Journal of Southeast University (English Edition) Vol. 20 No. 2 June 2004 ISSN 1003—7985

Optimal control strategy of institutional investor's execution cost

Zhong Liming Liu Hailong Wu Chongfeng

(School of Management, Shanghai Jiaotong University, Shanghai 200052, China)

Abstract: Optimal control of multi-assets liquidation in view of volatility risk was studied. The analytical solution of optimal strategy was achieved with the calculus of variation. Numerical examples and graphical illustrations were also given. The conclusion shows that the optimal strategy is the linear combination of time's hyperbolic sine and hyperbolic cosine. The investor's attitude towards risk can influence the optimal strategy. In order to avoid the uncertainty of the execution cost, the investor with high risk aversion liquidates assets rapidly in the early period. The decrease of liquidation loss is at the cost of the increase of the volatility level.

Key words: liquidation; execution cost; optimal control

During the past twenty years, institutional investors have played a greater role in security markets. While the 2001 value of all outstanding shares in the NYSE totaled 6.6 trillion dollars, institutional investors have held almost 50%[1].

Due to their large position, trades of institutional investors can significantly affect price dynamics[2-4]. This disequilibrium is due to the costliness of executing large trades[5].

Bertsimas and Lo defined the optimal control of execution cost as the trading strategy that provided the minimum expected cost of trading when trading a large block of shares in a fixed time horizon[6]. They showed that splitting the order evenly over time would be the optimal strategy. Almgren and Chriss pointed out that Bertsimas and Lo's approach ignored the volatility of revenues for different strategies[7]. They defined optimal strategy as the strategy that minimized the execution cost under the constraint of volatility risk over a fixed period of time. They transformed the problem into a second order differential equation and got the analytical solution of the equation. Almgren generalized the study to the nonlinear price impact function and introduced "trading-enhanced risk"[8]. Huberman and Stanzl also noted the limitation of Bertsimas and Lo's work[9]. They introduced the variance of execution costs into the utility function and deduced the optimal sequence with stochastic dynamic programming.

We generalize Almgren and Chriss's work with a focus on the portfolio liquidation in continuous time. Applying the calculus of variation, we get the analytical solution of the optimal strategy. The optimal strategy is the linear combination of the time's hyperbolic sine and hyperbolic cosine. Finally, we present the numerical and figure illustration using the practical transaction data.

Received 2003-08-19.

Foundation item: The National Natural Science Foundation of China (No.70025303, No.70173031).

Biographies: Zhong Liming (1976—), male, graduate; Liu Hailong (corresponding author), male, professor, hlliu@sjtu.edu.cn.

1 Relating Assumptions of the Model

Define that the portfolio position of an institutional invest is $\boldsymbol{\varphi}(t)=\{\varphi_1(t),\cdots,\varphi_n(t)\}^{\mathrm{T}}$ and the price vector is $\boldsymbol{p}(t)=\{p_1(t),\cdots,p_n(t)\}^{\mathrm{T}}$ at time t. At initial time 0, the investor holds $\boldsymbol{\varphi}(0)=\{\varphi_1(0),\cdots,\varphi_n(0)\}^{\mathrm{T}}$.

Because the investor can determine the trading strategy by controlling the position in the liquidation, the position will be equivalent to the trading strategy. And $\boldsymbol{v}(t)=-\dfrac{\mathrm{d}\boldsymbol{\varphi}(t)}{\mathrm{d}t}$ is the trading velocity.

Assume that the price follows standard r-dimensional arithmetic Brownian motion without drift:

$$\begin{Bmatrix}\mathrm{d}p_1(t)\\ \vdots\\ \mathrm{d}p_n(t)\end{Bmatrix}=\begin{bmatrix}a_{11} & \cdots & a_{1r}\\ \vdots & & \vdots\\ a_{n1} & \cdots & a_{nr}\end{bmatrix}\begin{Bmatrix}\mathrm{d}B_1(t)\\ \vdots\\ \mathrm{d}B_r(t)\end{Bmatrix}\tag{1}$$

The price impact comes into being when the investor executes a large liquidation. We separate the impact into two kinds[2]: the permanent impact and the instantaneous impact. The permanent impact adds a negative drift in the differential equation of price movement, which means the change of the equilibrium price. The instantaneous impact has a short-lived effect on the price and will vanish after the arrival of a counter-part order.

