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Hamiltonicity, neighborhood union and square graphs of claw-free graphs

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Abstract: Let G be a graph, the square graph G^2 of G is a graph satisfying $V(G^2)=V(G)$ and $E(G^2)=E(G)\cup\{uv:\ \mathrm{dist}_G(u,v)=2\}$. In this paper, we use the technique of vertex insertion on l-connected ($l=k$ or $k+1$, $k\geqslant 2$) claw-free graphs to provide a unified proof for G to be Hamiltonian, 1-Hamiltonian or Hamiltonian-connected. The sufficient conditions are expressed by the inequality concerning $\sum_{i=0}^{k}|N(Y_i)|$ and $n(Y)$ in G for each independent set $Y=\{y_0, y_1, \cdots, y_k\}$ of the square graph of G, where b ($0<b<k+1$) is an integer, $Y_i=\{y_i, y_{i-1}, \cdots, y_{i-(b-1)}\}\subseteq Y$ for $i\in\{0, 1, \cdots, k\}$, where subscriptions of y_j's will be taken modulo $k+1$, and $n(Y)=|\{v\in V(G):\ \mathrm{dist}(v, Y)\leqslant 2\}|$.

Key words: Hamiltonicity; claw-free graph; neighborhood union; vertex insertion; square graph

1 Preliminaries and Main Results

In this paper, the terminology and notation not defined will follow Ref.[1], and we consider simple finite graphs only. G will always stand for a graph. Let G be a graph, for any $u\in V(G)$, let $N(u)$ denote the neighborhood of u and $d(u)=|N(u)|$ be the degree of u. More generally, for any $U\subseteq V(G)$, let $N(U)=\bigcup_{u\in U}N(u)$.

Many results on Hamiltonicity of claw-free graphs have been discovered. We are especially interested in the following results.

Theorem 1[2] Let G be a k-connected claw-free graph of order $n\geqslant 3$, and $k\geqslant 2$. If $\sum_{i=0}^{k}d(y_i)>n-k-1$ for each independent set $\{y_0, y_1, \cdots, y_k\}$ of G, then G is Hamiltonian.

Theorem 2[3] Let G be a k-connected claw-free graph with $k\geqslant 2$. If $\alpha(G^2)\leqslant k$, then G is Hamiltonian.

Theorem 3[4] Let G be a $(k+1)$-connected claw-free graph with $k\geqslant 2$. If $\sum_{i=0}^{k}d(y_i)>n-k+2$ for each independent set $\{y_0, y_1, \cdots, y_k\}$ of G, then G is Hamiltonian-connected.

To understand the relation among these sufficient conditions on Hamiltonicity. We introduce the following notation.

Let $t>1$ be an integer. Denote $I_t(G)=\{Y:\ Y$ is an independent set of G, $|Y|=t\}$.

Let G be connected, $Y\subseteq V(G)$, and $v\in V(G)$. Denote $\mathrm{dist}(v, Y)=\min_{y\in Y}\{\mathrm{dist}(v, y)\}$, where $\mathrm{dist}(v, y)$ stands for the distance between v and y in G,

$$N_i(Y)=\{v\in V(G):\ \mathrm{dist}(v, Y)=i\}\quad i=0, 1, 2, \cdots$$

and

$$n(Y)=|N_0(Y)\cup N_1(Y)\cup N_2(Y)|=|\{v\in V(G):\ \mathrm{dist}(v, Y)\leqslant 2\}|$$

Clearly, $N(Y)=N_1(Y)$, and $n(Y)\leqslant|V(G)|$. For each $i\in\{0, 1, 2, \cdots, |Y|\}$, denote

$$S_i(Y)=\{v\in V(G):\ |N(v)\cap Y|=i\}$$

For $v\in V(G)$, denote $N[v]=N(v)\cup\{v\}$. Let $\{u, v\}\subseteq V(G)$. Set

$$J(u, v)=\{w\in N(u)\cap N(v):\ N(w)\subseteq N[u]\cup N[v]\}$$

The partially square graph G^{*}[5] of G is a graph satisfying $V(G^*)=V(G)$ and $E(G^*)=E(G)\cup\{uv:\ uv\notin E(G)$, and $J(u, v)\neq\varnothing\}$.

