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Fixed points on complete metric spaces

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Abstract: Two new fixed point theorems on two complete metric spaces are proved by using the concept of w-distance. One of the results is: let (X, d) and (Y, ρ) be two complete metric spaces, let p_1 be a w-distance on X and p_2 be a w-distance on Y. If T is a continuous mapping of X into Y and S is a mapping of Y into X, satisfying the inequalities: $p_1(STx, STx') \leqslant c\max\{p_1(x, x'), p_1(x, STx), p_1(x', STx'), p_1(x, STx')/2, p_2(Tx, Tx')\}$ and $p_2(TSy, TSy') \leqslant c\max\{p_2(y, y'), p_2(y, TSy), p_2(y', TSy'), p_2(y, TSy')/2, p_1(Sy, Sy')\}$ for all x, x' in X and y, y' in Y, where $0 \leqslant c < 1$. We have proved that ST has a unique fixed point z in X and TS has a unique fixed point w in Y. The two theorems have improved the fixed point theorems of Fisher and Namdeo, et al.

Key words: fixed point; complete metric space; w-distance

1 Preliminaries

Definition 1[1] ① A real-valued function f defined on a metric space X is said to be lower semicontinuous at a point t in X if either $\liminf\limits_{x\to t} f(x) = +\infty$ or $\liminf\limits_{x\to t} f(x) \geqslant f(t)$; ② A real-valued function f defined on a metric space X is said to be upper semicontinuous at a point t in X if either $\limsup\limits_{x\to t} f(x) = -\infty$ or $\limsup\limits_{x\to t} f(x) \leqslant f(t)$.

Definition 2[2] Let X be a metric space with a metric d. Then a function $p: X \times X \to [0, \infty)$ is called a w-distance on X if the following are satisfied: ① $p(x, z) \leqslant p(x, y) + p(y, z)$ for any $x, y, z \in X$; ② for any $x \in X$, $p(x, \cdot): X \to [0, \infty)$ is lower semicontinuous; ③ For any $\varepsilon > 0$, there exists $\delta > 0$ such that $p(z, x) \leqslant \delta$ and $p(z, y) \leqslant \delta$ imply $d(x, y) \leqslant \varepsilon$.

Example 1[2] Let X be a metric space with a metric d. Then $p = d$ is a w-distance on X.

Example 2[2] Let X be a metric space and let T be a continuous mapping from X into itself. Then a function $p: X \times X \to [0, \infty)$ defined by $p(x, y) = \max\{d(Tx, y), d(Tx, Ty)\}$ for every $x, y \in X$ is a w-distance on X.

Example 3[1] Let $X = R$ be a metric space with the usual metric. Then a function $p: X \times X \to [0, \infty)$ defined by $p(x,y) = \max\left\{\left|\frac{1}{2}x - y\right|, \frac{1}{2}|x - y|\right\}$ for every $x, y \in X$ is a w-distance on X.

Lemma 1[2] Let X be a metric space with a metric d and let p be a w-distance on X. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X, let $\{\alpha_n\}$ and $\{\beta_n\}$ be two sequences in $[0, \infty)$ converging to 0, and let $x, y, z \in X$. Then the following hold:

① If $p(x_n, y) \leqslant \alpha_n$ and $p(x_n, z) \leqslant \beta_n$ for any $n \in \mathbf{N}$, then $y = z$. In particular, if $p(x, y) = 0$ and $p(x, z) = 0$, then $y = z$.

② If $p(x_n, y_n) \leqslant \alpha_n$ and $p(x_n, z) \leqslant \beta_n$ for any $n \in \mathbf{N}$, then $\{y_n\}$ converges to z.

③ If $p(x_n, x_m) \leqslant \alpha_n$, for any $n, m \in \mathbf{N}$ with $m > n$, then $\{x_n\}$ is a Cauchy sequence.

④ If $p(y, x_n) \leqslant \alpha_n$ for any $n \in \mathbf{N}$, then $\{x_n\}$ is a Cauchy sequence.

Fisher, et al. proved fixed theorems in Refs. [3 − 5]. The following related fixed point theorem was proved in Ref. [3].