Let γ_{ij} denote the coefficient of permanent impact, which is the change of equilibrium price of stock j after one unit sale of stock i. Assume that γ_{ij} is constant in the liquidation period, and the new price dynamic is as follows:

$$\begin{Bmatrix} dp_1(t) \\ \vdots \\ dp_n(t) \end{Bmatrix} = -\begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & & \vdots \\ \gamma_{n1} & \cdots & \gamma_{nn} \end{bmatrix} \begin{Bmatrix} v_1(t) \\ \vdots \\ v_n(t) \end{Bmatrix} dt + \begin{bmatrix} a_{11} & \cdots & a_{1r} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nr} \end{bmatrix} \begin{Bmatrix} dB_1(t) \\ \vdots \\ dB_r(t) \end{Bmatrix} \tag{2}$$

At time t, the price vector of portfolio is

$$\begin{Bmatrix} p_1(t) \\ \vdots \\ p_n(t) \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{r} a_{1i} B_i(t) \\ \vdots \\ \sum_{i=1}^{r} a_{ni} B_i(t) \end{Bmatrix} + \begin{Bmatrix} p_1(0) \\ \vdots \\ p_n(0) \end{Bmatrix} - \begin{Bmatrix} \sum_{i=1}^{n} \gamma_{1i}(\varphi_i(0) - \varphi_i(t)) \\ \vdots \\ \sum_{i=1}^{n} \gamma_{ni}(\varphi_i(0) - \varphi_i(t)) \end{Bmatrix} \tag{3}$$

Let h_{ij} denote the coefficient of instantaneous impact. According to the definition, the instantaneous impact only persists in an interval of an instant. In the model, this impact causes the difference between the transaction price and the market price:

$$\begin{Bmatrix} \tilde{p}_1(t) \\ \vdots \\ \tilde{p}_n(t) \end{Bmatrix} = \begin{Bmatrix} p_1(t) \\ \vdots \\ p_n(t) \end{Bmatrix} - \begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & & \vdots \\ h_{n1} & \cdots & h_{nn} \end{bmatrix} \begin{Bmatrix} v_1(t) \\ \vdots \\ v_n(t) \end{Bmatrix} \tag{4}$$

Let $\boldsymbol{A} = \begin{bmatrix} a_{11} & \cdots & a_{1r} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nr} \end{bmatrix}$ denote the volatility matrix and $d\boldsymbol{B}(t) = \begin{Bmatrix} dB_1(t) \\ \vdots \\ dB_r(t) \end{Bmatrix}$ and $\boldsymbol{B}(t) = \begin{Bmatrix} B_1(t) \\ \vdots \\ B_r(t) \end{Bmatrix}$. $\boldsymbol{G} = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \vdots & & \vdots \\ \gamma_{n1} & \cdots & \gamma_{nn} \end{bmatrix}$ and $\boldsymbol{H} = \begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & & \vdots \\ h_{n1} & \cdots & h_{nn} \end{bmatrix}$ denote the coefficient matrix of permanent impact and the coefficient matrix of instantaneous impact, respectively.

Assume that the sale of one stock has no effect on the price of another stock:

$$\gamma_{ij} = \begin{cases} \gamma_{ii} & i=j,\ \gamma_{ii} \neq 0 \\ 0 & i \neq j \end{cases},\quad h_{ij} = \begin{cases} h_{ii} & i=j,\ h_{ii} \neq 0 \\ 0 & i \neq j \end{cases}$$

Then, we get the vector formation of Eqs.(2) to (4):

$$d\boldsymbol{p}(t) = -\boldsymbol{G}\boldsymbol{v}(t) + \boldsymbol{A}\,d\boldsymbol{B}(t) \tag{5}$$

$$\boldsymbol{p}(t) = \boldsymbol{p}(0) - \boldsymbol{G}(\boldsymbol{\varphi}(0) - \boldsymbol{\varphi}(t)) + \boldsymbol{A}\boldsymbol{B}(t) \tag{6}$$

$$\tilde{\boldsymbol{p}}(t) = \boldsymbol{p}(t) - \boldsymbol{H}\boldsymbol{v}(t) \tag{7}$$

2 Construction and Solution of the Model

In an arbitrary period $(t, t+\Delta t)$, the change of the investor's cash holding is

$$\Delta S(t) = \{\tilde{p}_1(t), \cdots, \tilde{p}_n(t)\} \begin{Bmatrix} -\Delta\varphi_1(t) \\ \vdots \\ -\Delta\varphi_n(t) \end{Bmatrix} \tag{8}$$

