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New method to compute the throughput capacity of HDR wireless networks

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Abstract: A new non-parameter method is proposed to compute the throughput capacity region of high data rate (HDR) wireless networks. We first transform the task of computing the throughput capacity region into a mathematical optimization problem without introducing any additional parameters. By using a greedy algorithm to solve the optimization problem, the non-parametric characterization of the throughput capacity region of HDR can be obtained. By using the new non-parameter method, the HDR throughput capacity region can be characterized by at most $N(M^2 - M + 1)^{N-1}$ linear constraints where N is the user number and M is the rate set size. The correctness of the new method is verified by several numerical examples.

Key words: capacity region; high data rate (HDR); resource allocation

Compared with wired networks, the time-varying nature of wireless networks makes providing reliable multimedia services a challenging problem. Due to path loss, shadowing and multipath fading, the strength of the received signal can fluctuate on the order of tens of decibels. To combat these effects, resources such as power, rate and slot should be dynamically allocated.

This paper will study the high data rate (HDR) wireless communication system, also known as IS-856 or 1x EV-DO[1,2]. On the HDR downlink, time is divided into fixed-size slots of 1.67 ms. In each slot, the base station (BS) can serve only one user. Each user constantly reports to the BS its “instantaneous” rate, at which data can be transmitted to the user if it is scheduled to transmit in the current slot. The date rate can be chosen from a finite set. A scheduler at the BS picks the next user to be served based on the reported data rate from the user.

Most of the existing work on HDR has been done to design specific wireless scheduling algorithms[1,2]. In this paper, we study HDR at a more fundamental level: what is the capacity region of an HDR system and how to compute it?

In Refs. [3, 4], the problem of computing the capacity region of the fading multi-access/broadcast channel is addressed, and the parametric representation of the boundary of the capacity region is obtained. Since HDR is a time-division (TD) system, the results presented in Refs. [3, 4] can therefore be directly used to compute its capacity region. But, the resulting parametric equations are not convenient to solve problems like call admission control (CAC). On the other hand, HDR is a special TD system (e.g., adaptive link and no power control), which can make computing the HDR capacity region from a new point of view, which is different from that of Refs.[3, 4] possible. In this paper, we will derive a new complete characterization of the HDR capacity region, which is more useful in practice than that of Refs. [3, 4] when dealing with problems like CAC. The main result of this paper is that the capacity region of HDR can be characterized by at most $N(M^2 - M + 1)^{N-1}$ linear constraints where N is the user number and M is the rate set size.

By Refs. [3, 4], at least two types of capacity region can be defined for fading channels: the throughput capacity and delay-limited capacity. This paper studies the throughput capacity.

1 Assumptions and System Model

Consider an adaptive time-division downlink consisting of N mobile users. Suppose that the system supports M rates and the rate set is $\{R_m, 1 \leqslant m \leqslant M\}$. We assume that $R_1 < R_2 < \cdots < R_M$. Let $R_{S_i[t]}$ be the feasible rate for user i $(1 \leqslant i \leqslant N)$ in slot t, where S_i is a random process taking on the values from $\{1, 2, \cdots, M\}$. Let $p_i(m)$ be the stationary probability that S_i is m $(1 \leqslant m \leqslant M)$. Further, we call $\boldsymbol{S} = \{S_1, \cdots, S_i, \cdots, S_N\}$ the joint channel process and denote Ξ as the set of all possible joint channel states. Let $p(\boldsymbol{s})$ be the stationary probability that the joint channel state is $\boldsymbol{s} \in \Xi$.