Theorem 1 Let (X, d) and (Y, ρ) be two complete metric spaces. If T is a continuous mapping of X into Y and S is a mapping of Y into X satisfying the inequalities:

$$d(STx, STx') \leqslant c\max\{d(x, x'), d(x, STx), d(x', STx'), \rho(Tx, Tx')\}$$
$$\rho(TSy, TSy') \leqslant c\max\{\rho(y, y'), \rho(y, TSy), \rho(y', TSy'), d(Sy, Sy')\}$$

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for all x, x' in X and y, y' in Y, where $0 \leqslant c < 1$. Then ST has a unique fixed point z in X and TS has a unique fixed point w in Y. Further, $Tz = w$ and $Sw = z$.

The following theorem is a generalization of theorem 1.

2 Main Results

Theorem 2 Let (X, d) and (Y, ρ) be two complete metric spaces. Let p_1 be a w-distance on X, p_2 be a w-distance on Y. If T is a continuous mapping of X into Y and S is a mapping of Y into X, satisfying the inequalities:

$$p_1(STx, STx') \leqslant c\max\{p_1(x, x'), p_1(x, STx), p_1(x', STx'), p_1(x, STx')/2, p_2(Tx, Tx')\} \quad (1)$$

$$p_2(TSy, TSy') \leqslant c\max\{p_2(y, y'), p_2(y, TSy), p_2(y', TSy'), p_2(y, TSy')/2, p_1(Sy, Sy')\} \quad (2)$$

for all x, x' in X and y, y' in Y, where $0 \leqslant c < 1$. Then

① For each $x \in X$, $\{(ST)^n x = x_n\}$ is a Cauchy sequence, $\{T(ST)^{n-1} x = y_n\}$ is a Cauchy sequence.

② Assume that $\inf\{p_1(x, u) + p_1(x, STx) : x \in X\} > 0$ for every $u \in X$ with $u \neq STu$. Then ST has a unique fixed point z in X.

Assume that $\inf\{p_2(y, v) + p_2(y, TSy) : y \in Y\} > 0$ for every $v \in Y$ with $v \neq TSv$. Then TS has a unique fixed point w in Y. Further, $Tz = w$ and $Sw = z$.

Proof ① Let x be an arbitrary point in X. We define the sequences $\{x_n\}$ in X and $\{y_n\}$ in Y by

$$(ST)^n x = x_n,\ T(ST)^{n-1} x = y_n$$

for every $n \in \mathbf{N}$. By (1), we obtain

$$\begin{aligned} p_1(x_n, x_{n+1}) &= p_1(STx_{n-1}, STx_n) \leqslant \\ &c\max\{p_1(x_{n-1}, x_n), p_1(x_{n-1}, x_n), p_1(x_n, x_{n+1}), p_1(x_{n-1}, x_{n+1})/2, p_2(y_n, y_{n+1})\} \leqslant \\ &c\max\{p_1(x_{n-1}, x_n), p_1(x_n, x_{n+1}), [p_1(x_{n-1}, x_n) + p_1(x_n, x_{n+1})]/2, p_2(y_n, y_{n+1})\} = \\ &c\max\{p_1(x_{n-1}, x_n), p_2(y_n, y_{n+1})\} \end{aligned} \quad (3)$$

Similarly, by (2), we obtain

$$p_2(y_n, y_{n+1}) \leqslant c\max\{p_2(y_{n-1}, y_n), p_1(x_{n-1}, x_n)\} \quad (4)$$