The square graph G^2 of G is a graph satisfying $V(G^2)=V(G)$ and $E(G^2)=E(G)\cup\{uv:\ \mathrm{dist}_G(u, v)=2\}$.

Clearly, $V(G)=V(G^*)=V(G^2)$, $E(G)\subseteq E(G^*)\subseteq E(G^2)$.

In this paper, we use the vertex inserting lemmas introduced in Ref.[6] to prove the following new results (theorems 4 to 6). In theorems 4 to 6, we assume that $Y=\{y_0, y_1, \cdots, y_k\}\in I_{k+1}(G)$ is an ordered set, $Y_i=\{y_i, y_{i-1}, \cdots, y_{i-(b-1)}\}\subseteq Y$, for $i\in\{0, 1, \cdots, k\}$, where the subscriptions of y_j's will be taken modulo $k+1$.

Theorem 4 Let G be a k-connected claw-free graph with $k\geqslant 2$, and let b be an integer with $0<b<k+1$. If

$$\sigma_b(Y)=\sum_{i=0}^{k}|N(Y_i)|>b(n(Y)-k-1)$$

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in G for each $Y \in I_{k+1}(G^2)$, then G is Hamiltonian.

Theorem 5 Let G be a $(k+1)$-connected claw-free graph with $k \geqslant 2$, and let b be an integer with $0 < b < k+1$. If

$$\sigma_b(Y) = \sum_{i=0}^{k} |N(Y_i)| > b(n(Y) - k - 1)$$

in G for each $Y \in I_{k+1}(G^2)$, then G is 1-Hamiltonian.

Theorem 6 Let G be a $(k+1)$-connected claw-free graph with $k \geqslant 3$, and let b be an integer with $0 < b < k+1$. If

$$\sigma_b(Y) = \sum_{i=0}^{k} |N(Y_i)| > b(n(Y) - k - 1)$$

in G for each $Y \in I_{k+1}(G^2)$, then G is Hamiltonian-connected.

For the case $b = 1$, we have the following results.

Corollary 1 Let G be a k-connected claw-free graph with $k \geqslant 2$. If

$$\sum_{i=0}^{k} d(y_i) > n(Y) - k - 1$$

in G for each $Y = \{y_0, y_1, \cdots, y_k\} \in I_{k+1}(G^2)$, then G is Hamiltonian.

Corollary 2 Let G be a $(k+1)$-connected claw-free graph with $k \geqslant 2$. If

$$\sum_{i=0}^{k} d(y_i) > n(Y) - k - 1$$

in G for each $Y = \{y_0, y_1, \cdots, y_k\} \in I_{k+1}(G^2)$, then G is 1-Hamiltonian.

Corollary 3 Let G be a $(k+1)$-connected claw-free graph with $k \geqslant 3$. If

$$\sum_{i=0}^{k} d(y_i) > n(Y) - k - 1$$

in G for each $Y = \{y_0, y_1, \cdots, y_k\} \in I_{k+1}(G^2)$, then G is Hamiltonian-connected.

From the above corollaries, it is easy to see that theorem 4 generalizes theorems 1 and 2, and theorem 6 generalizes theorem 3 with slight improvements.

2 Vertex Inserting Lemmas

In this section, we always assume that G is a connected non-Hamiltonian graph and C is a maximal cycle of G, i.e., there is no cycle C' in G, such that $V(C) \subset V(C')$, and H is a component of $G - V(C)$. Also assume $\{v_1, v_2, \cdots, v_m\} \subseteq N_C(H)$ and $v_1, v_2, \cdots, v_m$ occur on C in the order of their indices. The subscriptions of v_i's will be taken modulo m. If $x \in V(C)$, denote by x^+ and x^- the successor and the predecessor of x along the orientation of C, respectively.

For each $i \in \{1, 2, \cdots, m\}$, a vertex $u \in C(v_i, v_{i+1})$ is called insertible[6] if there is some vertex $w \in C[v_{i+1}, v_i)$ such that $\{w, w^+\} \subseteq N(u)$. Otherwise u is called non-insertible.

Lemma 1[6] Let $u \in C(v_i, v_{i+1})$ for some $i \in \{1, 2, \cdots, m\}$. If all vertices in $C(v_i, u)$ are insertible, then $u \notin N_C(H)$. Therefore, there exists a vertex in $C(v_i, v_{i+1})$, which is non-insertible.

