ are $\boldsymbol{n}_i$, $\boldsymbol{n}_{i+1}$, respectively. Then for a fixed i, let

$$\begin{aligned}
&\boldsymbol{P}(0,0)=\boldsymbol{P}_i-\boldsymbol{n}_i\\
&\boldsymbol{P}(0,1)=\boldsymbol{P}_i+\boldsymbol{n}_i\\
&\boldsymbol{P}(1,0)=\boldsymbol{P}_{i+1}-\boldsymbol{n}_{i+1}\\
&\boldsymbol{P}(1,1)=\boldsymbol{P}_{i+1}+\boldsymbol{n}_{i+1}\\
&\boldsymbol{P}_u(0,0)=\boldsymbol{P}_u(0,1)=\lambda_{i1}\boldsymbol{t}_i\\
&\boldsymbol{P}_u(1,0)=\boldsymbol{P}_u(1,1)=\lambda_{i2}\boldsymbol{t}_{i+1}\\
&\boldsymbol{P}_v(0,0)=\boldsymbol{P}_v(0,1)=\boldsymbol{n}_i\\
&\boldsymbol{P}_v(1,0)=\boldsymbol{P}_v(1,1)=\boldsymbol{n}_{i+1}
\end{aligned}$$

where λ_{i1} and λ_{i2} are positive constants (they denote magnitude of cross boundary tangent vectors of Coons patch in u direction and influence the shape of the Coons patch. As for how they work, readers can refer to the curve case in Ref.[11]). From Eq.(1) we get a Coons patch S_i as follows:

$$S_i(u,v)=\{F_0(u),F_1(u),G_0(u),F_1(u)\}\cdot\begin{bmatrix}\boldsymbol{P}_i-\boldsymbol{n}_i & \boldsymbol{P}_i+\boldsymbol{n}_i & \boldsymbol{n}_i & \boldsymbol{n}_i\\ \boldsymbol{P}_{i+1}-\boldsymbol{n}_{i+1} & \boldsymbol{P}_{i+1}+\boldsymbol{n}_{i+1} & \boldsymbol{n}_{i+1} & \boldsymbol{n}_{i+1}\\ \lambda_{i1}\boldsymbol{t}_i & \lambda_{i1}\boldsymbol{t}_i & \boldsymbol{0} & \boldsymbol{0}\\ \lambda_{i2}\boldsymbol{t}_{i+1} & \lambda_{i2}\boldsymbol{t}_{i+1} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}\begin{Bmatrix}F_0(v)\\ F_1(v)\\ G_0(v)\\ G_1(v)\end{Bmatrix} \tag{2}$$

Let $i=1, 2, \cdots, k-1$, we get a sequence of Coons patches S_i. Obviously, the composed Coons surface $\bigcup_{i=1}^{k-1} S_i$ has G^1 continuity along u direction and is a ruled surface. Moreover, proposition in subsection 1.1 guarantees the intersection curve

$$S\cap\left(\bigcup_{i=1}^{k-1} S_i\right) \tag{3}$$

has G^1 continuity. It is easy to prove that the intersection is in fact the desired interpolation curve and the degree of the parameter v in (2) can be reduced to one.

Remark 1 The constants λ_{i1} and λ_{i2}, $i=1, 2, \cdots, k-1$ in Eq.(2) provide some freedom in designing a Coons surface and can be used as control parameters to modify the intersection curve (3) so that its last shape meets our demand.

Remark 2 Interpolation curve (3) has some good local properties. Changing any one of $\boldsymbol{P}_i$ and $\boldsymbol{t}_i$ only influences the shape of two adjacent segments while altering λ_{i1} or λ_{i2} just results in deformation of one corresponding segment.

Remark 3 The method can be used in constructing a blending curve (transition curve) between two curves (implicit or parametric) on a surface. The resulting blending curve depends merely on the tangent directions and position vectors and has nothing to do with the global geometry and the representation of two given curves.

2 Experiments

Suppose that S is a parabola: $z=\frac{1}{8}(-x^2-y^2+2)+2$. Take $\boldsymbol{P}_1=\left\{-2.8, 0, \frac{5}{4}\right\}$, $\boldsymbol{t}_1=\frac{\{1, 1, 0.7\}}{\sqrt{2.49}}$; $\boldsymbol{P}_2=\left\{2, -2, \frac{5}{4}\right\}$, $\boldsymbol{t}_2=\frac{\{1, -2, -1.26\}}{\sqrt{6.59}}$. Furthermore we get corresponding normal vectors: $\boldsymbol{n}_1=\{-0.6, 0, 0.8\}$, $\boldsymbol{n}_2=\{0.4, -0.4, 0.8\}$. Then the interpolation curve passing $\boldsymbol{P}_1$, $\boldsymbol{P}_2$ with tangent directions $\boldsymbol{t}_1$, $\boldsymbol{t}_2$ is shown in Figs.1 and 2, where $\lambda_1=5\sqrt{2.49}$, $\lambda_2=2\sqrt{6.59}$.

