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Urban street network capacity reliability

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Abstract: A reliability measure of network capacity under node capacities has been introduced, using the concept of network reserve capacity, and compared with that based on link capacities. The measure incorporating node capacities suggested should be considered in reliability analysis of urban street networks. Note that the assumption that every origin-destination (OD) pair will have a uniform growth or decline in its OD demand is preserved, whereas relaxing this limitation can yield pictures regarding the spatial distribution of the demand pattern with non-uniform change, which can be especially useful in individual zone land-use development plans.

Key words: network capacity reliability; node capacity; link capacity; network reserve capacity

The capacity of a network is usually defined as the maximum origin-destination demand that can be accommodated into the network without violating the specified capacity of each link; an alternative concept is reserve network capacity[1,2]. The network, however, consists of elements including links and nodes, which can be translated naturally into basic segments and intersections, respectively. The emphasis on link capacities in network flow theory is appropriate for a motorway network where the nodes are not of direct significance for traffic. But in an urban street network, most bottlenecks can be observed at intersections rather than within the basic segments. Thus it is necessary to study the network capacity and its reliability under node capacity constraints.

A structural proportionality of an *a priori* origin-destination (OD) flow distribution is just the same as the reserve network capacity assumed, which is articulated in terms of an OD matrix with the total sum of all the proportional entries equal to one, because of the immutability of the OD flow pattern in one time period. The estimation of network capacity therefore becomes a mathematical programming problem, in which the maximum OD matrix multiplier is sought, subject to the pass-flow rate at the node, resulting from the network equilibrium, not exceeding the corresponding node capacities.

With the increasing demand for better and more reliable services, more attention has been concentrated on the reliability analysis of a road network. The reliability is generally defined as the probability that the system has the ability to perform its intended function. In this paper, the network capacity reliability under node capacity constraints is investigated, and compared with that under link capacity constraints[3]. To guarantee the quality of service provided by a road transportation system, the node capacity should not be underestimated in evaluating the performances of the road network.

1 Node Capacity Constraints

Traffic demand at intersections varies throughout the day and congestion frequently occurs during morning or afternoon peak periods due to the number of people commuting to or from work. The phenomenon of traffic waiting at the link exits occurs frequently. Thus the importance of a node capacity in network reliability analysis should be emphasized.

The transportation network is denoted as $G(N, A)$, where N is a set of nodes and A is a set of links. Consider a signalized intersection with three or more approaching links. Let g_n^i be the green time given to links approaching node n in the phase i, L_n be the total lost time of all phases per cycle, and C_n be the cycle length. Green time and lost time must satisfy the following relationship:

$$\sum_i g_n^i + L_n = C_n \text{ or } \sum_i \frac{g_n^i}{C_n} = 1 - \frac{L_n}{C_n} \leqslant 1$$

The term in the left hand side of the last equation signifies the proportion of the available green time to a signal cycle, which can be characterized as the equalization of the composite of flow ratios for all the critical flow movements. Let A_n^i denote the set of links approaching node n during signal phase i, and c_a be the saturation flow of the link, called link capacity. The access restriction of the signalized intersection may be written as

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$$\sum_i \max_a \left\{ \frac{x_a}{c_a} \middle| a \in A_n^i \right\} \leqslant 1 \qquad \forall n \in N$$

where x_a is the flow on link a.

This node pass inequality signifies a kind of access restriction of vehicular flow approaching a node at each signal cycle. The node alternatively allocates green time among conflicting traffic movements seeking use of the same physical space. The inspection is valid not only in each signal cycle but equally in an observed time period for studying traffic phenomena. A similar restriction inequality on node capacity also exits in unsignalized intersections, but the alternation of access right is no longer prearranged; rather it is adaptive: the priority is generally allowed for the early arrival. For a similar discussion about node capacity refer to Refs. [4, 5].