Substitute Eqs.(3) and (4) into Eq.(8):

$$\Delta S(t) = -\sum_{i=1}^{n} \Big[p_i(0) - \gamma_{ii}(\varphi_i(0) - \varphi_i(t)) + \sum_{j=1}^{r} a_{ij} B_j(t) - h_{ii} v_i(t) \Big] \Delta\varphi_j(t) \tag{9}$$

Let $\Delta t \to 0$ and resolve the integral of both sides of Eq.(9) in $(0, T)$:

$$S = \int_0^T dS(t) = \sum_{i=1}^{n} p_i(0)\varphi_i(0) - \sum_{i=1}^{n} \frac{\gamma_{ii}\varphi_i^2(0)}{2} + \sum_{i=1}^{n} \int_0^T h_{ii} v_i(t)\, d\varphi_i(t) - \sum_{i=1}^{n} \sum_{j=1}^{r} \int_0^T a_{ij} B_j(t)\, d\varphi_i(t) \tag{10}$$

Substituting $d\varphi_i(t) = -v_i(t)\,dt$ into Eq.(10) and applying the integration by parts to $\sum_{i=1}^{n} \sum_{j=1}^{r} \int_0^T a_{ij} B_j(t)\, d\varphi_i(t)$, we get

$$S = \sum_{i=1}^{n} p_i(0)\varphi_i(0) - \sum_{i=1}^{n} \frac{\gamma_{ii}\varphi_i^2(0)}{2} - \sum_{i=1}^{n} \int_0^T h_{ii} v_i^2(t)\, dt - \sum_{i=1}^{n} \sum_{j=1}^{r} \int_0^T a_{ij}\varphi_i(t)\, dB_j(t) \tag{11}$$

Define that the execution cost or liquidation loss L is the difference between the initial value $\boldsymbol{p}^{\mathrm{T}}(0) \cdot \boldsymbol{\varphi}(0)$ and the amount of cash S:

$$L = \boldsymbol{p}^{\mathrm{T}}(0)\boldsymbol{\varphi}(0) - S = \sum_{i=1}^{n} \frac{\gamma_{ii}\varphi_i^2(0)}{2} + \sum_{i=1}^{n} \int_0^T h_{ii} v_i^2(t)\, dt + \sum_{i=1}^{n} \sum_{j=1}^{r} \int_0^T a_{ij}\varphi_i(t)\, dB_j(t) \tag{12}$$

Eq.(12) can be rewritten in vector formation:

$$L = \frac{1}{2}\boldsymbol{\varphi}^{\mathrm{T}}(0)\boldsymbol{G}\boldsymbol{\varphi}(0) + \int_0^T \boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{H}\boldsymbol{v}(t)\, dt + \int_0^T \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{A}\, d\boldsymbol{B}(t) \tag{13}$$

Eq.(13) shows that the liquidation loss is a random variable relating to the trading strategy.

The expectation and variance of L is

$$E[L] = \frac{1}{2}\boldsymbol{\varphi}^{\mathrm{T}}(0)\boldsymbol{G}\boldsymbol{\varphi}(0) + \int_0^T \boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{H}\boldsymbol{v}(t)\, dt \tag{14}$$

$$V[L] = \int_0^T \boldsymbol{\varphi}^{\mathrm{T}}(\mathrm{t})\boldsymbol{C}\boldsymbol{\varphi}(t)\, dt \tag{15}$$

where $\boldsymbol{C} = \begin{bmatrix} c_{11} & \cdots & c_{1n} \\ \vdots & & \vdots \\ c_{n1} & \cdots & c_{nn} \end{bmatrix} = \boldsymbol{A}\boldsymbol{A}^{\mathrm{T}}$ is the covariance matrix of the portfolio, which is a real symmetric matrix.

Obviously, $E[L]$ and $V[L]$ are functions of

$\varphi_i(t)$.