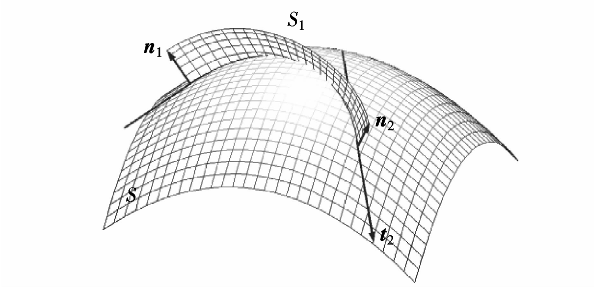

Fig.1 Intersection of parabola S and Coons patch S_1

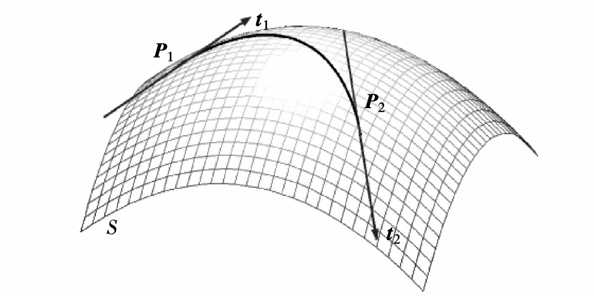

Fig.2 Desired interpolation curve

Take $\boldsymbol{P}_1 = \left\{-2.8, 0, \frac{5}{4}\right\}$, $\lambda_1 \boldsymbol{t}_1 = 8\{0, -1, 0\}$; $\boldsymbol{P}_2 = \left\{2, -2, \frac{5}{4}\right\}$, $\lambda_2 \boldsymbol{t}_2 = 5\{1, 1, 0\}$. $\boldsymbol{P}_3 = \{0, 0, \frac{9}{4}\}$, $\lambda_3 \boldsymbol{t}_1 = -2\{1, 1, 0\}$. We can get such interpolation curve shown in Figs.3 and 4, where the curve in Fig.3 is just the intersection curve shown in Fig.4.

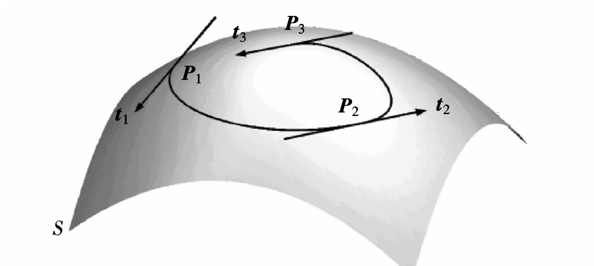

Fig.3 Desired interpolation curve

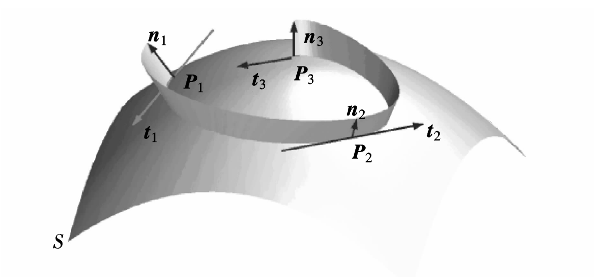

Fig.4 Composed Coons surface intersects with given surface

3 Conclusion

A new method for designing a surface curve (curve contained on a surface) is presented. With it, one can deal with the G^1 interpolation of an arbitrary sequence of points, in contrast to the limitation that the polygon made by data points must be of only right turn or only left turn in Hartmann's function blending method[7]. By introducing a number of free parameters, the interpolation process can be completed interactively. Compared with the method[7], the resulting interpolation curve has good local properties and control flexibility for every curve segment has two, instead of one, free parameters for interactive control and those free parameters have obvious geometric meaning unlike their counterparts in the other methods[7]. Moreover this method is applicable to interpolation on any surface (implicit or parametric), in contrast to the supposition in Ref.[8] that the surface should be parametric or could be parameterized. Furthermore, it can be directly used to construct a blending curve (transition curve) between two given curves on a surface regardless of the given curve's geometry and representation. The concept in this paper can be generalized to construct a G^2 interpolation curve on a surface. The detailed process will be reported in another paper.

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利用 Coons 曲面构造曲面上的 G^1 插值曲线

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摘要: 利用双三次 Coons 曲面, 给出了一种对隐式或参数曲面上任意点序列及切方向给定时的 G^1 连续插值方法.首先构造 G^1 连续的组合 Coons 曲面, 其次求该组合曲面片与已知曲面的交线. 插值曲线是组合 Coons 曲面片与给定曲面的交线. 由于引入了若干控制参数, 可对曲线进行交互修改使得其最终的形状更好地满足我们的要求. 实验证明该方法简单可行, 适用于计算机辅助设计、计算机图形学等领域.

关键词: 插值; Coons 曲面; G^1 连续; 曲面求交
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