2 Network Capacity and Reliability

2.1 Network capacity model

Network reserve capacity is defined as the largest multiplier applied to a base origin-destination demand matrix that can be allocated to a transportation network in a user-optimal way without violating the node capacities. It can be mathematically stated as follows:

$$\max \mu$$

$$\text{subject to } \sum_i \max_{a \in A} \left\{ \frac{x_a(\mu \boldsymbol{q})}{c_a} \middle| a \in A_n^i \right\} \leqslant 1$$

$$\forall n \in N$$

where $x_a(\mu \boldsymbol{q})$ is the equilibrium flow on arc $a \in A$; μ is the OD demand multiplier; $\boldsymbol{q}$ is the base OD demand vector; $\mu \boldsymbol{q}$ is the scaled demand, which is the base OD demands $\boldsymbol{q}$ scaled by μ. A network capacity problem under link capacity constraints[3] can be formulated if the node capacity constraints are substituted with the following inequality:

$$x_a(\mu \boldsymbol{q}) \leqslant c_a \qquad \forall a$$

Route choice behaviors are explicitly considered under equilibrium constraints, which bound equilibrium flows below their corresponding capacities. The pattern of equilibrium link flow is obtained by solving the following standard user-optimal traffic assignment problem:

$$\min \sum_a \int_0^{x_a} t_a(u, c_a) \, \mathrm{d}u$$

$$\text{subject to } \sum_{r \in R^w} f_r^w = \mu q_w \qquad \forall w$$

$$x_a = \sum_w \sum_r f_r^w \delta_{ar}^w \qquad \forall a$$

$$f_r^w \geqslant 0 \qquad \forall r, w$$

where R^w is the set of routes between OD pair, $w \in W$; $t_a(x_a, c_a)$ is the travel time on link, $a \in A$; q_w is the demand between OD pair, $w \in W$, W is the set of OD pairs in the network; f_r^w is the traffic value on route, $r \in R^w$; δ_{ar}^w is 1 if link a is included in route r, otherwise it is 0.

The problem of computing multiplier μ, is treated as a bi-level programming problem. At the upper level link use proportions or link flows are used as the inputs, which are the output of standard user equilibrium assignments enforced at the lower level. In consequence, route choice behavior and congestion effects are explicitly considered by the lower-level problem while the upper level problem determines the maximum OD matrix multiplier subject to the capacity constraints. As the scaled demand approaches the network capacity, equilibrium constraints will have a substantial effect on the distribution of traffic flow and on the network reserve capacity. Since the upper level problem has only one decision variable, it can be handled as a parameter in the lower level problem. And hence, the overall problem can be solved as a singular optimization, in which the multiplier μ is properly adjusted until at least one of the equilibrium element flows is approaching its upper bound, node or link capacity. Then, the multiplier μ, being greater than 1, shows that the network has reserve capacity amounting to $100(\mu - 1)$ percent of the base OD matrix, and the multiplier μ, the value of which is less than 1, shows that the network is overloaded by $100 \times (1 - \mu)$ percent of the base OD matrix.

2.2 Uncertain source and network capacity reliability

This paper proposes an analytical framework to evaluate the influence of variations in link capacity on the network capacity reliability. In reality, road network capacity is not deterministic, but subject to variations due to traffic accidents, weather conditions, roadside parking and so on. Clearly, the resultant network capacity depends on the link capacity vector $\boldsymbol{c}$. Let $\boldsymbol{c} = \boldsymbol{c}_0 - \varepsilon \boldsymbol{c}_0$, where $\boldsymbol{c}_0$ is a vector of normal link capacities, and ε is a parameter to determine the degree of deterioration of link capacity, and ranging from 0 (no degradation) to 1 (complete degradation). A probability distribution $p(\varepsilon)$ of perturbation occurrence is supposed to be available, which might be estimated from existing sources of data. As defined in section 1, the node capacity is associated with the capacities of its approaching links and hence is uncertain.

The methodology presented for evaluating network capacity addresses the capacity of the intersection as a whole, while the reliability analysis of network capacity is specifically on the basis of the capacity of the full intersection. Network capacity reliability is calculated as a probability that the maximum OD flow is greater than or equal to a required demand level when the capacity of links is subject to random variation. With the concept of reserve capacity, it can be given as

$$R(\mu^0,\ \varepsilon) = P_\varepsilon \qquad \mu^0 \leqslant \mu$$

The probability predicts how reliably the network with degraded links can accommodate a given demand level μ^0. The system is 100% reliable when the demand is zero, and 0% reliable when the demand is infinite. The employment of reserve capacity has provided a feasible approach to estimating network capacity reliability incorporating route choice behavior.