Let

$$W(\varphi_1, \cdots, \varphi_n) = E[L] + \lambda V[L] = \boldsymbol{\varphi}^{\mathrm{T}}(0) \cdot \boldsymbol{G}\boldsymbol{\varphi}(0) + \int_0^T [\boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{H}\boldsymbol{v}(t) + \lambda\boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{C}\boldsymbol{\varphi}(t)]\mathrm{d}t$$

where λ is the coefficient of risk aversion and $\lambda \geqslant 0$. Then, $W(\varphi_1, \cdots, \varphi_n)$ is also a function of $\varphi_i(t)$.

Let

$$F(t, \boldsymbol{\varphi}(t), \boldsymbol{v}(t)) = \boldsymbol{v}^{\mathrm{T}}(t)\boldsymbol{H}\boldsymbol{v}(t) + \lambda\boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{C}\boldsymbol{\varphi}(t)$$

To minimize $W(\varphi_1, \cdots, \varphi_n)$, $F(t, \boldsymbol{\varphi}(t), \boldsymbol{v}(t))$ should satisfy the following Euler equation:

$$\frac{\partial F}{\partial \varphi_k} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial F}{\partial \dot{\varphi}_k}\right) = 0 \qquad \forall k,\ k = 1, \cdots, n \tag{16}$$

Since $v_k(t) = -\frac{\mathrm{d}\boldsymbol{\varphi}(t)}{\mathrm{d}t}$, Eq.(16) can be rewritten as

$$\frac{\partial F}{\partial \varphi_k} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial F}{\partial v_k}\right) = 0 \tag{17}$$

We also note that

$$\frac{\partial F}{\partial \varphi_k} = \sum_{i=1}^{n} \lambda\varphi_i(t)(c_{ki} + c_{ik}) \tag{18}$$

With the symmetry of matrix $\boldsymbol{C}$, we rewrite Eq.(18):

$$\frac{\partial F}{\partial \varphi_k} = \sum_{i=1}^{n} 2\lambda\varphi_i(t)c_{ki} \tag{19}$$

And

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial F}{\partial v_k}\right) = \frac{\mathrm{d}(h_{kk}v_k(t))}{\mathrm{d}t} = -h_{kk}\frac{\mathrm{d}^2\varphi_k(t)}{\mathrm{d}t^2} \tag{20}$$

Substitute Eqs.(19) and (20) into Eq.(17):

$$\sum_{i=1}^{n} 2\lambda\varphi_i(t)c_{ki} - h_{kk}\frac{\mathrm{d}^2\varphi_k(t)}{\mathrm{d}t^2} = 0 \qquad \forall k,\ k = 1, \cdots, n \tag{21}$$

Eq.(21) is a second order linear differential equation. In the following, we will solve Eq.(21).

First, we rewrite Eq.(21) in vector formation:

$$2\lambda\boldsymbol{C}\boldsymbol{\varphi}(t) - \boldsymbol{H}\frac{\mathrm{d}^2\boldsymbol{\varphi}(t)}{\mathrm{d}t^2} = 0 \tag{22}$$

Let

$$\boldsymbol{\psi}(t) = \boldsymbol{H}^{\frac{1}{2}}\boldsymbol{\varphi}(t) \tag{23}$$

Substitute Eq.(23) into Eq.(22):

$$\frac{\mathrm{d}^2\boldsymbol{\psi}(t)}{\mathrm{d}t^2} = \lambda\boldsymbol{H}^{-\frac{1}{2}}\boldsymbol{C}\boldsymbol{H}^{-\frac{1}{2}}\boldsymbol{\psi}(t) \tag{24}$$

We note that $\boldsymbol{H}$ is a diagonal matrix and $\boldsymbol{C}$ is a real symmetric matrix, so $\boldsymbol{H}^{-\frac{1}{2}}\boldsymbol{C}\boldsymbol{H}^{-\frac{1}{2}}$ is also a real symmetric matrix. With a spectral decomposition theorem of real symmetric matrix, $\boldsymbol{H}^{-\frac{1}{2}}\boldsymbol{C}\boldsymbol{H}^{-\frac{1}{2}}$ can be decomposed to the following formation:

$$\boldsymbol{H}^{-\frac{1}{2}}\boldsymbol{C}\boldsymbol{H}^{-\frac{1}{2}} = \boldsymbol{P}^{\mathrm{T}}\boldsymbol{U}\boldsymbol{P}$$