By lemma 1, for each $i \in \{1, 2, \cdots, m\}$, let x_i be the first non-insertible vertex in $C(v_i, v_{i+1})$.

Let $X_m = \{x_1, x_2, \cdots, x_m\}$ and $X_M = \{x_0\} \cup X_m$ (where x_0 is an arbitrary vertex of H). Set

$$X' = \{x_{p_1}, x_{p_2}, \cdots, x_{p_k}\} \subseteq X_m$$

where $1 \leqslant p_1 < p_2 < \cdots < p_k \leqslant m$. For convenience, we always assume that $x_{p_t} = x'_t$ and $v_{p_t} = v'_t$ for $t \in \{1, 2, \cdots, k\}$. Set $X = X' \cup \{x'_0\}$ (where $x'_0 = x_0$).

Denote $J_X = \bigcup_{t=1}^{k} C[x'_t, v'_{t+1}]$, $K_X = V(G) \setminus J_X$.

Lemma 2[6] $X_M \in I_{m+1}(G)$, $X \in I_{k+1}(G)$, $K_X \subseteq S_0(X) \cup S_1(X)$, and $K_X \cap N_0(X) = \{x'_0\}$.

Lemma 3[5] $X_M \in I_{m+1}(G^*)$, therefore $X \in I_{k+1}(G^*)$.

A segment $C[z_1, z_2)$ ($\subseteq C[x'_t, v'_{t+1}]$, $t \in \{1, 2, \cdots, k\}$) is called a CX-segment if

① $C(z_1, z_2) \cap S_0(X) = \varnothing$;

② $z_1 \in N_2(X) \cup X$, $z_2 \in S_0(X) \cup \{v'^{+}_{t+1}\}$.

A CX-segment $C[z_1, z_2)$ is said to be simple if $C(z_1, z_2) \subseteq S_1(X)$.

Lemma 4[6] Let $C[z_1, z_2)$ ($\subseteq C[x'_t, v'_{t+1}]$, $t \in \{1, 2, \cdots, k\}$) be a CX-segment. If $L_i = N(x'_i) \cap C(z_1, z_2)$ ($i \in \{0, 1, \cdots, k\}$), then

$$L_t, L_{t-1}, \cdots, L_1, L_k, L_{k-1}, \cdots, L_{t+1}, L_0$$

(some of them may be empty) form consecutive subpaths of $C(z_1, z_2)$ which can only have their endvertices in common, and $|L_i| \leqslant 1$ for $i \in \{0, 1, \cdots, k\} \setminus \{t\}$.

3 Other Lemmas

Lemma 5 Let G be a claw-free graph. Then ① $G^* = G^2$[3]; ② $V(G) = S_0(X) \cup S_1(X)$[7]. So each CX-segment on J_X is simple.

We always assume that b is an integer ($0 < b < k+1$); $X_i = \{x'_i, x'_{i-1}, \cdots, x'_{i-(b-1)}\}$ ($\subseteq X$) (for $i \in \{0, 1, \cdots, k\}$, and the subscriptions of x'_j's will be taken modulo $k+1$).

Let $U \subseteq V(G)$. We always set

$$\sigma_b(U, X) = \sum_{i=0}^{k} |N(X_i) \cap U|$$

$$\sigma_b(X) = \sigma_b(V(G), X) = \sum_{i=0}^{k} |N(X_i)|$$

Lemma 6 ① If $w \in S_1(X)$, then $\sigma_b(\{w\}, X)$

$= b$; ② $\sigma_b(K_X, X) \leqslant b(|K_X| - 1 - |\bigcup_{l>2}(N_l(X) \cap K_X)|)$; ③ Let G be a claw-free graph, and λ be the number of CX-segments on C, then

$$\sigma_b(J_X, X) = b(|J_X| - \lambda - |\bigcup_{l>2}(N_l(X) \cap J_X)|)$$

Proof By the definition of $\sigma_b(Y)$ and lemma 2, it is not difficult to check that ① and ② hold.

We first prove two results.

1) Let $C[z_1, z_2)$ ($\subseteq C[x'_t, v'_{t+1}]$, $t \in \{1, 2, \cdots, k\}$) be a CX-segment, then

$$\sigma_b(C[z_1, z_2), X) = b(|C[z_1, z_2)| - 1)$$

In fact by lemma 5②, $C[z_1, z_2)$ is a simple CX-segment, so for $w \in C(z_1, z_2)$, $w \in S_1(X)$; and $z_1 \in S_0(X)$. Thus by ①,

$$\sigma_b(C[z_1, z_2), X) = \sum_{w \in C(z_1, z_2)} b = b(|C[z_1, z_2)| - 1)$$

so 1) holds.

