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Combined method for fast 3-D finite element modeling of nondestructive testing signal

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Abstract: A combined method for the fast 3-D finite element modeling of defect responses in nondestructive testing of electromagnetics is presented. The method consists of three numerical techniques: zoom-in technique, difference field technique and iterative solution technique. Utilizing the zoom-in technique, the computational zone focuses on a relatively small domain around the defect. Employing the difference field technique, the axisymmetrical field solution corresponding to the case with no defect can be used to simplify the mesh generation and obtain the modeling results quickly. Using the iterative solution technique, the matrix equation system in the 3-D finite element modeling of nondestructive probe signals can easily be solved. The sample calculation shows that the presented method is highly effective and can consequently save significant computer resources.

Key words: nondestructive testing; finite element method; modeling; computational technique

Numerical methods, such as finite element method (FEM) and boundary element method, have been successfully applied to study many nondestructive testing (NDT) problems in the past decades[1]. Most of these methods focus on the development of effective formulations instead of calculation techniques. However, better effects can be realized if efficient techniques and effective formulations are developed together. In most NDT modeling problems, such as defect response prediction and defect reconstruction, the linear algebraic equation systems have to be regenerated and solved repeatedly due to probe's moving or defect's change. Excessive computer resources, thus, are required in the multiple solutions of the equation systems by using available algorithms, especially in 3-D problems. The development of effective and efficient calculation techniques to simplify the numerical modeling would be highly useful. Indeed a very successful zoom-in technique has been reported, which requires very limited computer resource when studying the defect responses of the 3-D remote eddy current effect[2]. The difference field technique has also been applied in the fast simulation of eddy current testing signals[3]. The author's studies show that these techniques can be enhanced further to save additional computer resources in most 3-D FEM modeling of NDT problems by combining an iterative solution technique.

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This paper presents a combined method that saves a significant amount of computation cost in modeling 3-D NDT problems. The basic principle of the techniques is described in detail in this paper. The modeling of a benchmark problem provided by Ref. [4] shows that the proposed combined method is very promising in the simulation of electromagnetic NDT problems.

1 Combined Technique

1.1 Zoom-in technique

Most NDT problems have three common features from the viewpoints of both physics and geometry. The first one is that the field perturbation arising from probe's moving or defect's change occurs only in the local region around the probe or defect. The size of the real defect is generally very small compared with that of the specimen being studied. Hence the influence of the defect is only noticeable in the vicinity of the defect. A relatively small region where the perturbation is prominent and distinct may be taken as the calculation domain if utilizing the feature fully.

The second feature is that most NDT problems themselves are basically axisymmetrical in absence of defects, but with an exception at some local region, such as a fine crack on/in the detected plate or tube. Fig.1 shows two typical eddy current testing schemes for metal plate and tube, respectively. They can be approximately looked upon as axisymmetrical problems. If the axisymmetrical potential solution corresponding to the case without defect is taken as a basic solution, each 3-D field solution with defect will be the basic solution being added to a local perturbation. If one takes full advantage of "close to axisymmetrical" in numerical simulation, it will be a great savings of computer resources.

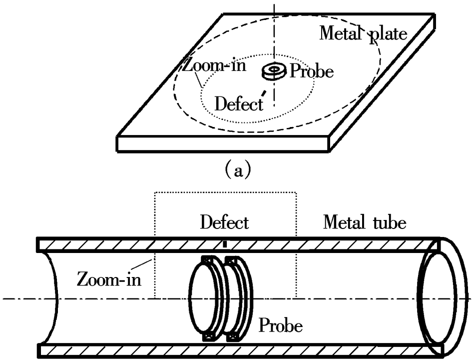

Fig.1 Two typical eddy current testing schemes. (a) Detecting metal plate with a pancake probe; (b) Detecting metal tube with a difference probe

The third feature of most NDT problems is that they can be considered as linear problems because the fields generated by some detecting probes are generally too weak to make ferromagnetic specimen saturated. This means that the detected specimen's non-linearity is negligible and the superposition theorem can accordingly be applied in these problems.