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In such a system, the time slot is the only resource to be allocated. For a time allocation policy τ, let $\tau_i(\boldsymbol{s})$ $(0 \leqslant \tau_i(\boldsymbol{s}) \leqslant 1)$ be the fraction of transmission time allocated to user i $(1 \leqslant i \leqslant N)$ given the joint channel state $\boldsymbol{s} = \{s_1, \cdots, s_i, \cdots, s_N\}$ for users. All feasible time allocation policies must satisfy

$$\sum_{i=1}^{N} \tau_i(\boldsymbol{s}) = 1 \qquad \forall \boldsymbol{s} \in \Xi \tag{1}$$

For a given time allocation strategy τ, the achievable throughput of user i $(1 \leqslant i \leqslant N)$ is

$$c_i = E_S[\tau_i(\boldsymbol{S}) R_{s_i}] = \sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) R_{s_i} \tag{2}$$

where $E[\cdot]$ denotes the expectation function and s_i is the i-th element of vector $\boldsymbol{s}$. Let us define

$$c_i^{\max} \triangleq \sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) R_{s_i} = \sum_{m=1}^{M} p_i(m) R_m \tag{3}$$

then the range of $c_i (1 \leqslant i \leqslant N)$ is $0 \leqslant c_i \leqslant c_i^{\max}$.

2 Conventional Method

In Ref. [4], the capacity region for the general TD fading broadcast channel is obtained. Since HDR is a TD system, we can directly apply the results of Ref. [4] to computing the HDR capacity region. Let Σ be the capacity region of HDR and Σ^* the boundary of the capacity region. By Ref. [4], Σ is convex. Therefore, if a throughput vector $\boldsymbol{c}^*$ is the solution to the optimization problem $\max\limits_{\boldsymbol{c} \in \Sigma} \boldsymbol{\mu} \cdot \boldsymbol{c}$, it will be on the boundary Σ^*, where $\boldsymbol{\mu} = \{\mu_1, \mu_2, \cdots, \mu_N\}$, $\mu_i \geqslant 0 (1 \leqslant i \leqslant N)$, and $\mu_1 + \mu_2 + \cdots + \mu_N = 1$. After some derivations similar to Ref. [4], the boundary of the capacity region can be characterized by the following parametrically defined surface:

$$\Sigma^* = \left\{ \boldsymbol{c}^*(\boldsymbol{\mu}) : \boldsymbol{\mu} \in \mathbf{R}_+^N, \sum_{i=1}^{N} \mu_i = 1 \right\} \tag{4}$$

where for $i = 1, 2, \cdots, N$,

$$c_i^*(\boldsymbol{\mu}) = \sum_{m=1}^{M} R_m \prod_{1 \leqslant k \leqslant N,\, k \neq i} \left(\sum_{j:\, R_j \leqslant (\mu_i/\mu_k) R_m} p_k(j) \right) p_i(m) \tag{5}$$

3 New Method

3.1 Motivation

The above method is based on the property that the capacity region is convex. Due to the convexity, the boundary of the capacity region can be parametrically expressed as (4), and the consequent task is to derive the analytical expression of $\boldsymbol{c}^*(\boldsymbol{\mu})$ for any feasible $\boldsymbol{\mu}$. Since this method introduces a new parameter variable $\boldsymbol{\mu}$ to help characterize the capacity region, it is called the parameter method in this paper.

However, the parameter method has some limitations. Although introducing $\boldsymbol{\mu}$ can help characterize Σ(and Σ^*), the resulting parametric expression like (4) is not convenient to be used for some problems. Take CAC as an example. A typical CAC problem is to determine if a given target rate vector $\boldsymbol{c} \in \Sigma$. In the case of using the parameter method, this problem is equivalent to checking if there exists a parameter $\boldsymbol{\mu}^*$ such that the inequalities $\boldsymbol{c} \leqslant \boldsymbol{c}^*(\boldsymbol{\mu}^*)$ can be satisfied. The corresponding computing process can be rather complicated (see Ref. [5] for an example).

To overcome such limitations, this paper will develop a new method which characterizes Σ in a non-parametric form like $\Sigma = \{\boldsymbol{c} \mid f(\boldsymbol{c}) \leqslant 0\}$. Since not introducing any additional parameter variable, it can be expected that the new method is more convenient than the conventional parameter method when dealing with problems like CAC. In this paper, we call this new method the non-parameter method, and will derive the non-parametric representation of Σ for the special case of HDR in this section.