2) If there are λ_t CX-segments on $C[x'_t, v'_{t+1}]$, then

$$\sigma_b(C[x'_t, v'_{t+1}], X) = b(|C[x'_t, v'_{t+1}]| - \lambda_t - |\bigcup_{l>2}(N_l(X) \cap C[x'_t, v'_{t+1}])|)$$

In fact for $t \in \{1, 2, \cdots, k\}$, divide

$$C[x'_t, v'_{t+1}] \setminus (\bigcup_{l>2}(N_l(X) \cap C[x'_t, v'_{t+1}])$$

into λ_t CX-segments

$$C[z_{11}^{(t)}, z_{12}^{(t)}), C[z_{21}^{(t)}, z_{22}^{(t)}), \cdots, C[z_{\lambda_t 1}^{(t)}, z_{\lambda_t 2}^{(t)})$$

Thus by 1)

$$\sigma_b(C[x'_t, v'_{t+1}], X) = \sum_{j=1}^{\lambda_t} \sigma_b(C[z_{j1}^{(t)}, z_{j2}^{(t)}), X) = \sum_{j=1}^{\lambda_t} b(|C[z_{j1}^{(t)}, z_{j2}^{(t)})| - 1) = b(|C[x'_t, v'_{t+1}]| - \lambda_t - |\bigcup_{l>2}(N_l(X) \cap C[x'_t, v'_{t+1}])|)$$

so 2) holds.

Now we prove ③. Consider that $J_X = \bigcup_{t=1}^{k} C[x'_t, v'_{t+1}]$, and $\sum_{t=1}^{k} \lambda_t = \lambda$. Thus by 2), it is easy to see that

$$\sigma_b(J_X, X) = \sum_{t=1}^{k} \sigma_b(C[x'_t, v'_{t+1}], X) = \sum_{t=1}^{k} b(|C[x'_t, v'_{t+1}]| - \lambda_t - |\bigcup_{l>2}(N_l(X) \cap C[x'_t, v'_{t+1}])|) = b(|J_X| - \lambda - |\bigcup_{l>2}(N_l(X) \cap J_X)|)$$

so ③ holds.

Lemma 7 Let G be a claw-free graph, and λ be the number of CX-segments on C, then

$$\sigma_b(X) \leqslant b(n(X) - \lambda - 1) \leqslant b(n(X) - k - 1)$$

Proof Note that $V(G) = J_X \cup K_X$, and $\lambda \geqslant k$. Thus by lemma 6② and lemma 6③,

$$\sigma_b(X) = \sigma_b(J_X, X) + \sigma_b(K_X, X) \leqslant b(|J_X| - \lambda - |\bigcup_{l>2}(N_l(X) \cap J_X)|) + b(|K_X| - 1 - |\bigcup_{l>2}(N_l(X) \cap K_X)|) = b(n(X) - \lambda - 1) \leqslant b(n(X) - k - 1)$$

Now we involve a graph G' other than G. In order to distinguish the notation such as $N(U)$, $S_i(X)$, $N_j(X)$, K_X, $n(X)$, $\sigma_b(X)$ introduced for G, we will simply add a prime to the notation with respect to G'. For example, $N'(U)$, $S'_i(X)$, etc.

By the proof of theorems 9 and 10 in Ref.[8], we have the following two lemmas.

Lemma 8 Assume that G is a $(k+1)$-connected graph with $k \geqslant 2$, and there exists some $w \in V(G)$ such that $G' = G - \{w\}$ is non-Hamiltonian. Choose a cycle C of G' such that

① $|N'_C(w)|$ is maximum;

② Subject to ①, C is maximal.

Let H be a component of $G' - V(C)$, and $N'_C(H) = \{v_1, v_2, \cdots, v_m\}$ with the convention that $v_1, v_2, \cdots, v_m$ occur on C in the order of their indices. Set x_i as the first non-insertible vertex in $C(v_i, v_{i+1})$ for each $i \in \{1, 2, \cdots, m\}$. Let $X_M = \{x_0, x_1, \cdots, x_m\}$, where x_0 is an arbitrary vertex of H. Then $X_M \in I_{m+1}((G')^*)$, and there exists some $X \subseteq X_M$, such that $x_0 \in X$ and $X \in I_{k+1}(G^*)$.