Let $M = \max\{p_1(x, x_1), p_2(y_1, y_2)\}$, it easily follows by induction that

$$p_1(x_n, x_{n+1}) \leqslant c^n \max\{p_1(x, x_1), p_2(y_1, y_2)\} = c^n M \quad (5)$$

$$p_2(y_n, y_{n+1}) \leqslant c^{n-1} \max\{p_1(x, x_1), p_2(y_1, y_2)\} = c^{n-1} M \quad (6)$$

for $n = 1, 2, \cdots$. If $n < m$, then (5) and (6) imply that

$$\begin{aligned} p_1(x_n, x_m) &\leqslant p_1(x_n, x_{n+1}) + p_1(x_{n+1}, x_{n+2}) + \cdots + p_1(x_{m-1}, x_m) \leqslant \\ &(c^n + c^{n+1} + \cdots + c^{m-1})M \leqslant c^n (1-c)^{-1} M \end{aligned} \quad (7)$$

$$\begin{aligned} p_2(y_n, y_m) &\leqslant p_2(y_n, y_{n+1}) + p_2(y_{n+1}, y_{n+2}) + \cdots + p_2(y_{m-1}, y_m) \leqslant \\ &(c^{n-1} + c^n + \cdots + c^{m-2})M \leqslant c^{n-1} (1-c)^{-1} M \end{aligned} \quad (8)$$

for $n = 1, 2, \cdots$. Since $0 \leqslant c < 1$, by lemma 1, $\{x_n\}$ is a Cauchy sequence with a limit z in X and $\{y_n\}$ is a Cauchy sequence with a limit w in Y. This completes the proof of ①.

② From (7) and definition 1, we have

$$p_1(x_n, z) \leqslant \liminf_{m\to\infty} p_1(x_n, x_m) \leqslant c^n (1-c)^{-1} M \quad (9)$$

Assume $z \neq STz$, then by hypotheses (5) and (9), we have

$$\begin{aligned} 0 &< \inf\{p_1(x, z) + p_1(x, STx) : x \in X\} \leqslant \inf\{p_1(x_n, z) + p_1(x_n, STx_n) : n \in \mathbf{N}\} = \\ &\inf\{p_1(x_n, z) + p_1(x_n, x_{n+1}) : n \in \mathbf{N}\} \leqslant \inf\{c^n (1-c)^{-1} M + c^n M : n \in \mathbf{N}\} = \\ &\inf\{(2-c) c^n (1-c)^{-1} M : n \in \mathbf{N}\} = 0 \end{aligned}$$

This is a contradiction. Therefore we have $STz = z$.

From (8) and definition 1, we have

$$p_2(y_n, w) \leqslant \liminf_{m\to\infty} p_2(y_n, y_m) \leqslant c^{n-1} (1-c)^{-1} M \quad (10)$$

Assume $w \neq TSw$, then by hypotheses (6) and (10), we have

$$\begin{aligned} 0 &< \inf\{p_2(y, w) + p_2(y, TSy) : y \in Y\} \leqslant \inf\{p_2(y_n, w) + p_2(y_n, TSy_n) : n \in \mathbf{N}\} = \\ &\inf\{p_2(y_n, w) + p_2(y_n, y_{n+1}) : n \in \mathbf{N}\} \leqslant \inf\{(2-c) c^{n-1} (1-c)^{-1} M : n \in \mathbf{N}\} = 0 \end{aligned}$$

This is a contradiction. Therefore we have $TSw = w$.

Since T is continuous, we have

$$w = \lim_{n\to\infty} y_n = \lim_{n\to\infty} Tx_{n-1} = Tz$$

Further $Sw = STz = z$.

To prove the uniqueness of the fixed point, suppose that ST has a second fixed point z' and TS has a second fixed point w'. Applying inequality (1), we have

$$p_1(z, z) = p_1(STz, STz) \leqslant c\max\{p_1(z, z), p_1(z, STz), p_1(z, STz), p_1(z, z)/2, p_2(Tz, Tz)\} = c\max\{p_1(z, z), P_2(Tz, Tz)\}$$

and so either

$$p_1(z, z) = 0 \text{ or } p_1(z, z) \leqslant cp_2(Tz, Tz) \tag{11}$$

Applying inequality (2), we have

$$p_2(Tz, Tz) = p_2(TSTz, TSTz) \leqslant c\max\{p_2(Tz, Tz), p_2(Tz, TSTz), p_2(Tz, TSTz), p_2(Tz, TSTz)/2, p_1(STz, STz)\} = c\max\{p_2(Tz, Tz), p_1(z, z)\}$$