where $\boldsymbol{P} = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & & \vdots \\ p_{n1} & \cdots & p_{nn} \end{bmatrix}$ is an orthogonal matrix and $\boldsymbol{U} = \begin{bmatrix} u_{11} & \cdots & u_{1n} \\ \vdots & & \vdots \\ u_{n1} & \cdots & u_{nn} \end{bmatrix}$ is a diagonal one. The columns of matrix $\boldsymbol{P}$ are formed by the orthogonal eigenvector of matrix $\boldsymbol{H}^{-\frac{1}{2}}\boldsymbol{C}\boldsymbol{H}^{-\frac{1}{2}}$. And $\boldsymbol{U} = \begin{bmatrix} u_{11} & \cdots & u_{1N} \\ \vdots & & \vdots \\ u_{N1} & \cdots & u_{NN} \end{bmatrix}$ is a diagonal matrix, the diagonal elements of which are the eigenvalue of matrix $\boldsymbol{H}^{-\frac{1}{2}}\boldsymbol{C}\boldsymbol{H}^{-\frac{1}{2}}$.

Let $\boldsymbol{\xi}(t) = \boldsymbol{P}\boldsymbol{\psi}(t)$ and substitute it into Eq.(24):

$$\frac{\mathrm{d}^2\boldsymbol{\xi}(t)}{\mathrm{d}t^2} = \lambda\boldsymbol{U}\boldsymbol{\xi}(t) \tag{25}$$

We can get

$$\frac{\mathrm{d}^2\xi_k(t)}{\mathrm{d}t^2} = \lambda u_{kk}\xi_k(t) \qquad k = 1, \cdots, n \tag{26}$$

The general solution of Eq.(26) is as follows:

① When $\lambda = 0$,

$$\xi_k(t) = \beta_k t + \theta_k \qquad k = 1, \cdots, n \tag{27}$$

where β_k and θ_k are determined by the boundary conditions and the linear relation between variables:

$$\begin{cases} \boldsymbol{\varphi}(0) = \boldsymbol{X},\ \boldsymbol{\varphi}(T) = \boldsymbol{0} \\ \boldsymbol{\psi}(t) = \boldsymbol{H}^{\frac{1}{2}}\boldsymbol{\varphi}(t),\ \boldsymbol{\xi}(t) = \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\psi}(t) \end{cases}$$

② When $\lambda > 0$,

$$\xi_k(t) = \beta_k \sinh(t\sqrt{\lambda u_{kk}}) + \theta_k \cosh(t\sqrt{\lambda u_{kk}}) \qquad k = 1, \cdots, n$$

where β_k and θ_k are determined by the boundary conditions and the linear relation between variables:

$$\begin{cases} \boldsymbol{\varphi}(0) = \boldsymbol{X},\ \boldsymbol{\varphi}(T) = \boldsymbol{0} \\ \boldsymbol{\psi}(t) = \boldsymbol{H}^{\frac{1}{2}}\boldsymbol{\varphi}(t),\ \boldsymbol{\xi}(t) = \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\psi}(t) \end{cases}$$

Obviously, when $\lambda \to 0$,

$$\lim_{\lambda \to 0} \xi_k(t) = -\frac{t}{T}\sum_{i=1}^{N} q_{ki}(X_i - Y_i) + \sum_{i=1}^{N} q_{ki}X_i$$

it is just the above general solution when $\lambda = 0$.

Finally, we get the solution of trading strategy:

$$\boldsymbol{x}(t) = \boldsymbol{H}^{-\frac{1}{2}}\boldsymbol{P}\boldsymbol{\xi}(t)$$

3 Numerical and Graphical Illustration of the Optimal Control Strategy

Both the Shanghai market and Shenzhen market fell on October 8, 2002. The consensus that the institutional investors' liquidations drive the market to fall has now been accepted. Therefore, we choose to study the Huaan Innovation Open-End Fund with the third-quarter announcement of the portfolio. Due to the absence of fund transaction account data, we assume that the fund liquidated four stocks which are Shanglingdianqi, Weifugaoke, Daxiangufen, and Shennengyuan in the portfolio and the liquidation period was from October 8th to 11th. The shares for liquidation are 150 thousand shares, 300 thousand shares, 300 thousand shares, 300 thousand shares, respectively. That means:

$$\boldsymbol{\varphi}(0) = \{150\,000,\ 300\,000,\ 300\,000,\ 300\,000\}^{\mathrm{T}}$$