Given the fact that the link capacity is a random variable following a certain probability distribution, reliability analysis focuses on knowing the resulting probabilistic fluctuations or reliabilities of maximum network flow and travel time between the specified origin and destination pairs.

3 Numerical Simulation and Analysis

The numerical examples of network capacity reliability are presented here with the use of the Monte Carlo method. The test network and base demand as well as link performances are the same as those[6] shown in Fig. 1. There are 8 nodes and 24 links. The link travel time is estimated by the standard BPR function.

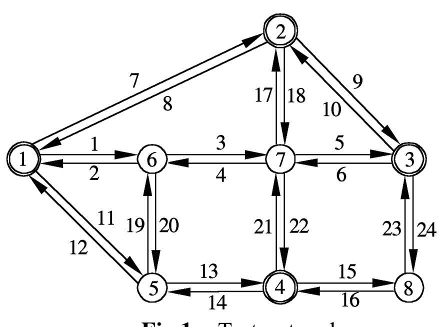

Fig.1 Test network

In the absence of link degradation data, a uniform distribution with an upper bound is assumed to generate the random capacities of all links. When the capacity of every link is fixed at the upper bound equal to its capacity, the largest multiplier is one, which means that the current network capacity is just enough to accommodate the base demand, corresponding to the non-degraded state. All the measures of capacity reliability are calculated from a Monte Carlo simulation of 5 000 samples.

Fig.2 shows the relationship between the network capacity reliability $R(\mu^0,\ \varepsilon)$ under link capacity constraints and indicates eleven series of $R(\mu^0,\ \varepsilon)$-curves with different values of the parameter ε. The $R(\mu^0,\ \varepsilon)$-curve in the extreme left hand side represents the capacity reliability in the case of $\varepsilon = 0$. It shows that in the normal state, the network can accommodate the base demand $\boldsymbol{q}$, and is 100% reliable for any given demand level μ^0. In contrast, the curve in the extreme right hand side represents the capacity reliability in the case of $\varepsilon = 1.0$. It shows the lowest capacity of the network in the worst case of degraded link capacities. Network capacity reliability under either link capacity or node capacity declines when the variation width of the link capacity grows.

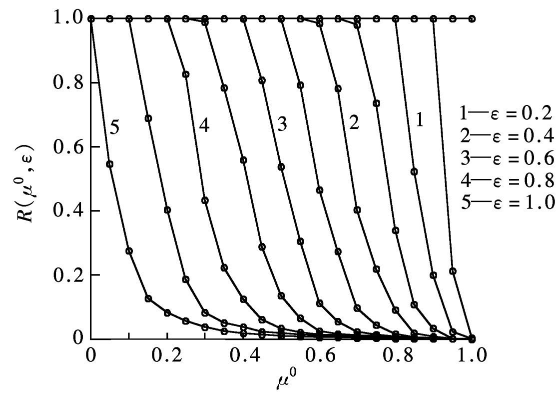

Fig.2 Capacity reliability under link capacity constraints

Fig. 3 shows the relationship between the network capacity reliability $R(\mu^0,\ \varepsilon)$ under node capacity constraints. As with Fig.2, the extreme left is the case of normal link capacities and the extreme right is the worst case of link capacity degradation. The capacity reliability under node constraints has the same tendency as that under link constraints and shows decline when link capacities vary greatly.

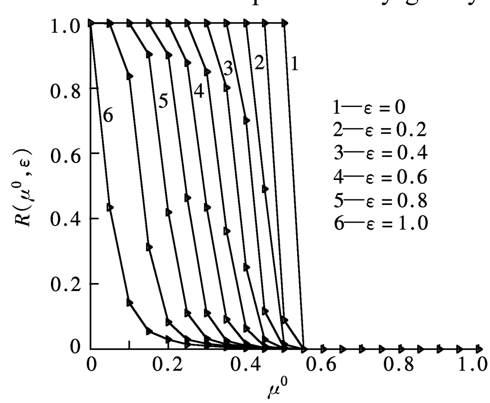

Fig.3 Capacity reliability under node capacity constraints

Under the same variation condition, however, the network capacity under link constraints is roughly double the one under node constraints. In other words, the assessment of the latter devalues the reliability of network capacity. There are no accidents because the shortage of element throughput usually occurs at the intersections rather than in the road segments over the street network. Therefore the reliability should also be considered under node constraints when network performance is studied.