3.2 Problem formulation

To derive the non-parametric representation of Σ, the key observation is that the convexity of Σ can be interpreted from a new point of view as follows. Given a point $\boldsymbol{c} = \{c_1, \cdots, c_{n-1}, c_n, c_{n+1}, \cdots, c_N\}$, we can define N lines $\{L_n, 1 \leqslant n \leqslant N\}$ through point $\boldsymbol{c}$ in which L_n: $x_i = c_i$, $1 \leqslant i \leqslant N$, $i \neq n$. Let the intersection point of line L_n and the boundary Σ^* be $\boldsymbol{p}_n = \{c_1, \cdots, c_{n-1}, c_n^*, c_{n+1}, \cdots, c_N\}$. Then, $\boldsymbol{c} \in \Sigma$ is equivalent to checking if

$c_n \leqslant c_n^*$ for all n. Therefore, the capacity region can be expressed as $\Sigma = \{\boldsymbol{c} \mid c_n \leqslant c_n^*,\ 1 \leqslant n \leqslant N\}$. By definition, c_n^* represents the maximum rate that user n can achieve given rates of all other $N-1$ users. Then, c_n^* is some function of $c_i(1 \leqslant i \leqslant N,\ i \neq n)$ and can be denoted as

$$c_n^* \triangleq f_n(c_1, \cdots, c_{n-1}, c_{n+1}, \cdots, c_N) \tag{6}$$

Hence, the capacity region can be rewritten as

$$\Sigma = \{\boldsymbol{c}:\ c_n \leqslant f_n(c_1, \cdots, c_{n-1}, c_{n+1}, \cdots, c_N),\ 1 \leqslant n \leqslant N\} \tag{7}$$

where the function $f_n(c_1, \cdots, c_{n-1}, c_{n+1}, \cdots, c_N)$ can be obtained by solving the following optimization problem:

$$\left.\begin{aligned} & f_n(c_1, \cdots, c_{n-1}, c_{n+1}, \cdots, c_N) = \max_{\boldsymbol{c} \in \Sigma} c_n \\ & \text{s.t. } c_i(1 \leqslant i \leqslant N,\ i \neq n) \text{ is fixed} \end{aligned}\right\} \tag{8}$$

It is interesting to compare (4) and (7) which interpret the convexity from two different points of view. Much existing work on determining capacity region is based on (4), while this paper will take (7) as the starting point. In the remainder of this paper, we will obtain the analytical expression of function $f_n(c_1, \cdots, c_{n-1}, c_{n+1}, \cdots, c_N)$ for HDR.

By Eqs. (1) – (3),

$$c_n = \sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) \tau_n(\boldsymbol{s}) R_{s_n} = \sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) \left(1 - \sum_{1 \leqslant i \leqslant N,\ i \neq n} \tau_i(\boldsymbol{s})\right) R_{s_n} = c_n^{\max} - \sum_{1 \leqslant i \leqslant N,\ i \neq n} \left(\sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) R_{s_n}\right) \tag{9}$$

Hence, the optimization problem (8) becomes

$$\left.\begin{aligned} & \max_{\{\tau_i(\boldsymbol{s}),\ 1 \leqslant i \leqslant N,\ \boldsymbol{s} \in \Xi\}} c_n^{\max} - \sum_{1 \leqslant i \leqslant N,\ i \neq n} \left(\sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) R_{s_n}\right) \\ & \text{s. t. } \sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) R_{s_i} = c_i,\ 1 \leqslant i \leqslant N,\ i \neq n \end{aligned}\right\} \tag{10}$$

Since the objective function and constraints in (10) are separable in i, to solve (10), we must first obtain the solution to the following optimization problem for $1 \leqslant i \leqslant N\ (i \neq n)$:

$$\left.\begin{aligned} & \min_{\{\tau_i(\boldsymbol{s}),\ \boldsymbol{s} \in \Xi\}} \sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) R_{s_n} \\ & \text{s. t. } \sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) R_{s_i} = c_i \end{aligned}\right\} \tag{11}$$