Lemma 9 Assume that G is a $(k+1)$-connected graph with $k \geqslant 3$, and there is some $\{u_1, u_2\} \subseteq V(G)$, G contains no (u_1, u_2)-Hamiltonian-path. If there exists a (u_1, u_2)-path P such that

① $V(P) \supseteq N(u_2)$;

② Subject to ①, $|N_P(u_1)|$ is maximum;

③ Subject to ① and ②, P is maximal.

Let H be a component of $G - V(P)$. Denote by G' the resulting graph obtained from G by adding a new vertex w and two new edges u_1w, u_2w. Then

① In G', $C = P[u_1, u_2]wu_1$ is a maximal (choose the orientation of C agree with that of P), but not Hamiltonian cycle of G'; H is a component of $G' - V(C)$.

② Let $\{v_1, v_2, \cdots, v_m\} = N'_C(H) = N_P(H)$. Then $v_m \neq u_2$, there exists the first non-insertible vertex x_i in $C(v_i, v_{i+1})$ for $i \in \{1, 2, \cdots, m\}$, where $m \geqslant k + 1 \geqslant 4$; $X_M = \{x_0, x_1, \cdots, x_m\} \in I_{m+1}((G')^*)$, where x_0 is arbitrarily chosen in $V(H)$.

③ There exists $X \subseteq X_M$, such that $x_0 \in X$ and $X \in I_{k+1}(G^*)$.

4 Proofs of Theorems

Proof of Theorem 4 Suppose that graph G

satisfies the conditions but is non-Hamiltonian. Let C be the longest cycle of G, and H a component of $G-V(C)$. Since G is a k-connected graph with $k \geqslant 2$, $|N_C(H)| \geqslant k$. Let $\{v_1, v_2, \cdots, v_k\} \subseteq N_C(H)$ (where $k = m$). Thus by lemma 1, for each $i \in \{1, 2, \cdots, k\}$, choose x_i the first non-insertible vertex in $C(v_i, v_{i+1})$. Set $X = \{x_0, x_1, \cdots, x_k\}$ (where x_0 is an arbitrary vertex of H).

Note that G is a claw-free graph. By lemma 3 and lemma 5 ①, $X = \{x_0, x_1, \cdots, x_k\} \in I_{k+1}(G^*) = I_{k+1}(G^2)$. On the other hand, by lemma 7, we have

$$\sigma_b(X) = \sum_{i=0}^{k} |N(X_i)| \leqslant b(n(X) - k - 1)$$

a contradiction.

Proof of Theorem 5 Suppose that G satisfies the conditions but is not 1-hamiltonian. Then there exists a $w \in V(G)$ such that $G' = G - \{w\}$ is non-Hamiltonian. Since G is claw-free, G' is also claw-free. Choose a cycle C of G' such that

① $|N'_C(w)|$ is maximum;

② Subject to ①, C is maximal.

Let H be a component of $G' - V(C)$, and $N'_C(H) = \{v_1, v_2, \cdots, v_m\}$ with the convention that $v_1, v_2, \cdots, v_m$ occur on C in the order of their indices. Set x_i as the first non-insertible vertex in $C(v_i, v_{i+1})$ for each $i \in \{1, 2, \cdots, m\}$. Let $X_M = \{x_0, x_1, \cdots, x_m\}$, where x_0 is an arbitrary vertex of H. By lemma 8, there is $X \subseteq X_M$, such that $x_0 \in X$ and $X \in I_{k+1}(G^*)$.

Note that G is a claw-free graph. By lemma 5 ①, $X \in I_{k+1}(G^*) = I_{k+1}(G^2)$. On the other hand, since G and G' are claw-free graphs, $w \in S_0(X) \cup S_1(X)$ and $V(G') = S'_0(X) \cup S'_1(X)$. Set $\xi = 0$ if $w \in S_0(X)$, $\xi = 1$ if $w \in S_1(X)$; so $n'(X) + \xi \leqslant n(X)$. Thus by lemma 7, we have

$$\sigma_b(X) = \sum_{i=0}^{k} |N(X_i)| = \sum_{i=0}^{k} |N'(X_i)| + b\xi = \sigma'_b(X) + b\xi \leqslant b(n'(X) - k - 1 + \xi) \leqslant b(n(X) - k - 1)$$

a contradiction.