and so either

$$p_2(Tz, Tz) = 0 \text{ or } p_2(Tz, Tz) \leqslant cp_1(z, z) \tag{12}$$

By (11) and (12), it follows that

$$p_1(z, z) = 0, \ p_2(Tz, Tz) = 0 \tag{13}$$

Similarly, applying inequality (1) and (2), we get

$$p_1(z', z') = 0, \ p_2(Tz', Tz') = 0 \tag{14}$$

Applying (1), (13) and (14), we get

$$p_1(z, z') = p_1(STz, STz') \leqslant c\max\{p_1(z, z'), p_1(z, STz), p_1(z', STz'), p_1(z, z')/2, p_2(Tz, Tz')\} = c\max\{p_1(z, z'), p_2(Tz, Tz')\}$$

and so either

$$p_1(z, z') = 0 \text{ or } p_1(z, z') \leqslant cp_2(Tz, Tz') \tag{15}$$

Applying (2), (13) and (14), we get

$$p_2(Tz, Tz') = p_2(TSTz, TSTz') \leqslant c\max\{p_2(Tz, Tz'), p_2(Tz, TSTz), p_2(Tz', TSTz'), p_2(Tz, TSTz')/2, p_1(STz, STz')\} = c\max\{p_2(Tz, Tz'), p_1(z, z')\}$$

and so either

$$p_2(Tz, Tz') = 0 \text{ or } p_2(Tz, Tz') \leqslant cp_1(z, z') \tag{16}$$

By (15) and (16), it follows that

$$p_1(z, z') = 0, \ p_2(Tz, Tz') = 0 \tag{17}$$

Thus we obtain $p_1(z, z) = p_1(z, z') = 0$.

By lemma 1 we have $z = z'$, proving that z is the unique fixed point of ST.

Now $TSw' = w'$ implies that $STSw' = Sw'$ and so $Sw' = z$. Thus $w = Tz = TSw' = w'$, which proves that w is the unique fixed point of TS. **This completes the proof of the theorem.**

Example 4 Let $X = R = Y$ be complete metric spaces with the usual metric $d(x,y) = |x - y|$. Define mappings $S, T: X \to X$ by

$$Sx = -1 \text{ for } -\infty < x \leqslant 0, \ \mathrm{Sx} = \mathrm{x} \text{ for } 0 < x < +\infty, , \ Tx = x \text{ for } -\infty < x \leqslant 0, \ Tx = 0 \text{ for } 0 < x < +\infty$$

respectively.

Let $p_1: X \times X \to [0, +\infty)$ be a mapping such that $p_1(x,y) = \max\{d(Tx,y), d(Tx,Ty)\}$ for every $x, y \in X$.

Let $p_2: Y \times Y \to [0, +\infty)$ be a mapping such that $p_2(x,y) = \max\left\{\left|\frac{1}{2}x - y\right|, \frac{1}{2}|x - y|\right\}$ for every $x, y \in X$.

From examples 2 and 3, we know p_1 is a w-distance on X, p_2 is a w-distance on Y. Then, clearly p_1, p_2, S and T satisfy all conditions in theorem 2, ST has a unique fixed point -1 in X and TS has a unique fixed point -1 in Y.

Corollary 1 Let (X, d) be a complete metric space and p be a w-distance on X. Let T be a continuous mapping of X into itself. Suppose that there exists $c \in [0, 1)$ such that

$$p(T^2x, T^2x') \leqslant c\max\{p(x, x'), p(x, T^2x), p(x', T^2x')\}$$

for every $x, x' \in X$ and that

$$\inf\{p(x, u) + p(x, T^2x) : x \in X\} > 0$$

for every $u \in X$ with $u \neq T^2u$. Then z is the unique common fixed point of T^2 and T.

Proof Applying lemma 1 and by the method similar to the proof of theorem 1, T^2 has a unique fixed point z. Then $T^2(Tz) = T(T^2 z) = Tz$, so we see that Tz is also a fixed point of T^2. Since the fixed point is unique, we must have $Tz = z$.