Fig. 4 shows comparisons in network capacity reliability between the cases of link and node capacity constraints in three kinds of variation width. Under node capacity constraints, the network is 100% reliable in Fig. 4(b) for lower demand levels up to 25% of the base demand, while under link capacity constraints the reliability of 100% will last until to 50% of the base demand. As the demand level increases, both measures of capacity reliability decline

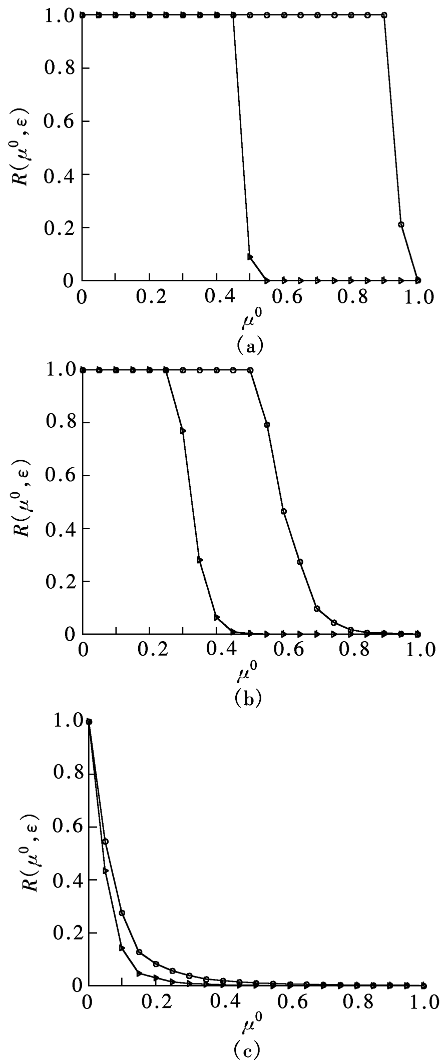

Fig.4 Comparisons of reliability measures under different variation widths. (a) $\varepsilon = 0.1$ (Variation width = 0.1 × link capacity); (b) $\varepsilon = 0.6$ (Variation width = 0.6 × link capacity); (c) $\varepsilon = 1.0$ (Variation width = 1.0 × link capacity)

and finally fail. It further verifies that reliability measures are rather different from vary visual points, and that the measure in node capacity constraints is conservative relative to that in link capacity constraints. Figs.4(a) and (c) describe the two extreme situations of variation width being 0.1 or 1.0 times link capacity. It exhibits that the difference of two assessments is the largest when the variation is small, and that the two reliability curves are approximately the same; in other words, the differences of the two measures tend towards equality as the amplitude of link capacity variation grows. This comparison manifests that the network capacity reliability under node capacity might not be dispensable in network reliability analysis, especially in traffic demand subject to daily fluctuations.

4 Conclusion

A reliability measure of network capacity under node capacities using the concept of network reserve capacity has been introduced, and compared with that based on link capacities. The measure suggested which incorporates node capacities should be considered in reliability analysis on road network. Note that the assumption that every OD pair will have a uniform growth or decline in its OD demand is preserved, whereas relaxing this limitation can yield pictures regarding the spatial distribution of the demand pattern with non-uniform change, which can be especially useful in individual zone land-use development plans.

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城市道路网络容量可靠性

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摘要: 运用交通网络保留容量的概念, 引入交通网络容量可靠性, 分析比较了基于路段容量的网络可靠性和基于结点容量的网络可靠性. 结果表明: 基于路段的网络容量可靠性通常小于基于结点的网络容量可靠性, 在城市街路网络容量可靠性分析中, 应当充分考虑街路交叉口的容量. 网络保留容量拥有统一的 OD 结构, 即单一的比例因子, 如果每一个 OD 拥有各自的比例因子, 交通网络容量将会更加确切地描述交通量的空间分布, 在城市土地利用中将会有极其重要的意义.

关键词: 网络容量可靠性; 结点容量; 路段容量; 网络保留容量

中图分类号: U113; U491.1^{+}3; O157.6