Define

$$x_{m_1, m_2}^{i, n} \triangleq \sum_{\boldsymbol{s}:\ s_i = m_1,\ s_n = m_2} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) \tag{12}$$

then the objective function and constraints in (12) become

$$\left.\begin{aligned} & \sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) R_{s_n} = \sum_{m=1}^{M} \sum_{m_1=1}^{M} \sum_{\boldsymbol{s}: s_i = m,\ s_n = m_1} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) R_{m_1} = \sum_{m=1}^{M} \sum_{m_1=1}^{M} R_{m_1} x_{m, m_1}^{i, n} \\ & \sum_{\boldsymbol{s} \in \Xi} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) R_{s_i} = \sum_{m=1}^{M} \sum_{m_1=1}^{M} \sum_{\boldsymbol{s}: s_i = m,\ s_n = m_1} p(\boldsymbol{s}) \tau_i(\boldsymbol{s}) R_m = \sum_{m=1}^{M} \sum_{m_1=1}^{M} R_m x_{m, m_1}^{i, n} = c_i \end{aligned}\right\} \tag{13}$$

Additionally, since $0 \leqslant \tau_i(\boldsymbol{s}) \leqslant 1$, we know that

$$0 \leqslant x_{m, m_1}^{i, n} \leqslant \sum_{\boldsymbol{s}:\ s_i = m,\ s_n = m_1} p(\boldsymbol{s}) \triangleq h_{m, m_1}^{i, n} \tag{14}$$

Thus, the optimization problem (11) reduces to the following one:

$$\left.\begin{aligned} & e_{i, n}(c_i) \triangleq \min_{\{x_{m, m_1}^{i, n},\ 1 \leqslant m,\ m_1 \leqslant M\}} \sum_{m=1}^{M} \sum_{m_1=1}^{M} R_{m_1} x_{m, m_1}^{i, n} \\ & \text{s. t. } \sum_{m=1}^{M} \sum_{m_1=1}^{M} R_m x_{m, m_1}^{i, n} = c_i,\ 0 \leqslant x_{m, m_1}^{i, n} \leqslant h_{m, m_1}^{i, n} \end{aligned}\right\} \tag{15}$$

Given the solution to (15), then by (8) – (10), (11) and (15), the solution to (8) can be given by

$$f_n(c_1, \cdots, c_{n-1}, c_{n+1}, \cdots, c_N) = c_n^{\max} - \sum_{1 \leqslant i \leqslant N,\ i \neq n} e_{i, n}(c_i) \tag{16}$$

3.3 Solution to optimization problem (15)

Define set $E = \{R_{m_1}/R_m,\ 1 \leqslant m,\ m_1 \leqslant M\}$ and vector $\{\{\alpha_1, \beta_1\}, \{\alpha_2, \beta_2\}, \cdots, \{\alpha_{M^2}, \beta_{M^2}\}\}$ is a permutation on set E such that

$$\frac{R_{\beta_1}}{R_{\alpha_1}} \leqslant \frac{R_{\beta_2}}{R_{\alpha_2}} \leqslant \cdots \leqslant \frac{R_{\beta_{M2}}}{R_{\alpha_{M2}}} \tag{17}$$

Then the solution to (15) can be obtained from the following greedy algorithm[3]:

① Initialization: set $x_{m, m_1}^{i, n} = 0$ for all m and m_1. Set $k = 1$.

② Step k: increase the value of $x_{\alpha_k, \beta_k}^{i, n}$ until a constraint becomes tight. Goto step $k + 1$.

③ After M^2 steps, the optimal solution is reached.

The results obtained by performing the above greedy algorithm are summarized as follows.