Theorem 3 Let (X, d) and (Y, ρ) be two complete metric spaces. Let p_1 be a w-distance on X and p_2 be a w-distance on Y. If T is a continuous mapping of X into Y and S is a mapping of Y into X satisfying the inequalities

$$p_1(sy, sy')p_1(STx, STx') < c\max\{p_1(sy, sy')p_2(Tx, Tx'), [p_1(sy, x')]^2, p_1(x, x')p_1(sy, sy'), p_1(sy, STx)p_1(sy', STx')\} \tag{18}$$

$$p_2(Tx, Tx')p_2(TSy, TSy') < c\max\{p_1(sy, sy')p_2(Tx, Tx'), [p_2(Tx, y')]^2, p_2(y, y')p_2(Tx, Tx'), p_2(Tx, TSy)p_2(Tx', TSy')\} \tag{19}$$

for all $x, x' \in X$ and $y, y' \in Y$, where $0 < c < 1$. Then

① For each $x \in X$, both $\{(ST)^n x = x_n\}$ and $\{T(ST)^{n-1}x = y_n\}$ are Cauchy sequences;

② Assume that $\inf\{p_1(x, u) + p_1(x, STx): x \in X\} > 0$ for every $u \in X$ with $u \neq STu$. Then ST has a fixed point z in X.

Assume that $\inf\{p_2(y, v) + p_2(y, TSy): y \in Y\} > 0$ for every $v \in Y$ with $v \neq TSv$. Then TS has a fixed point w in Y. Further, $Tz = w$ and $Sw = z$.

Proof ① Let x be an arbitrary point in X. Define the sequences $\{x_n\}$ in X and $\{y_n\}$ in Y by

$$(ST)^n x = x_n,\ T(ST)^{n-1}x = y_n$$

for every $n \in \mathbf{N}$. Applying inequality (18), we get

$$p_1(x_{n-1}, x_n)p_1(x_n, x_{n+1}) = p_1(sy_{n-1}, sy_n)p_1(STx_{n-1}, STx_n) < c\max\{p_1(x_{n-1}, x_n)p_2(y_n, y_{n+1}), [p_1(x_{n-1}, x_n)]^2, [p_1(x_{n-1}, x_n)]^2, p_1(x_{n-1}, x_n)p_1(x_n, x_{n+1})\}$$

from which it follows that

$$p_1(x_n, x_{n+1}) < c\max\{p_1(x_{n-1}, x_n), p_2(y_n, y_{n+1})\} \tag{20}$$

Applying inequality (19), we get

$$p_2(y_{n-1}, y_n)p_2(y_n, y_{n+1}) = p_2(Tx_{n-2}, Tx_{n-1})p_2(TSy_{n-1}, TSy_n) < c\max\{p_1(x_{n-1}, x_n)p_2(y_{n-1}, y_n), [p_2(y_{n-1}, y_n)]^2, [p_2(y_{n-1}, y_n)]^2, p_2(y_{n-1}, y_n)p_2(y_n, y_{n+1})\}$$

from which it follows that

$$p_2(y_n, y_{n+1}) < c\max\{p_1(x_{n-1}, x_n), p_2(y_{n-1}, y_n)\} \tag{21}$$

Let $M = \max\{p_1(x, x_1), p_2(y_1, y_2)\}$, it easily follows by induction that

$$p_1(x_n, x_{n+1}) < c^n \max\{p_1(x, x_1), p_2(y_1, y_2)\} = c^n M \tag{22}$$

$$p_2(y_n, y_{n+1}) < c^{n-1} \max\{p_1(x, x_1), p_2(y_1, y_2)\} = c^{n-1} M \tag{23}$$

for $n = 1, 2, \cdots$. If $n < m$, then by (22) and (23)

$$p_1(x_n, x_m) \leqslant p_1(x_n, x_{n+1}) + p_1(x_{n+1}, x_{n+2}) + \cdots + p_1(x_{m-1}, x_m) < (c^n + c^{n+1} + \cdots + c^{m-1})M \leqslant c^n(1-c)^{-1}M \tag{24}$$

$$p_2(y_n, y_m) \leqslant p_2(y_n, y_{n+1}) + p_2(y_{n+1}, y_{n+2}) + \cdots + p_2(y_{m-1}, y_m) < (c^{n-1} + c^n + \cdots + c^{m-2})M \leqslant c^{n-1}(1-c)^{-1}M \tag{25}$$

for $n = 1, 2, \cdots$. Since $0 < c < 1$, by lemma 1, $\{x_n\}$ is a Cauchy sequence with a limit z in X and $\{y_n\}$ is a Cauchy sequence with a limit w in Y. This is the proof of ①.