With the actual transaction data in the period, we estimate the corresponding parameters with the method proposed by Sadka[10]:

$$\boldsymbol{G} = \begin{bmatrix} 2.3 & 0 & 0 & 0 \\ 0 & 3.2 & 0 & 0 \\ 0 & 0 & 1.3 & 0 \\ 0 & 0 & 0 & 0.48 \end{bmatrix} \times 10^{-8}$$

$$\boldsymbol{H} = \begin{bmatrix} 2.9 & 0 & 0 & 0 \\ 0 & 4.1 & 0 & 0 \\ 0 & 0 & 1.6 & 0 \\ 0 & 0 & 0 & 0.6 \end{bmatrix} \times 10^{-7}$$

$$\boldsymbol{C} = \begin{bmatrix} 0.999 & -0.217 & -0.028 & -0.095 \\ -0.217 & 2.168 & 0.022 & 0.365 \\ -0.028 & 0.022 & 0.374 & 0.207 \\ -0.095 & 0.365 & 0.207 & 0.501 \end{bmatrix} \times 10^{-2}$$

The optimal liquidation strategy is shown in Tab.1 and Fig.1, where the coefficient of risk aversion $\lambda = 10^{-6}$.

If the investor has different risk aversions, λ has different values. The optimal strategy in different risk aversions is shown in Fig.2. Obviously, the investor with high-risk aversion will liquidate most shares in the very early period to control volatility risk. The linear liquidation strategy will be chosen by the risk neutral investor to minimize the execution cost.

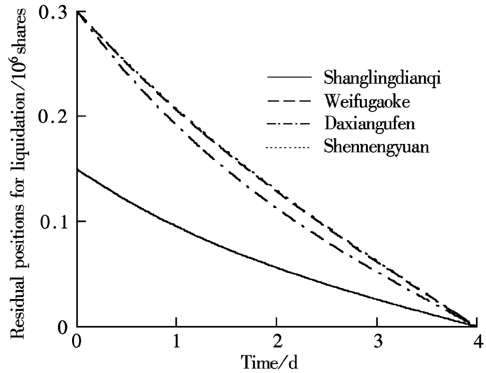

Fig.1 Optimal control strategy of Huaan Innovation Open-End Fund liquidation

The expectation and variance of the liquidation loss in different risk aversion attitudes are presented in Tab.2. We can easily observe that the decrease of the cost is at the expense of the increase of uncertainty.

Tab.1 The arrangement of Huaan Innovation Open-End Fund liquidation 10^4

Stock schedule name	Liquidated shares on Oct. 8th	Liquidated shares on Oct. 9th	Liquidated shares on Oct. 10th	Liquidated shares on Oct. 11th	Total liquidated shares
Shanglingdianqi	5.432 9	3.944 0	3.016 3	2.606 8	15
Weifugaoke	9.325 9	7.737 7	6.689 2	6.247 2	30
Daxiangufen	9.210 3	7.731 5	6.738 4	6.319 8	30
Shennengyuan	5.432 9	3.944 0	3.016 3	2.606 8	30