Since c_i appears in the constraint in the (15), the solution $e_{i, n}(c_i)$ depends on c_i. Since $R_m = R_n$ for $m = n$, (17) can be further written as

$$\frac{R_{\beta_1}}{R_{\alpha_1}} \leqslant \cdots \leqslant \frac{R_{\beta_{M(M-1)/2}}}{R_{\alpha_{M(M-1)/2}}} < \overbrace{1 = \cdots = 1}^{M} < \frac{R_{\beta_{M(M+1)/2+1}}}{R_{\alpha_{M(M+1)/2+1}}} \leqslant \cdots \leqslant \frac{R_{\beta_{M2}}}{R_{\alpha_{M2}}} \tag{18}$$

Correspondingly, we can divide the range of c_i into $M^2 - M + 1$ disjoint intervals $\{I_j^{i, n}\}$ such that

$$(0, c_i^{\max}] = \bigcup_{j=1}^{M^2 - M + 1} I_j^{i, n} \tag{19}$$

where

$$I_j^{i, n} = \begin{cases} \left(\sum_{k=1}^{j-1} R_{\alpha_k} h_{\alpha_k, \beta_k}^{i, n}, \ \sum_{k=1}^{j} R_{\alpha_k} h_{\alpha_k, \beta_k}^{i, n} \right] & 1 \leqslant j \leqslant \dfrac{M(M-1)}{2} \\ \left(\sum_{k=1}^{M(M-1)/2} R_{\alpha_k} h_{\alpha_k, \beta_k}^{i, n}, \ \sum_{k=1}^{M(M+1)/2} R_{\alpha_k} h_{\alpha_k, \beta_k}^{i, n} \right] & j = \dfrac{M(M-1)}{2} + 1 \\ \left(\sum_{k=1}^{j+M-2} R_{\alpha_k} h_{\alpha_k, \beta_k}^{i, n}, \ \sum_{k=1}^{j+M-1} R_{\alpha_k} h_{\alpha_k, \beta_k}^{i, n} \right] & \dfrac{M(M-1)}{2} + 2 \leqslant j \leqslant M^2 - M + 1 \end{cases} \tag{20}$$

All these $M^2 - M + 1$ intervals can be further classified into three types as follows.

Type 1 $\{I_j^{i, n}, 1 \leqslant j \leqslant M(M-1)/2\}$. If c_i belongs to $I_j^{i, n}$ of this type, then

$$(x_{\alpha_k, \beta_k}^{i, n})^* = \begin{cases} h_{\alpha_k, \beta_k}^{i, n} & k < j \\ 0 & k > j \end{cases}, \quad (x_{\alpha_j, \beta_j}^{i, n})^* = \frac{c_i - \sum_{k=1}^{j-1} R_{\alpha_k} h_{\alpha_k, \beta_k}^{i, n}}{R_{\alpha_j}} \tag{21}$$

The corresponding solution to (15) is given by

$$e_{i, n}(c_i) = \frac{R_{\beta_j}}{R_{\alpha_j}} \left(c_i - \sum_{k=1}^{j-1} R_{\alpha_k} h_{\alpha_k, \beta_k}^{i, n} \right) + \sum_{k=1}^{j-1} R_{\beta_k} h_{\alpha_k, \beta_k}^{i, n} \tag{22}$$

Type 2 $\{I_j^{i, n}, j = M(M-1)/2 + 1\}$. If c_i belongs to $I_j^{i, n}$ of this type, then

$$(x_{\alpha_k, \beta_k}^{i, n})^* = \begin{cases} h_{\alpha_k, \beta_k}^{i, n} & k \leqslant \dfrac{M(M-1)}{2} \\ 0 & k \geqslant \dfrac{M(M+1)}{2} + 1 \end{cases} \tag{23}$$

The corresponding solution to (15) is given by

$$e_{i, n}(c_i) = c_i + \sum_{k=1}^{M(M-1)/2} (R_{\beta_k} - R_{\alpha_k}) h_{\alpha_k, \beta_k}^{i, n} \tag{24}$$