The above three common features of most NDT problems are just the basis of zoom-in technique. This technique can be used to confine the 3-D modeling to a relatively small domain around the defect, out of which the perturbation arising from the defect is ignored. It has successfully been applied in 3-D computations of remote field eddy current effects with relatively few computer resources[2]. In the previous applications of zoom-in technique, the axisymmetric potential solution was directly utilized as the boundary condition of the 3-D modeling. In this paper, it will be used as an equivalent perturbation source instead of boundary condition, as shown in section 1.2, to simplify numerical calculations further.

1.2 Difference field technique

If the magnetic vector potential $\boldsymbol{A}$ and electric scalar potential φ are employed to analyze the typical eddy current testing problems as shown in Fig.2 with FEM, the governing equations describing the low frequency eddy current phenomenon neglecting the displacement current and surface charge are as[5]

$$\left.\begin{aligned} &\nabla \times \frac{1}{\mu} \nabla \times \boldsymbol{A} + \nabla \frac{1}{\mu} \nabla \cdot \boldsymbol{A} + \mathrm{j}\omega\sigma(\boldsymbol{A} + \nabla\varphi) = \boldsymbol{J}_{\mathrm{S}} \\ &\nabla \cdot \mathrm{j}\omega\sigma(\boldsymbol{A} + \nabla\varphi) = 0 \end{aligned}\right\} \tag{1}$$

where μ is the material permeability, σ is the material conductivity, and $\boldsymbol{J}_{\mathrm{S}}$ is the current density imposed in eddy current probe.

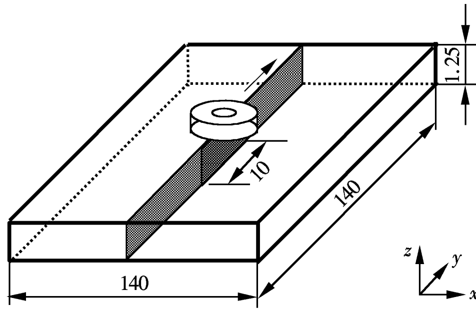

Fig.2 Analyzed model (unit: mm)

Applying Galerkin weighted residual method to discretize (1), the final simultaneous FEM equation systems for the two cases with and without defects can easily be obtained as

$$\boldsymbol{K}^0 \boldsymbol{U}^0 = \boldsymbol{K}^0 \begin{Bmatrix} \boldsymbol{A} \\ \boldsymbol{\varphi} \end{Bmatrix} = \boldsymbol{P}^0 \tag{2}$$

and

$$\boldsymbol{K}^{\mathrm{d}} \boldsymbol{U}^{\mathrm{d}} = \boldsymbol{K}^{\mathrm{d}} \begin{Bmatrix} \boldsymbol{A}^{\mathrm{d}} \\ \boldsymbol{\varphi}^{\mathrm{d}} \end{Bmatrix} = \boldsymbol{P}^{\mathrm{d}} \tag{3}$$

where $\boldsymbol{K}$ is the finite element (FE) coefficient matrix, $\boldsymbol{U}$ is the unknown vector, $\boldsymbol{P}$ is the source vector, while the superscript 0 and d denote the two cases with and without defect, respectively.

It is not difficult to understand that the matrices in (2) and (3) have the same structure and dimension number when the FE meshes are the same in the two cases with and without defects[6]. Furthermore, the source vector $\boldsymbol{P}$ is a constant whether there is a defect or not, provided that both exciting current and the FE mesh keep unchanged.

Define the difference between the two coefficient matrices in the two cases with and without defects as

$$\Delta\boldsymbol{K} = \boldsymbol{K}^{\mathrm{d}} - \boldsymbol{K}^0 \tag{4}$$

It reflects the change of the FE coefficient matrix due to defect, and is only relevant to those elements and nodes within the defect region[3]. The real dimension of $\Delta\boldsymbol{K}$ is therefore very small so that less computer memory is needed to save it. Similarly, define difference field between the two potential solutions in the two cases as

$$\Delta\boldsymbol{U} = \boldsymbol{U}^{\mathrm{d}} - \boldsymbol{U}^0 = \begin{Bmatrix} \boldsymbol{A}^{\mathrm{d}} \\ \boldsymbol{\varphi}^{\mathrm{d}} \end{Bmatrix} - \begin{Bmatrix} \boldsymbol{A}^0 \\ \boldsymbol{\varphi}^0 \end{Bmatrix} \tag{5}$$