② From (24) and definition 1, we have

$$p_1(x_n, z) \leqslant \lim_{m\to\infty} p_1(x_n, x_m) \leqslant c^n(1-c)^{-1}M \tag{26}$$

Assume $z \neq STz$, then by hypotheses (22) and (26), we have

$$0 < \inf\{p_1(x, z) + p_1(x, STx): x \in X\} \leqslant \inf\{p_1(x_n, z) + p_1(x_n, STx_n): n \in \mathbf{N}\} \leqslant \inf\{c^n(1-c)^{-1}M + c^n M: n \in \mathbf{N}\} = \inf\{(2-c)c^n(1-c)^{-1}M: n \in \mathbf{N}\} = 0$$

This is a contradiction. Therefore we have $STz = z$.

From (25) and definition 1, we have

$$p_2(y_n, w) \leqslant \lim_{m\to\infty} p_2(y_n, y_m) \leqslant c^{n-1}(1-c)^{-1}M \tag{27}$$

Assume $w \neq TSw$, then by hypothesis (23) and (27), we have

$$0<\inf\{p_2(y, w)+p_2(y, TSy): y\in Y\}\leqslant\inf\{p_2(y_n, w)+p_2(y_n, TSy_n): n\in\mathbf{N}\}\leqslant$$
$$\inf\{c^{n-1}(1-c)^{-1}M+c^{n-1}M: n\in\mathbf{N}\}=\inf\{(2-c)c^{n-1}(1-c)^{-1}M: n\in\mathbf{N}\}=0$$

This is a contradiction. Therefore we have $TSw=w$.

By using the continuity of T, we now have $w=\lim_{n\to\infty}y_n=\lim_{n\to\infty}Tx_{n-1}=Tz$.

Further, $Sw=STz=z$. This completes the proof of the theorem.

Corollary 2 Let (X, d) be a complete metric space and let p be a w-distance on X. Let T be a mapping of X into itself. Suppose that there exists $c\in(0, 1)$ such that

$$p(Ty, Ty')p(T^2x, T^2x')<c\max\{[p(Ty, x')]^2, p(x, x')p(Ty, Ty'), p(Ty, T^2x)p(Ty', T^2x')\}$$

for all x, x', y, y' in X and that

$$\inf\{p(x, u)+p(x, T^2x): x\in X\}>0$$

for every $u\in X$ with $u\neq T^2u$. Then there exists $z\in X$ such that $z=T^2z$.

Proof By the method similar to the proof of theorem 2, the results follow.

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完备度量空间中的不动点

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摘要: 使用 w −距离概念, 证明了在 2 个完备度量空间中 2 个新的不动点定理, 其中之一的结果为: 设(X, d)和(Y, ρ)是 2 个完备度量空间, 设 p_1 是 X 上 w-距离和 p_2 是 Y 上 w-距离. 如果 T 是一个从 X 到 Y 的连续映射和 S 是一个从 Y 到 X 的映射, 对 X 中所有 x, x' 和 Y 中所有 y, y'以及$o<c<1$, 满足不等式 $p_1(STx, STx')\leqslant c\max\{p_1(x, x'), p_1(x, STx), p_1(x', STx'), p_1(x, STx')/2, p_2(Tx, Tx')\}$和 $p_2(TSy, TSy')\leqslant c\max\{p_2(y, y'), p_2(y, TSy), p_2(y', TSy'), p_2(y, TSy')/2, p_1(Sy, Sy')\}$. 证明了 ST 在 X 中有惟一不动点 z 和 TS 在 Y 中有惟一不动点 w. 这 2 个定理推广了 Fisher 和 Namdeo 等人的不动点定理.

关键词: 不动点; 完备度量空间; w-距离

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