Type 3 $\{I_j^{i, n}, M(M-1)/2 + 2 \leqslant j \leqslant M^2 - M + 1\}$. If c_i belongs to $I_j^{i, n}$ of this type, then

$$(x_{\alpha_k, \beta_k}^{i, n})^* = \begin{cases} h_{\alpha_k, \beta_k}^{i, n} & k < j + M - 1 \\ 0 & k > j + M - 1 \end{cases}, \quad (x_{\alpha_{j+M-1}, \beta_{j+M-1}}^{i, n})^* = \frac{c_i - \sum_{k=1}^{j+M-2} R_{\alpha_k} h_{\alpha_k, \beta_k}^{i, n}}{R_{\alpha_{j+M-1}}} \tag{25}$$

The corresponding solution to (15) is given by

$$e_{i, n}(c_i) = \frac{R_{\beta_{j+M-1}}}{R_{\alpha_{j+M-1}}} \left(c_i - \sum_{k=1}^{j+M-2} R_{\alpha_k} h_{\alpha_k, \beta_k}^{i, n} \right) + \sum_{k=1}^{j+M-2} R_{\beta_k} h_{\alpha_k, \beta_k}^{i, n} \tag{26}$$

Based on the above results (22), (24) and (26), we can conclude that the solution to the optimization problem (15) consists of $M^2 - M + 1$ line segments:

$$e_{i, n}(c_i) = a_j^{i, n} c_i + b_j^{i, n} \qquad c_i \in I_j^{i, n}, \ 1 \leqslant j \leqslant M^2 - M + 1 \tag{27}$$

where the coefficient pair $(a_j^{i, n}, b_j^{i, n})$ is given by

$$(a_j^{i,n},\ b_j^{i,n}) \triangleq \begin{cases} \left(\dfrac{R_{\beta_j}}{R_{\alpha_j}},\ \displaystyle\sum_{k=1}^{j-1}\left(R_{\beta_k} - \dfrac{R_{\beta_j}}{R_{\alpha_j}}R_{\alpha_k}\right)h_{\alpha_k,\beta_k}^{i,n}\right) & 1 \leqslant j \leqslant \dfrac{M(M-1)}{2} \\ \left(1,\ \displaystyle\sum_{k=1}^{M(M-1)/2}(R_{\beta_k} - R_{\alpha_k})h_{\alpha_k,\beta_k}^{i,n}\right) & j = \dfrac{M(M-1)}{2} + 1 \\ \left(\dfrac{R_{\beta_{j+M-1}}}{R_{\alpha_{j+M-1}}},\ \displaystyle\sum_{k=1}^{j+M-2}\left(R_{\beta_k} - \dfrac{R_{\beta_{j+M-1}}}{R_{\alpha_{j+M-1}}}R_{\beta_k}\right)h_{\alpha_k,\beta_k}^{i,n}\right) & \dfrac{M(M-1)}{2} + 2 \leqslant j \leqslant M^2 - M + 1 \end{cases} \tag{28}$$

3.4 Final result

Substituting (27) into (16), the solution to (8) is composed of $(M^2 - M + 1)^{N-1}$ line parts:

$$f_n(c_1, \cdots, c_{n-1}, c_{n+1}, \cdots, c_N) = c_n^{\max} - \sum_{1 \leqslant i \leqslant N,\ i \neq n}(a_{j_i}^{i,n}c_i + b_{j_i}^{i,n}) \qquad c_i \in I_{j_i}^{i,n} \tag{29}$$

where $1 \leqslant j_i \leqslant M^2 - M + 1$. Then by (7), the capacity region is given by

$$\Sigma = \left\{\boldsymbol{c}:\ c_n \leqslant c_n^{\max} - \sum_{1 \leqslant i \leqslant N,\ i \neq n}(a_{j_i}^{i,n}c_i + b_{j_i}^{i,n}),\ 1 \leqslant j_i \leqslant M^2 - M + 1,\ 1 \leqslant n \leqslant N\right\} \tag{30}$$