Subtracting (3) from (2), we have

$$\lfloor \boldsymbol{K}^0 + \Delta\boldsymbol{K} \rfloor \Delta\boldsymbol{U} = -\Delta\boldsymbol{K}\boldsymbol{U}^0 \tag{6}$$

It is obvious that if $\Delta\boldsymbol{U}$ can be worked out in terms of (6), the potential solution vector with defect presence $\boldsymbol{U}^{\mathrm{d}}$ can be easily obtained by appending $\boldsymbol{U}^0$. The magnetic vector potential vector in absence of defect,

$\boldsymbol{A}^0$, a sub-vector of $\boldsymbol{U}^0$ can rapidly be solved through the corresponding axisymmetrical field computation. While $\boldsymbol{\varphi}^0$ can accordingly be taken as a zero vector if the reference value of the electric scalar potential φ is taken as zero in the 3-D analysis.

The comprehensive physical meaning of (6) is that the perturbation potential $\Delta\boldsymbol{U}$ results from the perturbation source $-\Delta\boldsymbol{K}\boldsymbol{U}^0$, which is essentially derived from the medium change due to defect. The fact that the source vector $\boldsymbol{P}$ disappears off (6) will be remarkably helpful in simplifying the mesh generation in the 3-D analysis since the detecting coil need no longer be considered. This is a distinct advantage of the difference field technique. Besides, the treatment of the Dilichlet boundary condition of $\Delta\boldsymbol{A}$ becomes easier in the 3-D calculations, in which the difference field $\Delta\boldsymbol{U}$ is taken as the unknown vector. The reason for this is that $\Delta\boldsymbol{A}$ can simply be set as zero on its Dilichlet boundary when applying the zoom-in technique.

1.3 Iterative solution technique

According to section 1.2, (6) has to still be repeatedly solved to model different defects or probe moving so as to get whole NDT signals. Although the 3-D numerical calculation can be limited in a relatively small surrounding domain by the zoom-in technique, the linear algebraic equation system arising from (6) is still enormous. Their multiple solutions will require huge computer resources. To cope with this problem, an iterative solution technique is presented to solve (6).

Rewriting (6), we obtain

$$\boldsymbol{K}^0\Delta\boldsymbol{U} = -\Delta\boldsymbol{K}\,\boldsymbol{U}^0 - \Delta\boldsymbol{K}\,\Delta\boldsymbol{U} \tag{7}$$

This equation can be solved by the common relaxation iteration method. Note that the coefficient matrix on the left of the equation, $\boldsymbol{K}^0$, never changes with different defect cases, which is, in fact, corresponding to the case in absence of defects. Once $\boldsymbol{K}^0$ is decomposed by some decomposition algorithm, the decomposed coefficient matrices can be saved and be utilized directly in the modeling of other defective cases. Hence, large computer resources can be saved in the modeling of whole NDT signals.

The iterative process is implemented by the following weighted combination of the unknown vectors at successive steps:

$$\Delta\boldsymbol{U}_{i+1} = \Delta\boldsymbol{U}_i + \beta(\Delta\boldsymbol{U}_i - \Delta\boldsymbol{U}_{i-1}) \tag{8}$$

where β, the relaxation factor, is generally taken in (0, 2). A zero vector can be taken as a reasonable initial value of $\Delta\boldsymbol{U}$. The iterative process terminates once two successive solutions agree within some prescribed tolerance.

Since the real defect is usually very fine, $\Delta\boldsymbol{K}$ is typically a slight perturbation to $\boldsymbol{K}^0$. Some example calculations have shown that when the above iterative algorithm is applied to the FEM equation system (6), the iteration process is rapidly convergent. The multiple solution of whole matrix equation system stemming from the 3-D FEM modeling can eventually be avoided by applying the iterative solution technique.