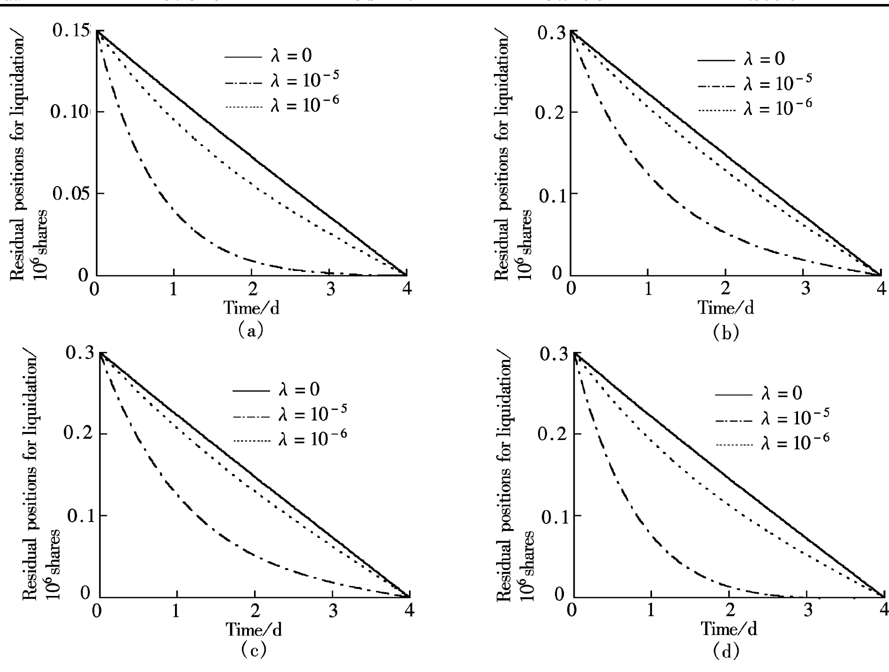

Fig.2 Optimal control strategy of different risk aversions. (a) Shanglingdianqi; (b) Weifugaoke; (c) Daxiangufen; (d) Shennengyuan

Tab.2 The expectation and variance of liquidation loss in different risk aversion attitudes

Coefficient of relative risk aversion	Expectation of liquidation loss/Yuan	Variance of liquidation loss/Yuan^2
$\lambda = 10^{-5}$	3.35×10^4	1.92×10^9
$\lambda = 10^{-6}$	1.90×10^4	4.14×10^9
$\lambda = 0$	1.83×10^4	4.98×10^9

4 Conclusion and Future Research Direction

This paper studies the optimal control of multi-asset liquidation in view of volatility and gets the analytical solution with calculus of variation. The conclusion shows that the optimal strategy is the linear combination of time's hyperbolic sine and hyperbolic cosine and is influenced by the degree of investor's risk aversion. The investor with high risk aversion would liquidate assets rapidly in the early period to avoid the uncertainty of the execution cost. The decrease of liquidation loss is at the cost of the increase of the volatility level.

To get an analytical solution, we assume that stock prices follow arithmetic Brownian motion. Loosening this assumption to geometric Brownian motion is more practical and it is undoubtedly a challenge.

Furthermore, we also assume a constant coefficient of price impact and the diagonal formation of the coefficient matrices. In practice, price impacts are time variable and there are cross effects when the investors liquidate the portfolio. The optimal strategy can be acquired with numerical methods when the above assumptions are violated.

Finally, the optimal strategy we get is time-independent. Obviously, the optimal strategy that can be modified between the liquidation is a "true" optimal strategy. This problem awaits future research.

References

[1] NYSE. Fact book 2001[R]. New York: NYSE, 2001.

[2] Kraus A, Stoll H R. Price impacts of block trading on the New York Stock Exchange [J]. *Journal of Finance*, **1972, 27**(2): 569 − 588.

[3] Keim D, Madhavan A. The upstairs market for large-block transactions: analysis and measurement of price effects [J]. *Review of Financial Studies*, **1996, 9**(1): 1 − 36.

[4] Chan L, Lakonishok J. The behavior of stock prices around institutional trades [J]. *Journal of Finance*, **1995, 50**(4): 1147 − 1174.

[5] Perold A. The implementation shortfall: paper versus reality [J]. *Journal of Portfolio Management*, **1988, 14**(1): 4 − 9.

[6] Bertsimas D, Lo A W. Optimal control of execution costs [J]. *Journal of Financial Markets*, **1998, 1**(1): 1 − 50.

[7] Almgren R, Chriss N. Optimal execution of portfolio transactions[J]. *Journal of Risk*, **2000, 3**(1), 5 − 39.

[8] Almgren R. Optimal execution with nonlinear impact functions and trading-enhanced risk [J]. *Applied Mathematical Finance*, **2003, 10**(1): 1 − 18.

[9] Huberman G, Stanzl W. Optimal liquidity trading [R]. Northwestern University, working paper, 2001.

[10] Sadka R. Momentum, liquidity risk and limit to arbitrage [R]. Northwestern University, working paper, 2003.

机构投资者执行成本的最优控制策略

仲黎明 刘海龙 吴冲锋

(上海交通大学管理学院, 上海 200052)

摘要: 研究了考虑波动性风险时多个资产变现的最优控制策略, 并利用变分法求出最优策略的解析形式.研究表明, 最优控制策略是时间的双曲正弦和双曲余弦函数的线性组合, 并且与投资者的风险厌恶程度有关.当投资者风险厌恶程度较高时, 他会在早期就迅速降低头寸, 以规避风险.在变现过程中, 变现成本的减少是以变现成本波动水平的提高为代价的.

关键词: 变现; 执行成本; 最优控制

中图分类号: F830