To give a brief expression of Σ, let $\boldsymbol{j}^n = \{j_1, \cdots, j_{n-1}, 0, j_{n+1}, \cdots, j_N\}$ where $1 \leqslant j_i \leqslant M^2 - M + 1$ for $i \neq n$ and denote J^n as the set of all possible $\boldsymbol{j}^n$. Further, define

$$\left.\begin{aligned} \boldsymbol{a}^n(\boldsymbol{j}^n) &\triangleq \{a_{j_1}^{1,n}, \cdots, a_{j_{n-1}}^{n-1,n}, 1, a_{j_{n+1}}^{n+1,n}, \cdots, a_{j_N}^{N,n}\} \\ b^n(\boldsymbol{j}^n) &\triangleq c_n^{\max} - \sum_{1 \leqslant i \leqslant N,\ i \neq n} b_{j_i}^{i,n} \end{aligned}\right\} \tag{31}$$

Hence, the capacity region of HDR can be finally given by

$$\Sigma = \{\boldsymbol{c}:\ \boldsymbol{a}^n(\boldsymbol{j}^n) \cdot \boldsymbol{c} \leqslant b^n(\boldsymbol{j}^n),\ \boldsymbol{j}^n \in J^n,\ 1 \leqslant n \leqslant N\} \tag{32}$$

By (31), there are totally $N(M^2 - M + 1)^{N-1}$ possible $(\boldsymbol{a}^n(\boldsymbol{j}^n),\ b^n(\boldsymbol{j}^n))$ pairs. Therefore, we can conclude that the capacity region of HDR is the set of rate vectors which satisfy $N(M^2 - M + 1)^{N-1}$ linear constraints at the same time. Compared with the conventional method (4)-(5), since the new method (7) does not introduce any additional parameter variable, the new characterization (32) is more convenient than (4)-(5) to solve problems like CAC.

4 Numerical Examples

In this section, we present numerical results for the capacity region of HDR under various parameter settings. The capacity region obtained analytically in (32) leads to $N(M^2 - M + 1)^{N-1}$ linear equations that can be computed easily to obtain the numerical results.

In Fig. 1, we plot the capacity region of HDR with the user number $N = 2$. The rate set is {10, 20, ⋯, 90} (kbit/s) and the rate set size M is 9. The pdf of channel state process of user i is $p_i(m) = 1/M$ $(1 \leqslant i \leqslant N,\ 1 \leqslant m \leqslant M)$. Under the parameter method (4)-(5), the HDR capacity region is characterized by parameter $\boldsymbol{\mu} = \{\mu_1, \mu_2\}$ where $\mu_1 + \mu_2 = 1$. As μ_1(or μ_2) varies from 0 to 1, we get all points on the boundary of the region. On the other hand, under the non-parameter method (32), the capacity region is characterized by $N(M^2 - M + 1)^{N-1} = 146$ linear constraints. In Fig. 1, we see that these two methods give the same result, which has been expected since they characterized the same region. Additionally, although the non-parameter method (32) specifies 146 linear constraints, only 55 of them are linear independent. In Fig. 2, we plot the capacity region of HDR as M varies from 1 to 9. The rate set is $\{R_0, 2R_0, \cdots, MR_0\}$ (kbit/s). The pdf of the channel state process of user i is $p_i(m) = 1/M$ $(1 \leqslant i \leqslant N,\ 1 \leqslant m \leqslant M)$ such that both $c_1^{\max}$ and $c_2^{\max}$

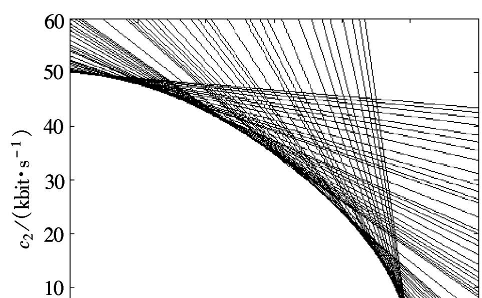

Fig.1 HDR capacity region ($N = 2$, $M = 9$)