2 Application in a Benchmark Problem

A benchmark problem provided by Ref. [4] has been studied to validate the proposed method. In the problem a pancake coil is used to inspect cracks with different depths in a square Inconel plate as shown in Fig.2[3]. The inner and outer diameters of the coil are 1.2 mm and 3.2 mm, respectively. The coil height is 0.8 mm. The coil lift-off is 1.0 mm. The frequency of the applied current is 300 kHz. The probe is very small in size compared with the detected metal plate. The objective of the modeling is to compute the change in impedance as the coil moves along the cracks and to compare the values with the corresponding experimental results provided by Ref. [4].

By ignoring its edge effect on the probe, the square metal plate being inspected can be approximately considered as a conducting disc with 140 mm diameter. As a result, an axisymmetrical analysis can easily be used to get the basic solution of the potential in the case with no defect, $\boldsymbol{A}^0$. Fig. 3 gives the flux distributions at $\omega t = 0°$ and $\omega t = 90°$. Because the axisymmetrical mesh is allowed to be denser than the 3-D one, they generally do not coincide with each other. The potential values on the nodes of the 3-D mesh can be estimated through the

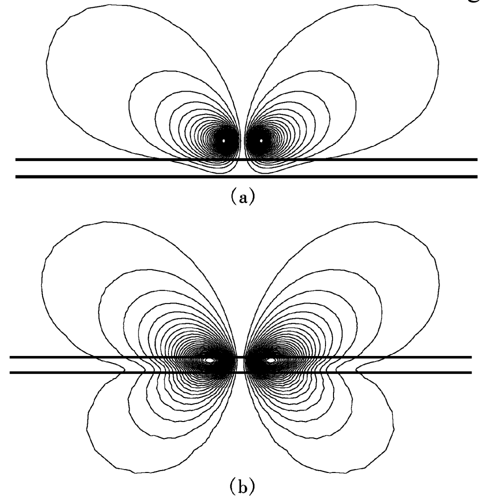

Fig.3 Flux distributions by axisymmetrical analysis. (a) $\omega t = 0°$; (b) $\omega t = 90°$

linear interpolation of $\boldsymbol{A}^0$.

Due to the symmetry of the problem, half of the field is taken as the 3-D analysis domain. In the calculations a 25 mm × 25 mm × 30 mm hexahedral mesh was used, which was carefully generated to make it accommodate to different depths of cracks. The number of unknown variables is 45 136. The LDU algorithm is used to decompose the coefficient matrix formed in terms of (6), only the upper triangular part of which needs to be saved due to the symmetry of the coefficient matrix. In total about 55 M of computer memory is occupied to solve the FE equation system. The CPU times for the decomposition of the coefficient matrix is about 19 min, and 8 s for one-step iteration of (6) on a Pentium IV/1.0G PC. The above numbers show that such a large NDT engineering problem can be analyzed on a common PC by applying the proposed combined method.

Fig.4 shows the eddy current distributions on the surface of the metal plate for four coil positions: y = 0.0, 2.5, 5.0 and 10 mm. It can be seen that the eddy current distributions are quite reasonable.

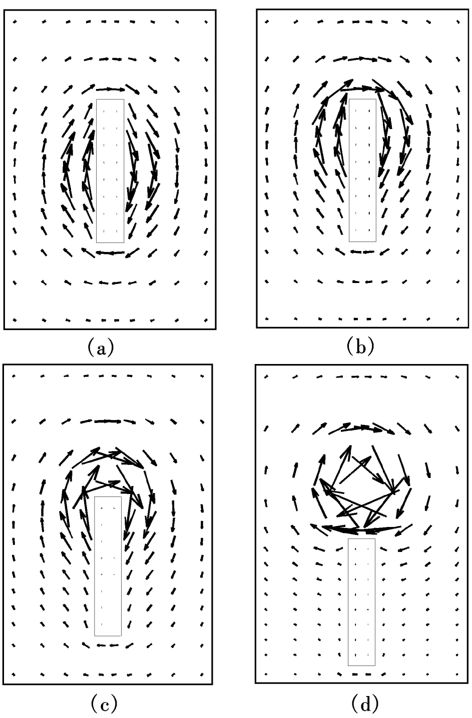

Fig.4 Eddy currents around defect on the surface of the plate. (a) y = 0.0 mm; (b) y = 2.5 mm; (c) y = 5.0 mm; (d) y = 10 mm

Fig. 5 shows the impedance perturbations (Δz) for the cases with inner defects and outer defects, respectively.