Proof of Theorem 6 Suppose that graph G satisfies the conditions but is not Hamiltonian-connected. Then there is some $\{u_1, u_2\} \subseteq V(G)$, G contains no (u_1, u_2)-Hamiltonian-path. By theorem 5, there is a Hamiltonian cycle C' in $G - u_2$. Choose an orientation of C', let $C'(u'_2, u_1) \cap N(u_2) = \varnothing$ and $u'_2 \in N_C(u_2)$. Then the (u_1, u_2)-path $C'[u_1, u'_2]u_2$ contains the set $N(u_2)$. Thus one can choose a (u_1, u_2)-path P such that

① $V(P) \supseteq N(u_2)$;

② Subject to ①, $|N_P(u_1)|$ is maximum;

③ Subject to ① and ②, P is maximal.

Let H be a component of $G - V(P)$. Add a new vertex w and two new edges u_1w, u_2w to G and denote by G' the resulting graph. By lemma 9①, $C = P[u_1, u_2]wu_1$ is a maximal cycle in G' (choose the orientation of C agree with that of P), but not Hamiltonian cycle of G'; H is a component of $G' - V(C)$. Let $\{v_1, v_2, \cdots, v_m\} = N'_C(H) = N_P(H)$. By lemma 9②, $v_m \neq u_2$, there exists the first non-insertible vertex x_i in $C(v_i, v_{i+1})$ for $i \in \{1, 2, \cdots, m\}$, where $m \geqslant k+1 \geqslant 4$; $X_M = \{x_0, x_1, \cdots, x_m\} \in I_{m+1}((G')^*)$, where x_0 is arbitrarily chosen in $V(H)$. By lemma 9 ③, there exists $X \subseteq X_M$, such that $x_0 \in X$ and $X \in I_{k+1}(G^*)$.

Note that G is a claw-free graph. By lemma 5 ①, $X \in I_{k+1}(G^*) = I_{k+1}(G^2)$. On the other hand, by the construction of G', $n'(X) \leqslant n(X) + 1$ and $w \in S'_0(X) \cup S'_1(X)$. Set $\xi = 0$ if $w \in S'_0(X)$, $\xi = 1$ if $w \in S'_1(X)$. Since $|N'_C(H)| \geqslant k+1$, it is easy to see that $\lambda' \geqslant k + 1 - \xi$ (where λ' is the number of CX-segments on J'_X). Thus by lemma 7, it is easy to see that

$$\sigma_b(X) = \sum_{i=0}^{k} |N(X_i)| = \sum_{i=0}^{k} |N'(X_i)| - \xi b = \sigma'_b(X) - \xi b \leqslant b(n'(X) - \lambda - 1 - \xi) \leqslant b(n(X) - (k + 1 - \xi) - \xi) = b(n(X) - k - 1)$$

a contradiction.

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哈密尔顿性、邻域并和无爪图的平方图

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摘要: 设 G 是一个图, G 的平方图 G^2 满足 $V(G^2)=V(G)$, $E(G^2)=E(G)\cup\{uv:\mathrm{dist}_G(u,v)=2\}$. 本文利用插点方法, 给出了关于 k 或 $(k+1)$-连通 $(k\geqslant 2)$ 无爪图 G 是哈密尔顿的、1-哈密尔顿的或哈密尔顿连通的统一证明. 其充分条件是 G 中关于 $\sum_{i=0}^{k}|N(Y_i)|$ 与 $n(Y)$ 的不等式, 这里 $Y=\{y_0,y_1,\cdots,y_k\}$ 是图 G^2 的任一独立集, 对于 $i\in\{0,1,\cdots,k\}$, $Y_i=\{y_i,y_{i-1},\cdots,y_{i-(b-1)}\}\subseteq Y$ (y_j 的下标将取模 $k+1$); b 是一个整数, 且 $0<b<k+1$; $n(Y)=|\{v\in V(G):\mathrm{dist}(v,Y)\leqslant 2\}|$.

关键词: 哈密尔顿性; 无爪图; 邻域并; 插点; 平方图

中图分类号: O157.5