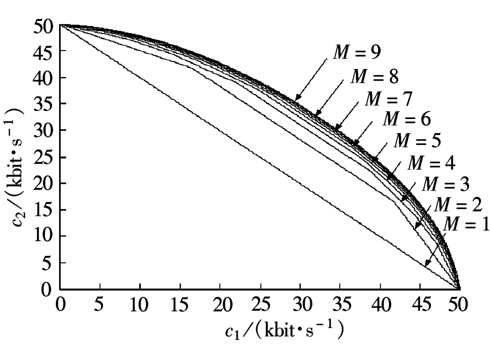

Fig.2 HDR capacity region ($N = 2$, $M = 1$, 2, ⋯, 9)

equal $[(M+1)/2]R_0$. Let $R_0 = 100/(M+1)$, then $c_1^{\max}$ and $c_2^{\max}$ are fixed to be 50 kbit/s under different M. In Fig.2, we can also see that the results given by the two methods are the same.

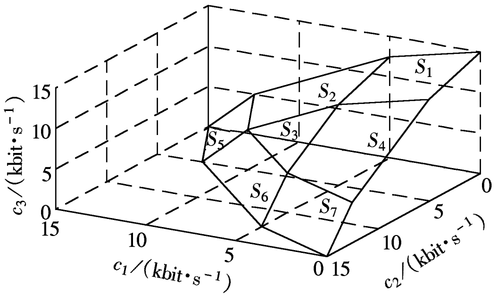

Fig.3 HDR capacity region ($N=3$, $M=2$)

In Fig.3, we plot the three-dimensional capacity region of HDR with the user number $N=3$. The rate set is $\{10, 20\}$ (kbit/s) and the rate set size $M=2$. The pdf of the channel state process of user i is $p_i(m) = 1/M$ ($1 \leqslant i \leqslant N$, $1 \leqslant m \leqslant M$). Under the non-parameter method (32), the capacity region can be characterized by $N(M^2 - M + 1)^{N-1} = 27$ linear constraints. Checking these constraints, we find that only seven of them are linear independent as follows: S_1: $c_1 + c_2 + 2c_3 \leqslant 30$, S_2: $2c_1 + c_2 + 2c_3 \leqslant 35$, S_3: $c_1 + c_2 + c_3 \leqslant 20$, S_4: $c_1 + 2c_2 + 2c_3 \leqslant 35$, S_5: $2c_1 + c_2 + c_3 \leqslant 30$, S_6: $2c_1 + 2c_2 + c_3 \leqslant 35$, and S_7: $c_1 + 2c_2 + c_3 \leqslant 30$. On the other hand, since the conventional method characterizes the HDR capacity region by parameter $\boldsymbol{\mu} = \{\mu_1, \mu_2, \mu_3\}$ where $\mu_1 + \mu_2 + \mu_3 = 1$, it is not easy for the parameter method to plot the same figure. Further, since above seven equations can be directly used to determine if a given rate vector $\boldsymbol{c} \in \Sigma$, the new method is more convenient than the conventional one to solve problems like CAC.

5 Conclusion

This paper studies the HDR capacity region. Starting from a new point of view which is different from existing work, we show the HDR capacity region can be characterized by at most $N(M^2 - M + 1)^{N-1}$ linear constraints. The new method is more useful than the conventional one to solve problems like CAC.

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一种新的 HDR 无线网络吞吐率容量计算方法

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摘要: 基于一种新的非参数方法计算了 HDR (high data rate)无线网络的吞吐率容量区域. 首先把 HDR 吞吐率容量区域的计算问题转化为一个不带参数的数学优化问题, 然后通过贪婪算法求解得到该优化问题的解, 从而最终给出了 HDR 吞吐率容量区域的非参数解析刻划. 与传统的参数化方法的庞大计算量相比, 该方法的计算复杂度降低很多, 最多只需要计算 $N(M^2 - M + 1)^{N-1}$ 个线性约束即可, 其中 N 为用户数, M 为系统支持速率数. 数值计算例子验证了这种非参数方法的正确性.

关键词: 容量区域; HDR; 资源分配

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