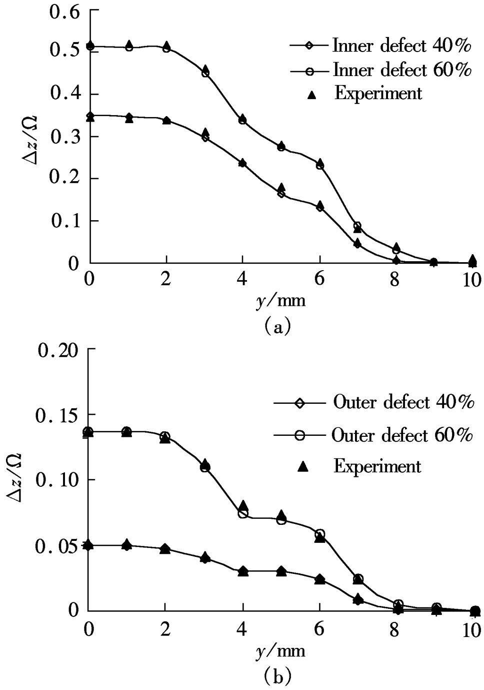

Fig.5 Impedance perturbations for inner and outer defects. (a) Inner defect; (b) Outer defect

The convergence of the iterative algorithm has also been investigated numerically in the computation of this benchmark problem. Tab. 1 shows the iterative numbers to different inner defect depths and relaxation factors. The entries indicate that the iteration of (6) has satisfied convergence, and [0.75, 1.25] is a proper region for taking β.

3 Conclusion

A combined calculation method for the fast 3-D FEM modeling of NDT probe signals has been successfully presented. Based on the combined method, the calculated region can be limited to a relatively small domain around the defect, instead of **the whole field domain used in conventional methods.**

Tab. 1 Iteration numbers to different inner defect depths and relaxation factors

β	Inner defect/%										
	0	10	20	30	40	50	60	70	80	90	100
0.00	8	20	38	34	35	38	43	45	50	57	45
0.25	7	21	36	39	37	45	44	46	48	43	48
0.50	6	23	29	35	41	44	49	41	55	57	50
0.75	7	19	27	28	33	31	45	44	57	58	49
1.00	6	19	27	27	35	34	39	38	51	51	45
1.25	5	14	30	31	32	29	37	40	45	51	41
1.50	6	20	35	34	45	41	37	49	52	54	47
1.75	7	22	34	36	37	39	41	56	57	50	59
2.00	9	25	37	33	40	39	47	55	53	58	61

The mesh generation can also be simplified in the 3-D modeling. Furthermore, an iterative algorithm can be used to obtain the solution of the matrix equation system quickly. A great amount of computer resources can thus be saved. The proposed method has been applied to the derivation of the 3-D solution of a benchmark problem with relatively few computer resources. It has been shown that the proposed combined method is highly effective and efficient in electromagnetic NDT simulations, particularly for problems having only small perturbations.

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无损检测信号快速三维有限元模拟的组合方法

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摘要: 提出了一种基于三维有限元的电磁无损检测缺损响应快速模拟的组合方法, 该方法由三项技术构成, 分别是局部计算技术、差场技术和迭代解技术. 采用局部计算技术, 可以将三维计算区域限制在缺损附近的一个较小范围; 使用差场技术, 无缺损情形下的轴对称解可以用来简化三维网格生成以及快速获得三维解答; 而应用迭代解技术, 可以加速无损探头信号有限元模拟的矩阵方程求解. 实例计算表明本文方法是正确有效的, 可以节省大量的计算机资源.

关键词: 无损检测; 有限元法; 模拟; 计算技术

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